Homework Set 3

Exercise 1 *Derive the Black-Scholes equation for a general system of stocks* $S(t) \in \mathbf{R}^d$ *of the form*

$$dS_i(t) = a_i(t, S(t))dt + \sum_{j=1}^{d} b_{ij}(t, S(t))dW_j(t), \ i = 1, \ldots, d$$

and the European option with final payoff $f(T, S(T)) = g(S(T))$. *Here* $g : \mathbf{R}^d \to \mathbf{R}$ *is a given function e.g.* $g(s) = \max\left(\frac{\sum_{i=1}^{d} s_i}{d} - K, 0\right)$.
Hint: *Generalize the classroom derivation considering a self financing portfolio with all stocks and the option.*

Exercise 2 *Assume that* $S(t)$ *is the price of a single stock and that the assumptions made in the derivation of the Black-Scholes equation hold.*

(i) *Using a hedging argument, derive a PDE method to determine the price of a contingent claim with with the (path dependent) payoff*

$$\int_0^T h(t, S(t))dt,$$

for a given function h.

Hint: *Introduce an auxiliary variable,* $Y(t)$, *with* $Y(0) = 0$ *and* $dY_t = h(t, S(t))dt$. *Let the price of the option be* $f(t, S(t), Y(t))$ *and use a replicating portfolio argument.*

(ii) *Use the Feyman-Kac representation to the option price above and express it as an expected value. What is the dynamics of the stock in such a representation?*