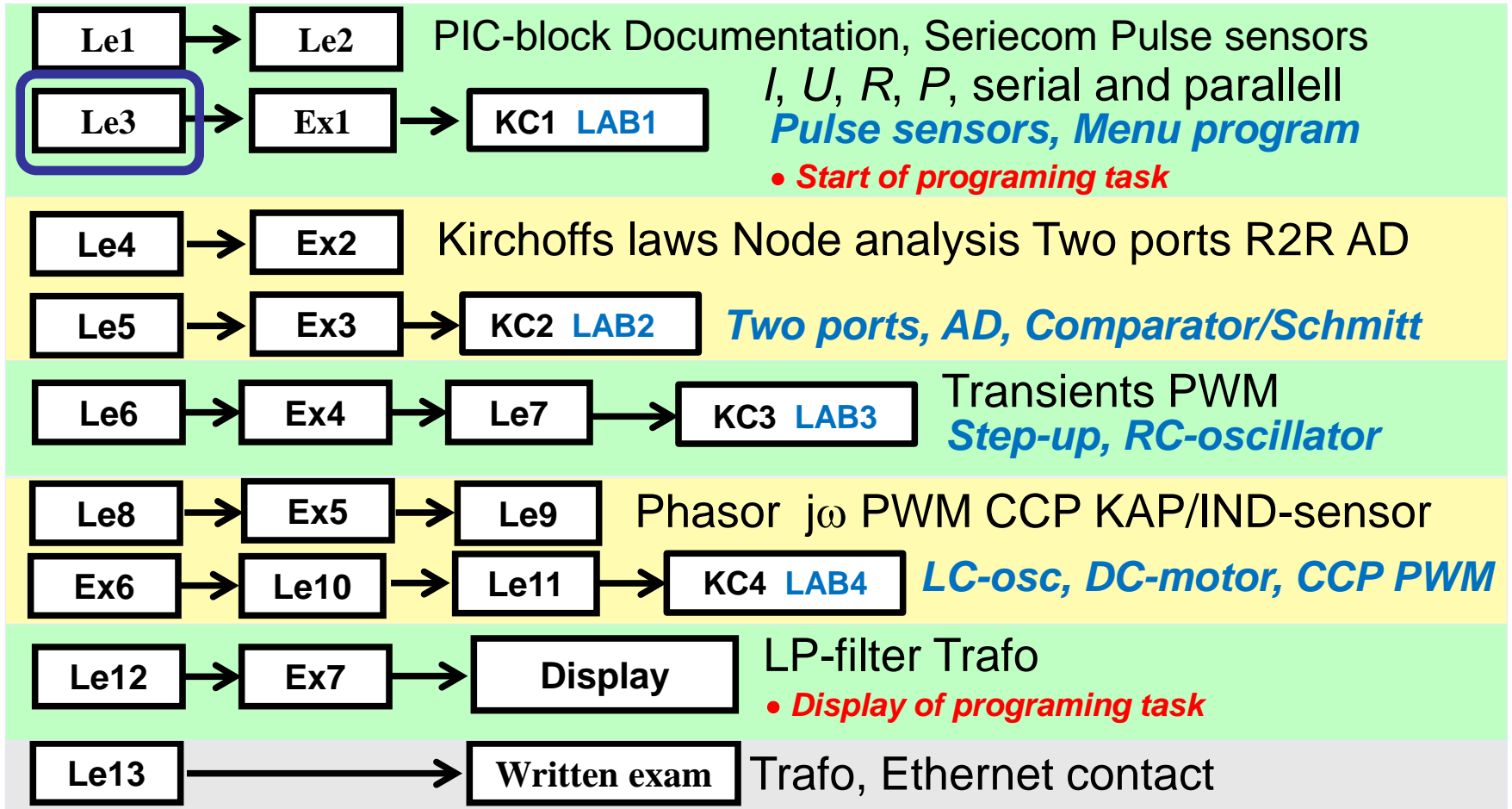


IE1206 Embedded Electronics



Scircuit analysis

resistors

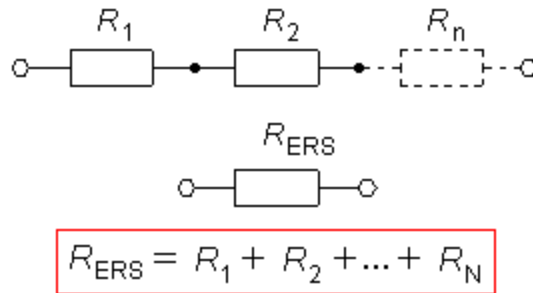
connected in series and parallel

Series-connected resistors – equivalent resistance

Series connected resistors – equivalent resistance

Series connected resistors $R_1 R_2 \dots R_n$ can in calculating be replaced by a **equivalent resistance** R_{ERS} which is the sum of the resistors.

The sum is obviously larger than the largest of the resistors.

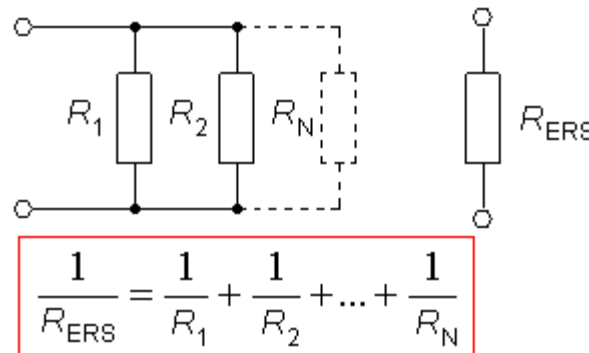


Series-connected components, are characterized in that they are interconnected in **one** point.

Parallel connected resistors – equivalent resistans

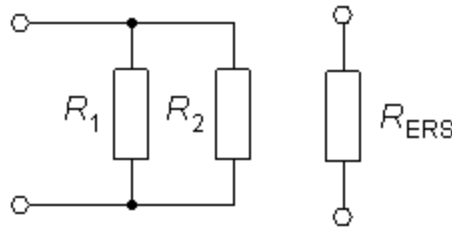
Parallel connected resistors – equivalent resistance

Parallel connected components have *both* connections in common with each other. Parallel resistors $R_1 R_2 \dots R_n$ can in calculating be replaced by a **equivalent resistance** R_{ERS} .



Parallel connected components, are characterized in that they have **both connections** in common with each other.

Two Parallel connected resistors



If one particularly has two parallel resistors R_1 and R_2 the formula can be reformulated as :

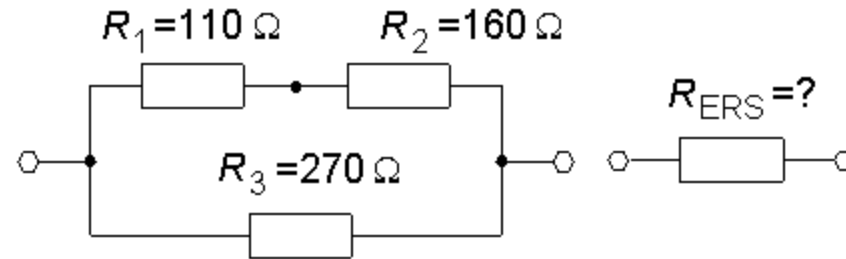
$$\frac{1}{R_{ERS}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2}{R_2} \cdot \frac{1}{R_1} + \frac{R_1}{R_1} \cdot \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 \cdot R_2}$$

$$R_{ERS} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Do you have more parallel resistors than two, repeat this formula for two resistors at a time until you get the equivalent resistance for all.

In parallel connection the equivalent resistance always becomes smaller than the smallest of the constituent parallel connected resistors.

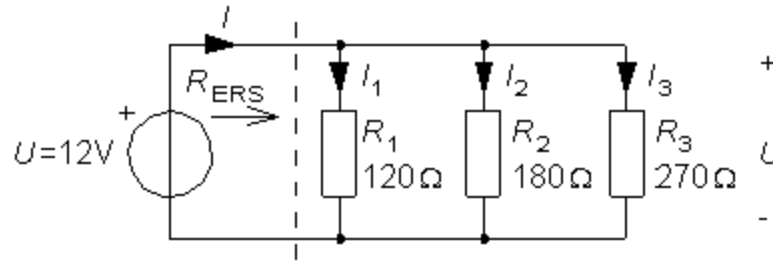
Example – series and parallel connection



$$R_{ERS} = \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3} = \frac{270 \cdot (110 + 160)}{110 + 160 + 270} = 135 \Omega$$

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Parallel circuit

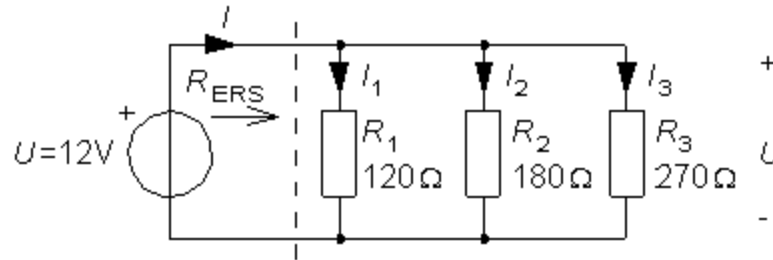


Same U over all resistors!

$$I_1 = \frac{U}{R_1} = \frac{12}{120} = 0,1 \quad I_2 = \frac{U}{R_2} = \frac{12}{180} = 0,067 \quad I_3 = \frac{U}{R_3} = \frac{12}{270} = 0,044$$

$$I = I_1 + I_2 + I_3 = 0,1 + 0,067 + 0,044 = 0,21 \text{ A}$$

Equivalent resistance



From the emf U one only sees the current I , it could likely go to a lonely resistor, an equivalent resistance R_{ERS} . Ohms law gives:

$$R_{ERS} = \frac{U}{I} = \frac{U}{I_1 + I_2 + I_3} = \frac{U}{\frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{120} + \frac{1}{180} + \frac{1}{270}} = 56,8 \Omega$$

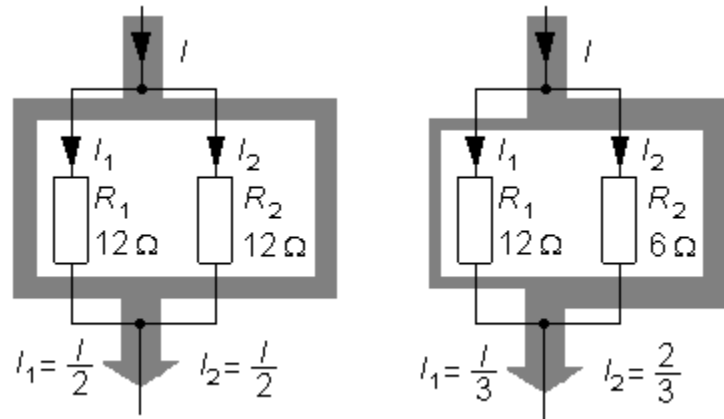
$$I = \frac{U}{R_{ERS}} = \frac{12}{56,8} = 0,21 \text{ A} \qquad \frac{1}{R_{ERS}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The calculated equivalent resistor $R_{ERS} = 56,8 \Omega$ gives the same total current $I = 0,21 \text{ A}$ as calculated earlier.

This is how to derive the expression for the equivalent resistance.

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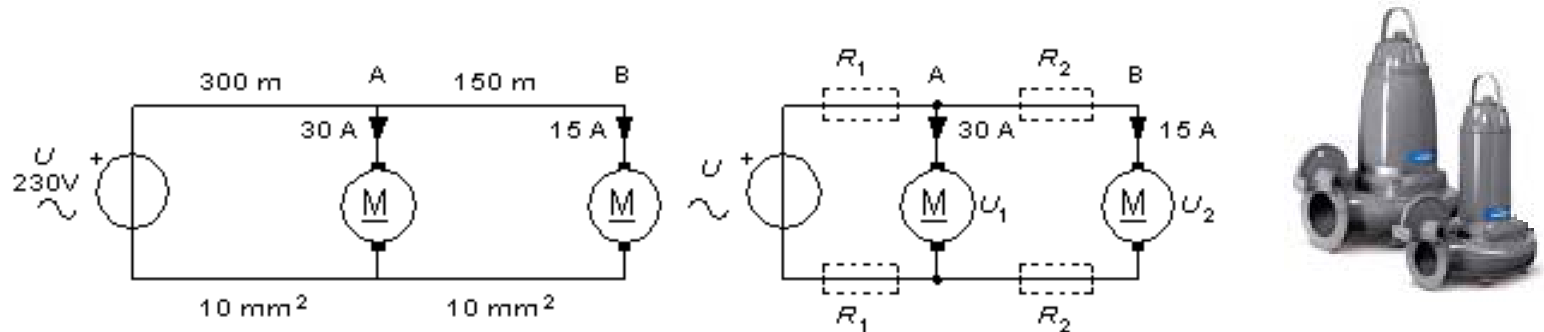
Current branching



The current is divided between parallel branches inversely with the branch resistance (follows the least resistance).

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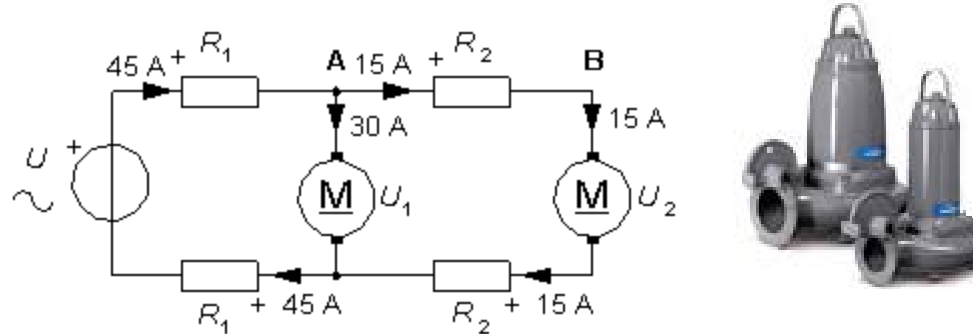
Example – *not* a parallel circuit



Two electric pumps A and B are placed 150 m from each other. A and then B, are powered by 230V from a socket 300 m away. A pump draws the current 30 A and B 15 A. See figure.

On paper it looks as if the motors are connected in parallel, but then you have not counted the resistance found in long lines. To the right of the figure, it is complemented with resistance symbols for the wiring resistances.

Example – *not* a parallel circuit



When the engines work, and thus consumes current, there will be a voltage drop in the cables : $U > U_1 > U_2$

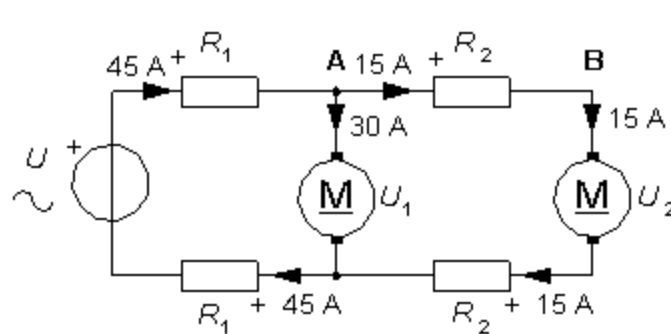
How big will the voltages U_1 and U_2 be when *both* pumps are working?

The wires are of copper with the resistivity $0,018 [\Omega\text{mm}^2/\text{m}]$. $R = \rho \cdot l / A$

$$R_1 = 0,018 \times 300 / 10 = 0,54 \Omega$$

$$R_2 = 0,018 \times 150 / 10 = 0,27 \Omega$$

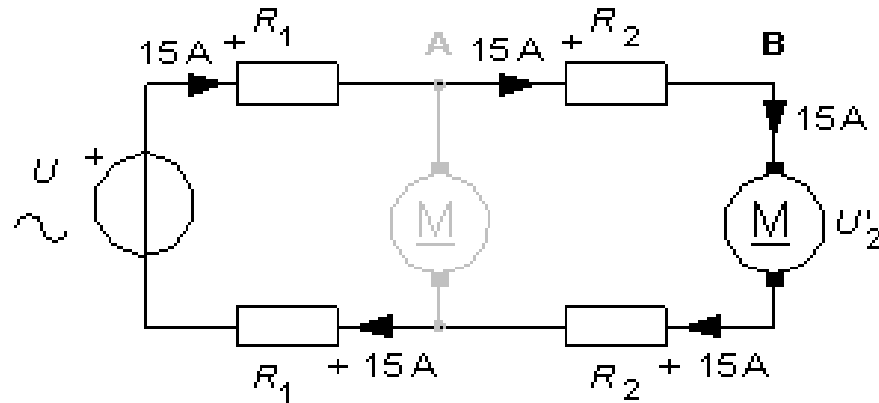
Example – *not* a parallel circuit



$$U_1 = U - 2 \times R_1 \times 45 = 230 - 2 \times 0,54 \times 45 = 181,4 \text{ V}$$

$$U_2 = U_1 - 2 \times R_2 \times 15 = 181,4 - 2 \times 0,27 \times 15 = 173,3 \text{ V}$$

If pump A is off?



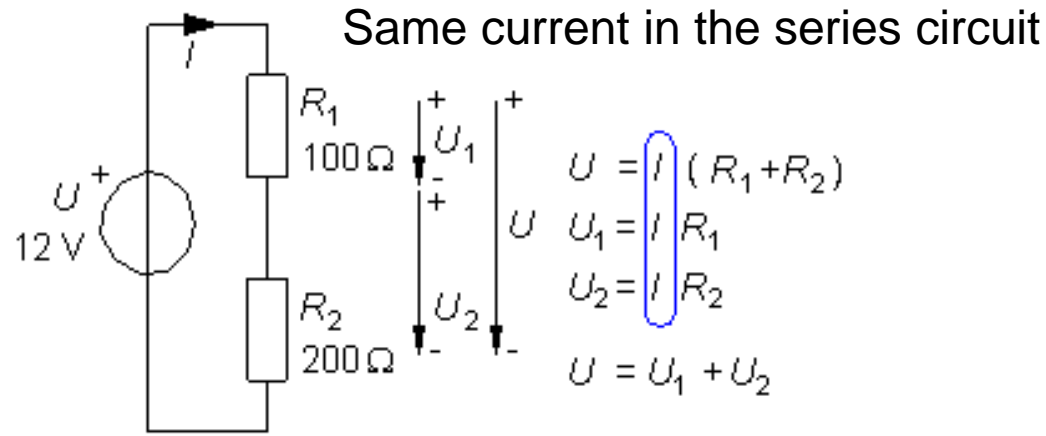
How big will the voltage be at pump B, U'_2 , when pump A is off?

$$U'_2 = U - 2 \times 15 \times (R_1 + R_2) = 230 - 2 \times 15 \times (0,54 + 0,27) = 205,7 \text{ V}$$

$$U'_2 = 205,7 \text{ V} \quad (U_2 = 173,3 \text{ V}) \quad - \text{ this change will be noticed!}$$

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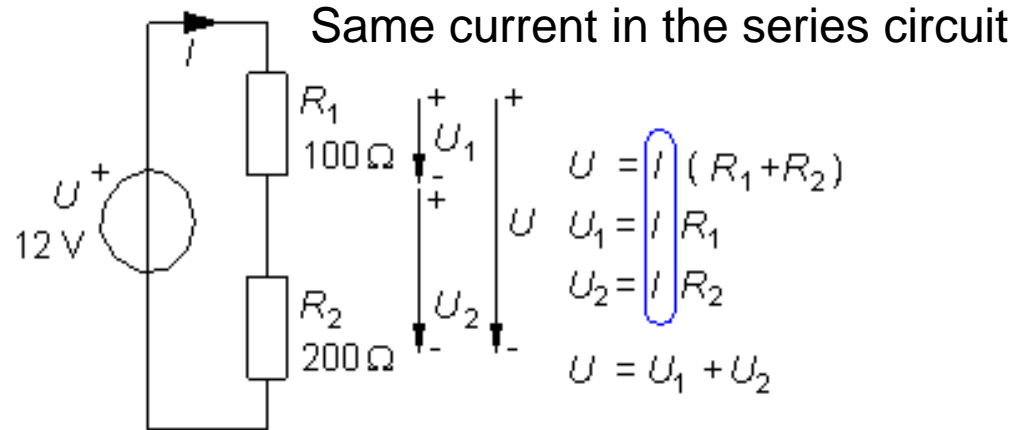
Series circuit



Same I through all resistors.

Series circuit is being characterized that it's the same current that goes through all the resistors. One example is the Christmas tree lights. If a bulb is broken so it is of course no current through it, and because it is a series circuit in this case same current in all resistors will mean that no other lamp will light!

Series circuit



How big are voltages U_1 and U_2 ?

$$R_{\text{ERS}} = R_1 + R_2 = 100 + 200 = 300$$

$$I = U/R_{\text{ERS}} = 12/300 = 0,04 \text{ A}$$

$$U_1 = I \times R_1 = 0,04 \times 100 = 4 \text{ V}$$

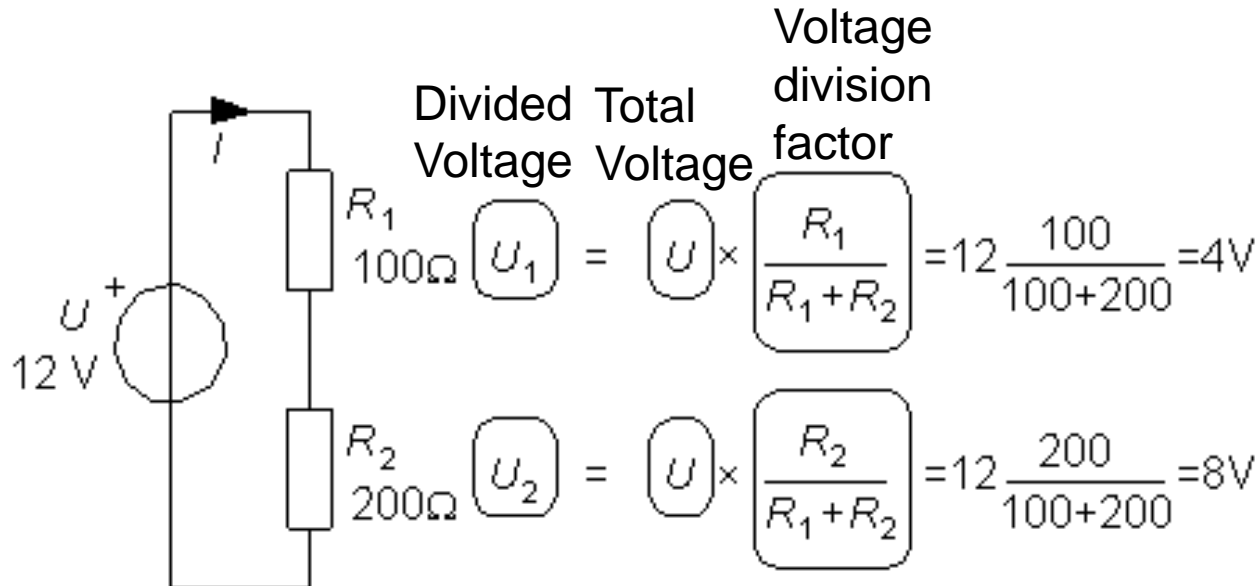
$$U_2 = I \times R_2 = 0,04 \times 200 = 8 \text{ V}$$

$$U = U_1 + U_2 = 4 + 8 = 12 \text{ V}$$

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Voltage division formula

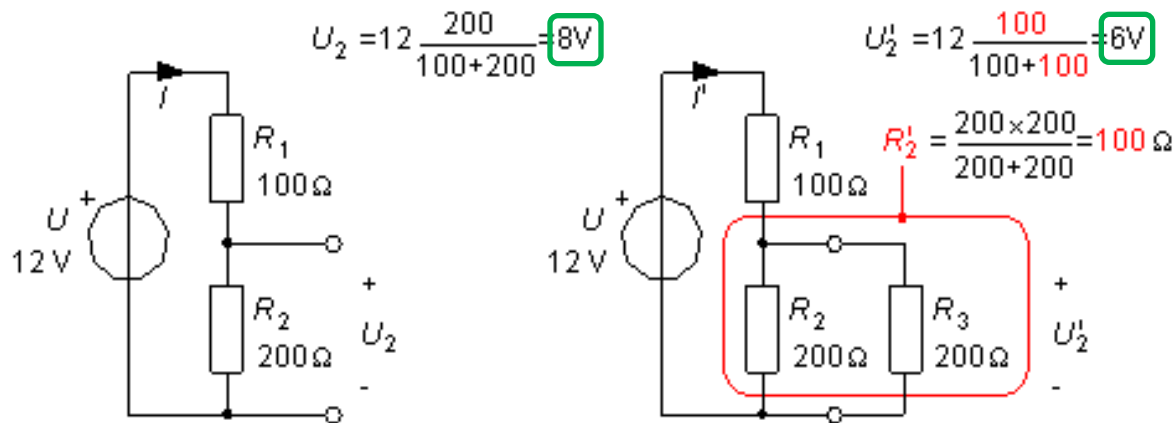
Since all resistors have the same current for series connection, the voltage falls proportional to their resistances. Using Ohm's law (twice), one can develop a formula, the voltage division formula, which can be used to quickly find out the voltage drop across a resistor in series with other resistors.



According to the voltage divider formula you get a divided voltage, for example U_1 across the resistor R_1 , by multiplying the total voltage U with a voltage division factor. This voltage division factor is the resistance R_1 divided by the sum of all the resistors that are in the series connection.

Voltage divider with a load

In cars the battery voltage is 12 V. Suppose you need voltage 8 V to an electronic equipment in a car. One can then lower the voltage with a voltage divider.

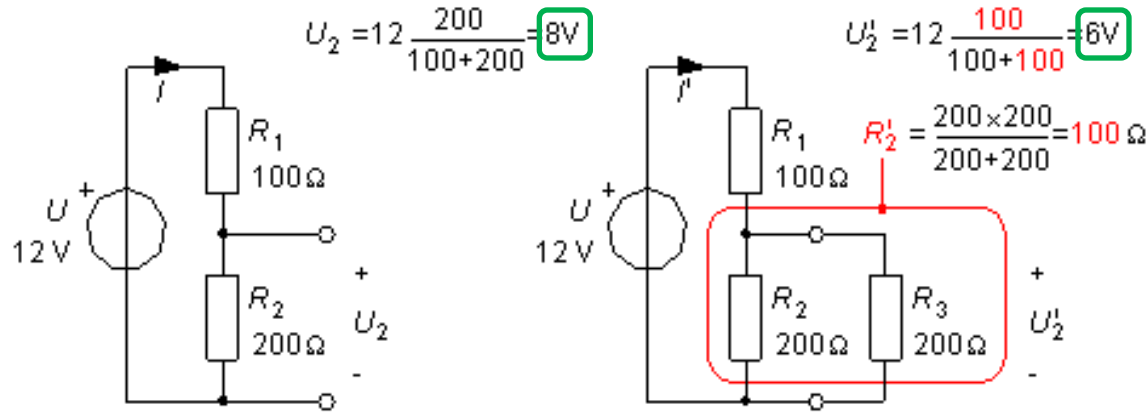


In the figure above to right, the resistor $R_3 = 200\Omega$ symbolizes the electronic equipment.

To use the voltage division formula one now has to see R_2 and R_3 as parallel connected. It is this equivalent resistance R'_2 that is in series with R_1 .

The divided voltage U'_2 for a voltage divider with load is now calculated to 6 V, 2 V lower than for the voltage divider without load.

Voltage divider with a load



For a voltage divider to maintain its divided voltage when it is loaded, it is required that the connected load has a much higher resistance than the resistors included in the voltage divider.

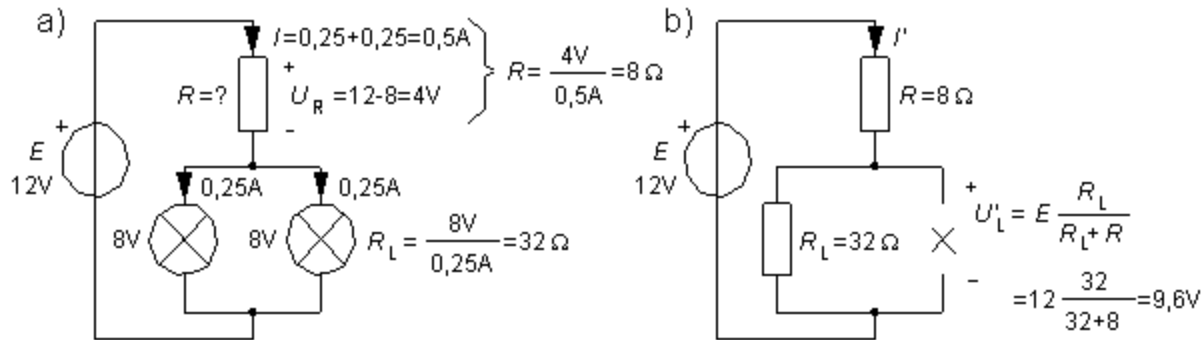
(In example $R_3 = 2000\ \Omega$ would give $U_2 = 7,74$ which is closer to the unloaded value 8,0) .

($R_3 = 20000\ \Omega$ would give $U_2 = 7,97$ which is even closer to 8,0).

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Example – voltage divider for lamps

Two 8 V 0,25 A lamps are used in a car that has a 12 V battery. Lamps are parallel connected and then in series via a resistor to the 12 V battery.



a) Calculate the series resistor R so that the lamp voltage will be correct, 8 V.

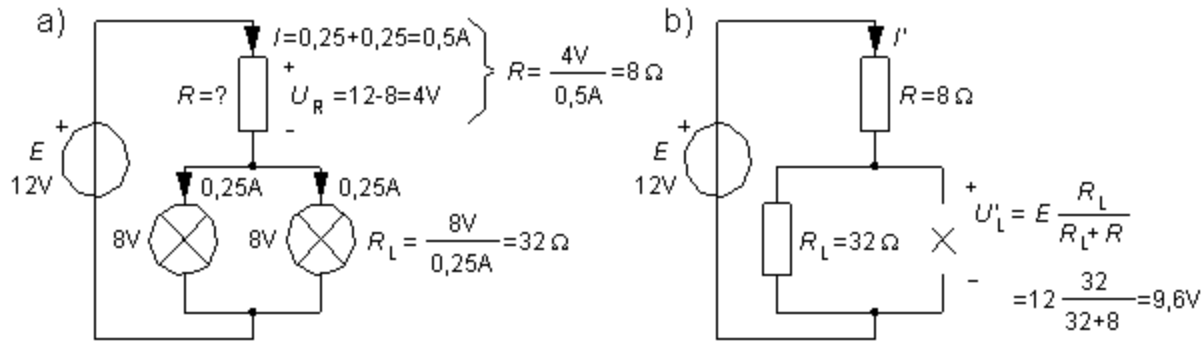
Current through series resistor will be the sum of the currents to the lamps.

$$I = 0,25 + 0,25 = 0,5 \text{ A}$$

Voltage drop over resistor shall be $12 - 8 = 4 \text{ V}$

Ohms law gives: $R = 4/0,5 = 8 \Omega$

Example – voltage divider for lamps



b) Suppose that one of the lamps breaks – how big will then the voltage over the other lamp be?

The lamp resistance is calculated from rated data: $R_L = 8/0,25 = 32 \Omega$

The series resistor and the working lamp forms a voltage divider. The lamp voltage is calculated with the voltage divider formula :

$$U'_L = E \times R_L / (R + R_L) = 12 \times 32 / (32 + 8) = 9,6 V$$

What do you think will happen to the single lamp?

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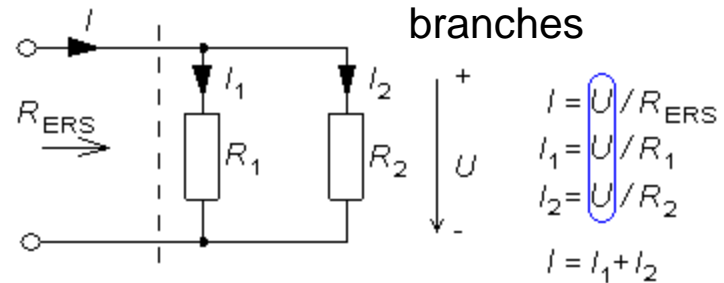
Current branching formula

In the same way as with the voltage division formula, one can derive a current branching formula.

In practice, however, one has less advantage of a current branching formula.

$$I_1 = \frac{I \cdot \frac{R_1 \cdot R_2}{R_1 + R_2}}{R_1} = I \cdot \frac{R_2}{R_1 + R_2}$$

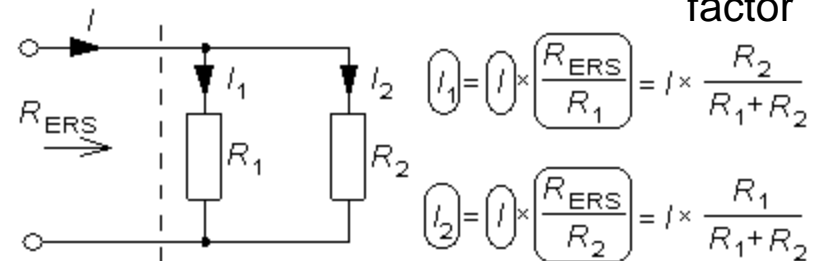
Same voltage over parallel branches



$$U = I \times R_{ERS}$$

Branch Total
current current

Current
Branching
factor

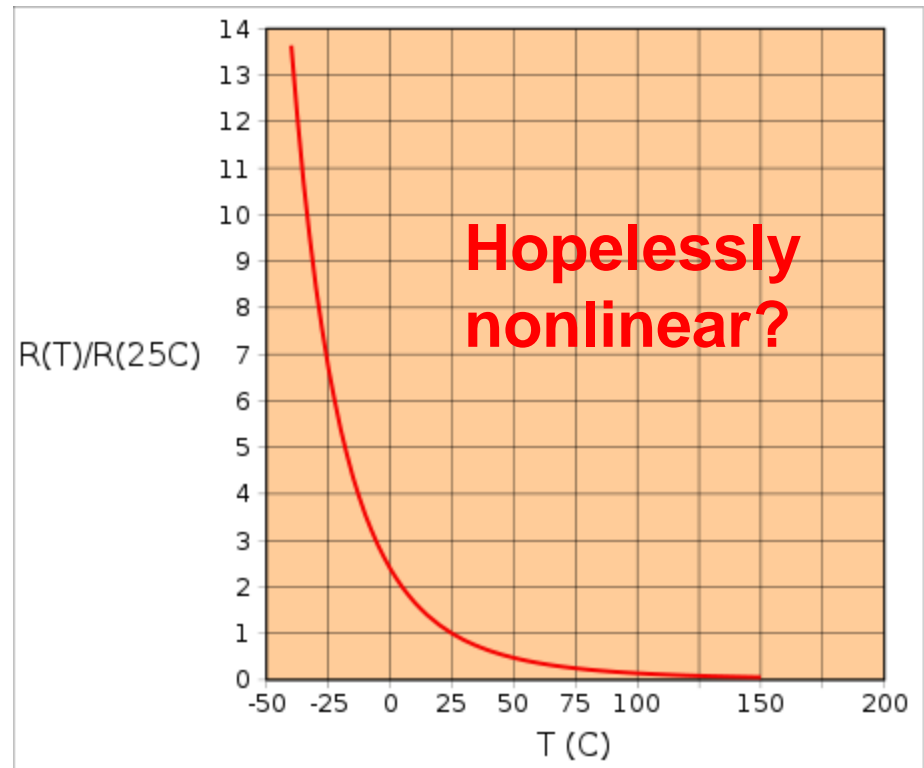


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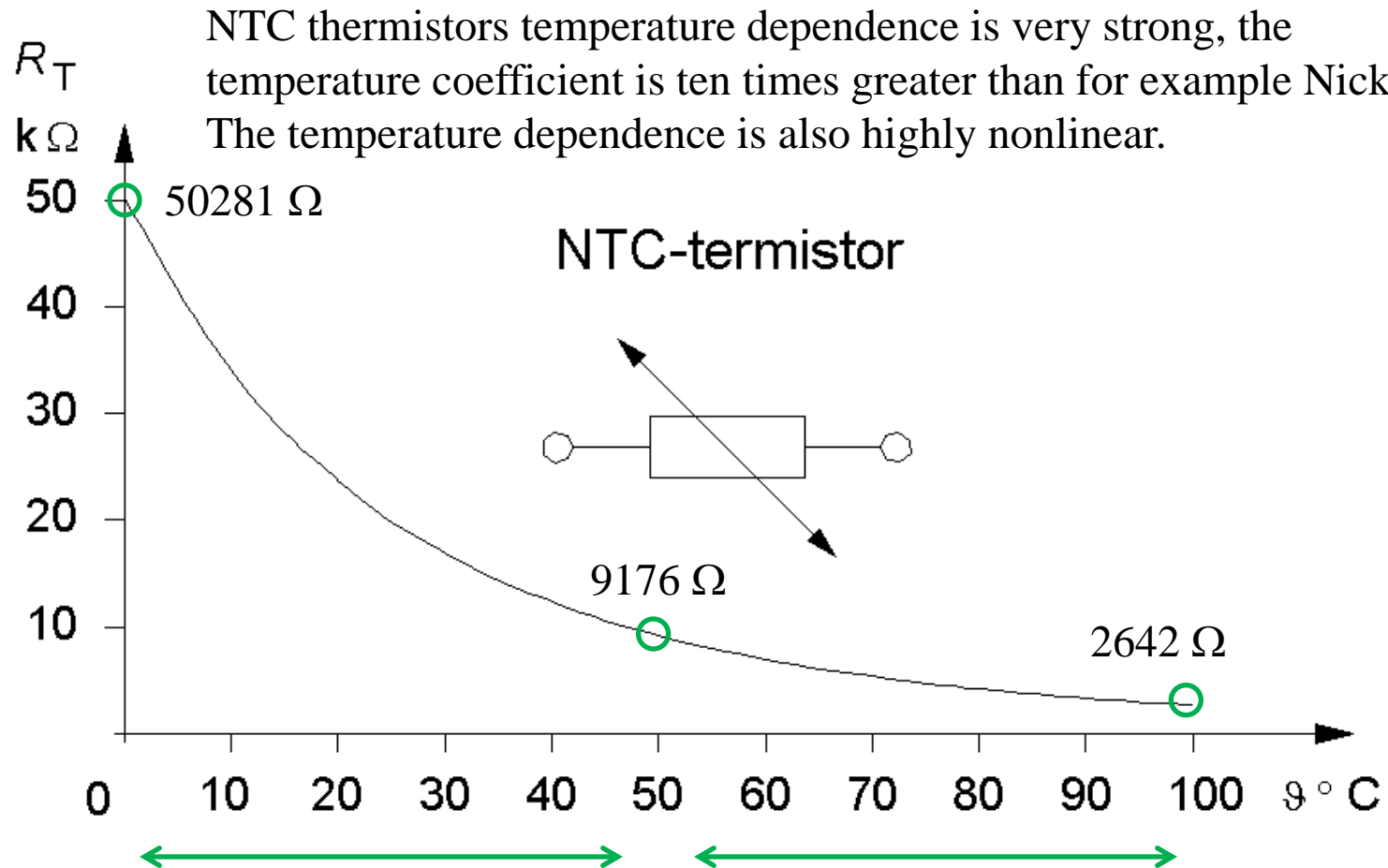
An application of the voltage divider formula

NTC-termistor

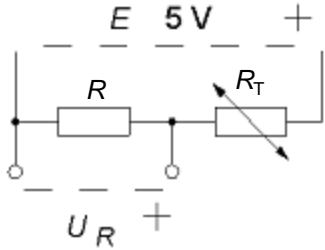
Now you know the voltage division formula - then it's time to show the Linearisation method ...



Linearization of NTC thermistor



Linearization



If a fixed resistor R is connected in series with the NTC thermistor R_T this combination will have a lesser nonlinearity than the thermistor alone. We will let the two resistors form a voltage divider.

If the temperature increases, the thermistor's resistance decrease, and then the portion of the voltage drops across the fixed resistor U_R increases and therefore gives a proportional measure of temperature.

Voltage division formula:
$$U_R = E \frac{R}{R_T + R}$$

$U_R(R_T)$ is a monotonic *decreasing* function of temperature, and $R_T(\vartheta)$ is also monotonic *decreasing*. The combined function $R_T(\vartheta)$ therefore has the potential of being somewhat linear, if you give R a suitable value.

Linearization example

We measure thermistor resistance at three **evenly distributed** temperatures, eg. 0 °C, 50 °C, och 100 °C. $R_{T_0} = 50281 \Omega$, $R_{T_{50}} = 9176 \Omega$, och $R_{T_{100}} = 2642 \Omega$.

If there is linearity then the voltages from the voltage divider U_{R_0} , $U_{R_{50}}$, and $U_{R_{100}}$ also be "evenly distributed". From voltage division we get:

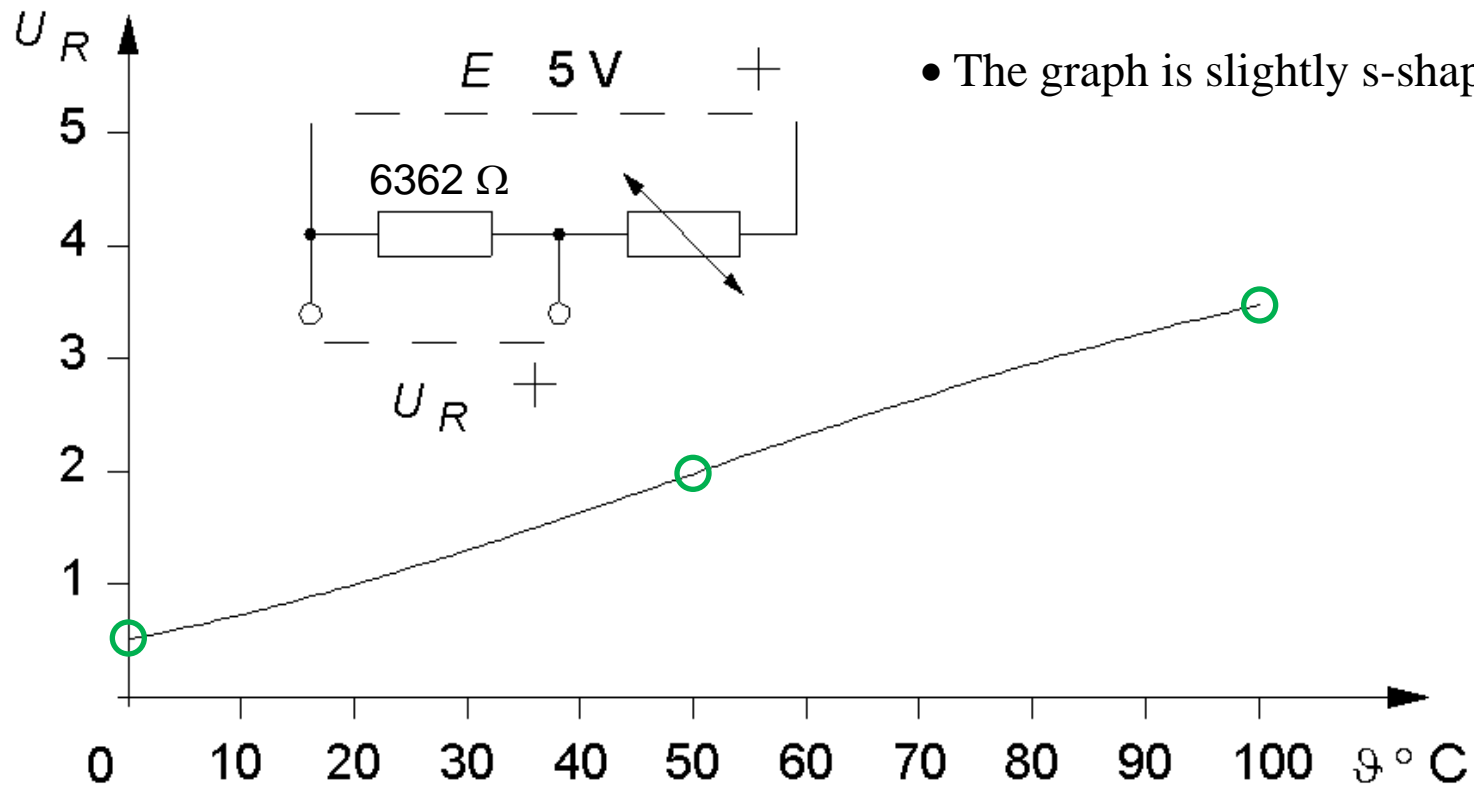
$$\cancel{E} \frac{\cancel{R}}{R_{T_{50}} + R} - \cancel{E} \frac{\cancel{R}}{R_{T_0} + R} = \cancel{E} \frac{\cancel{R}}{R_{T_{100}} + R} - \cancel{E} \frac{\cancel{R}}{R_{T_{50}} + R}$$

If R is solved:

$$R = \frac{R_{T_0} R_{T_{50}} + R_{T_{50}} R_{T_{100}} - 2R_{T_0} R_{T_{100}}}{R_{T_0} + R_{T_{100}} - 2R_{T_{50}}}$$

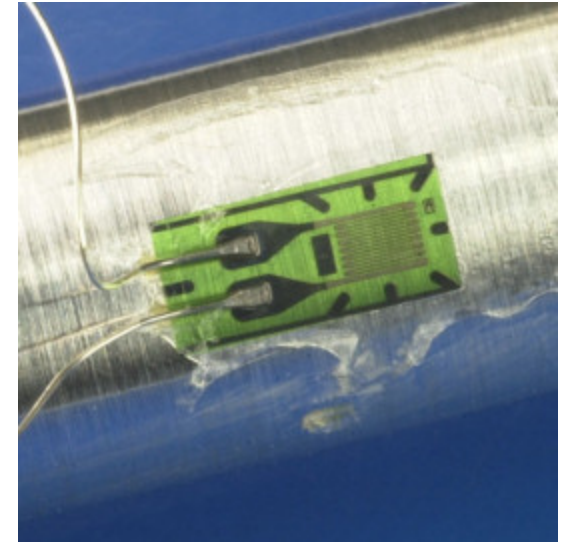
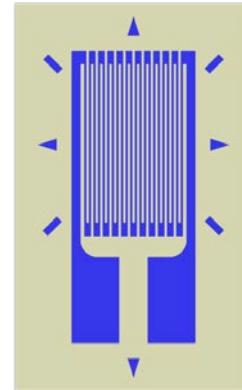
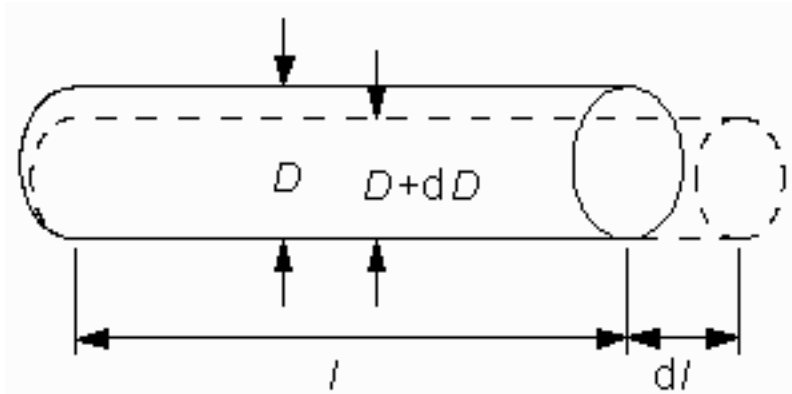
After insertion of our numerical values, we get $R = 6362 \Omega$.

The result - surprisingly linear!

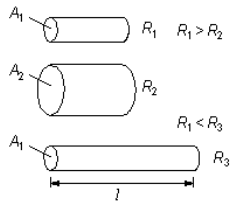


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Example - strain gauge

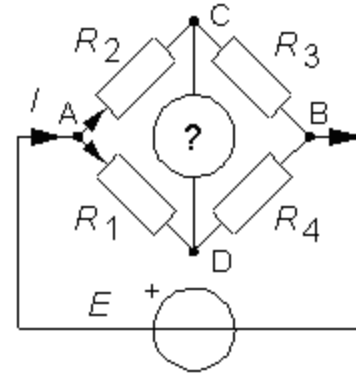
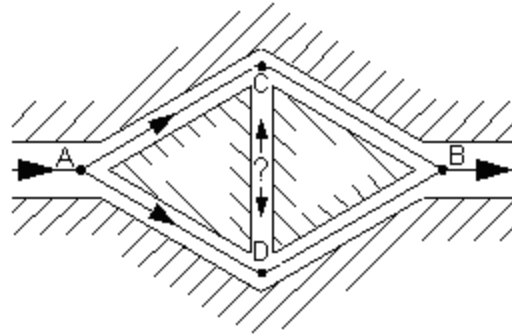


How do you measure such small resistance changes ΔR ?



$$R = \rho \frac{l \cdot 4}{D^2 \cdot \pi} \quad \Delta R \approx k \cdot dl$$

Wheatstone bridge – branched river



Suppose
 $R_4 = R_x$

Balance, no current through the indicator $U_{R4} = U_{R3}$.
Voltage division formula gives us:

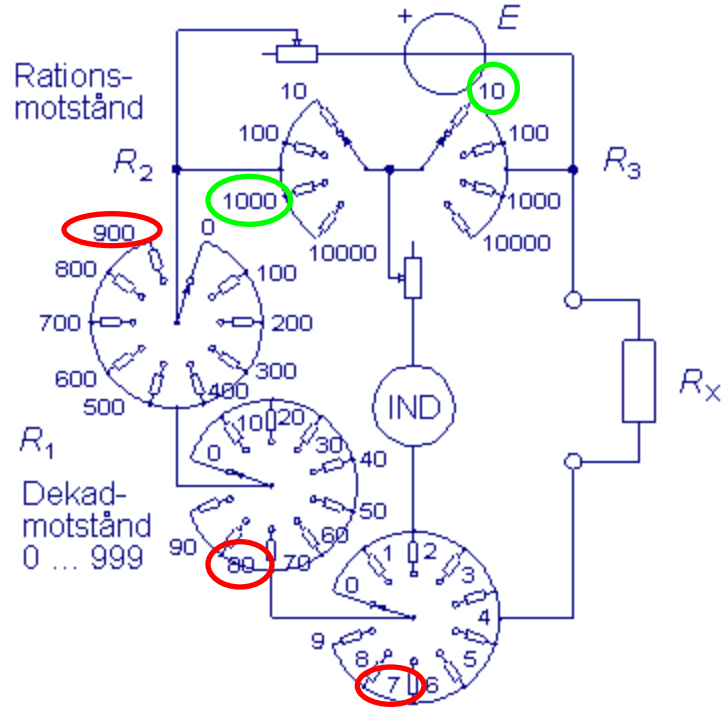
$$\cancel{E} \frac{R_4}{R_1 + R_4} = \cancel{E} \frac{R_3}{R_2 + R_3} \Leftrightarrow \cancel{R_4} R_2 + \cancel{R_4} R_3 = R_3 R_1 + \cancel{R_3} R_4$$

At balance:

$$R_4 = R_1 \frac{R_3}{R_2}$$

$$R_x = ?$$

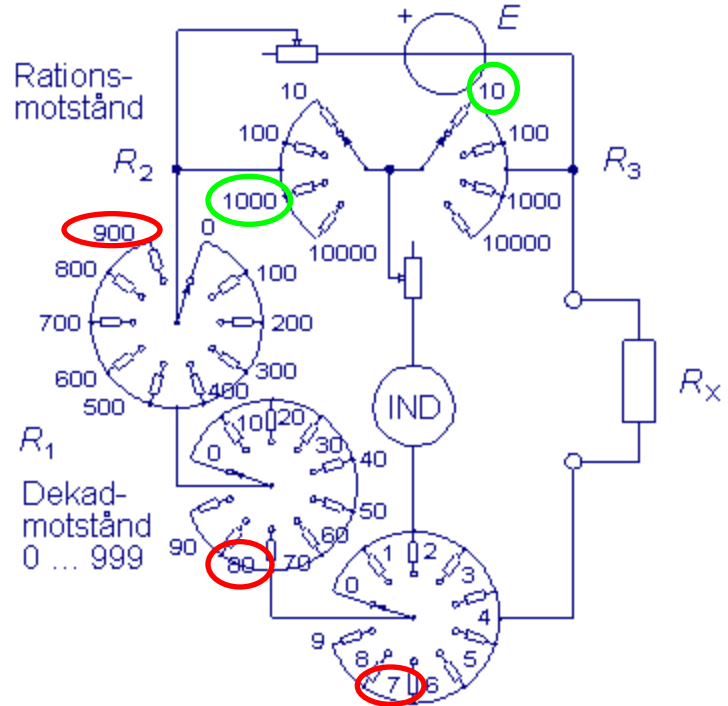
$$R_x = R_1 \frac{R_3}{R_2}$$



$$R_x = ?$$

$$R_x = R_1 \frac{R_3}{R_2}$$

$$R_x = \frac{987 \cdot 10}{1000} = 9,87 \Omega$$



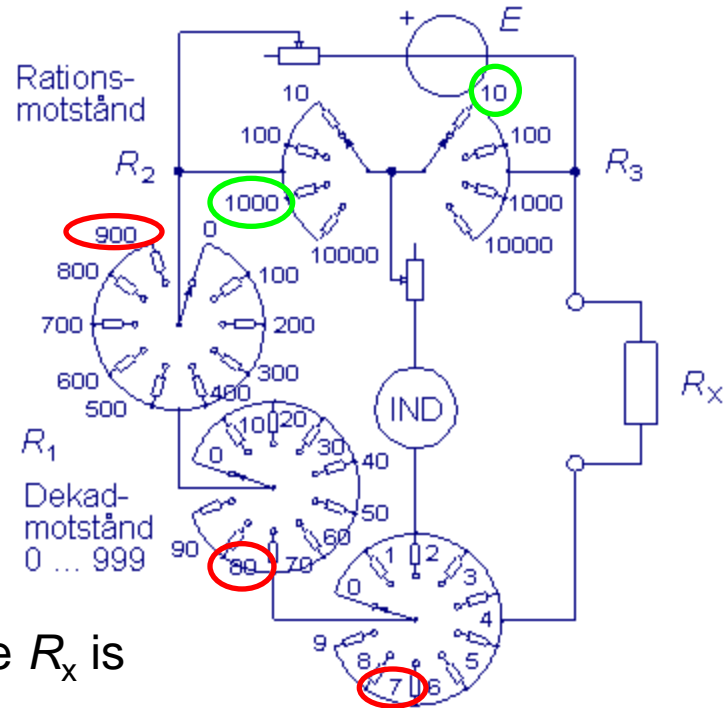
$$R_x = ?$$

$$R_x = R_1 \frac{R_3}{R_2}$$

$$R_x = \frac{987 \cdot 10}{1000} = 9,87 \Omega$$

The balance method to determine R_x is simple to use but slow.

It will work for temperature, but not if one would like to follow a resistive sensor that is used for measuring a faster dynamical process.



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