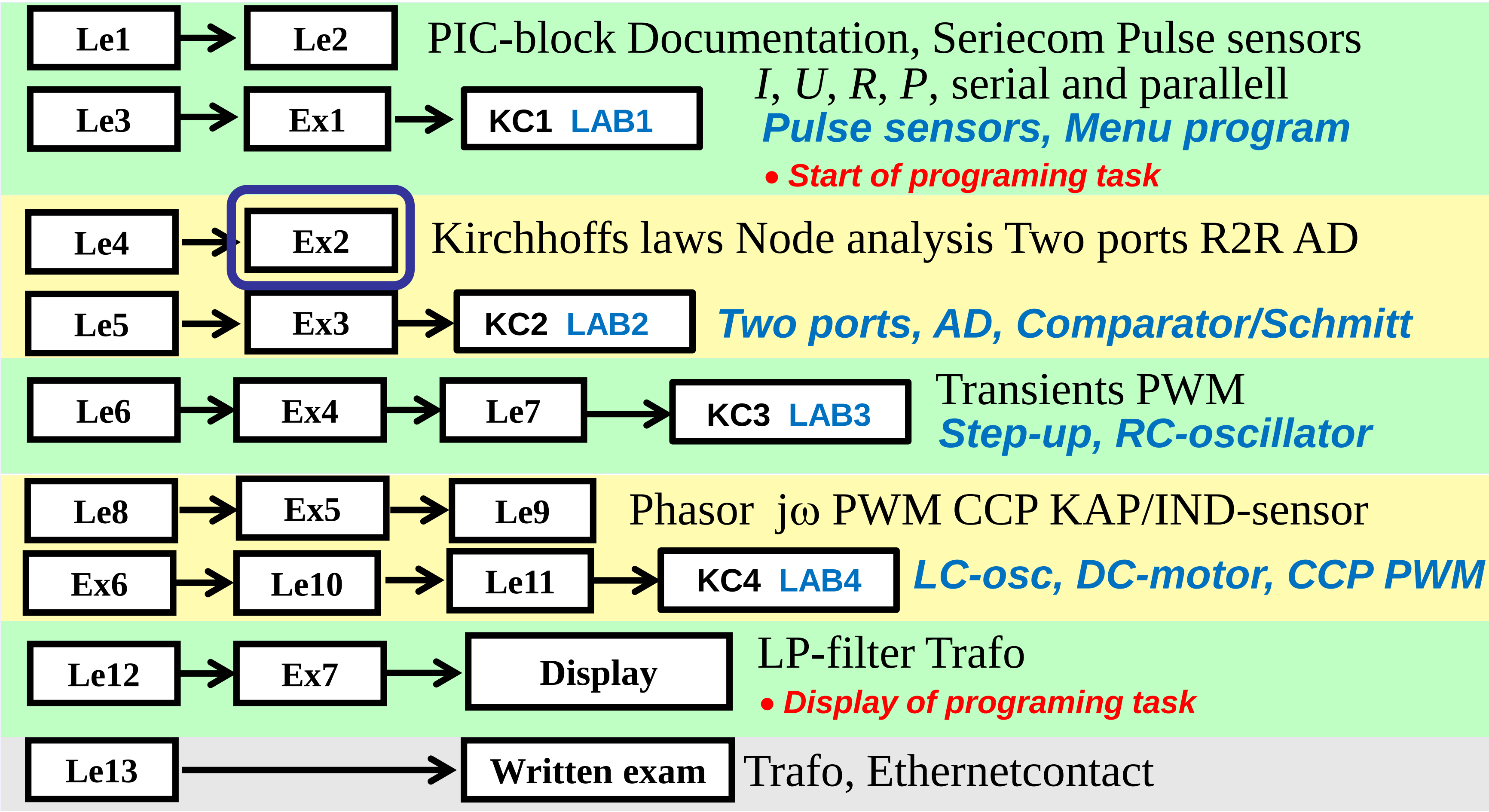
IE1206 Embedded Electronics



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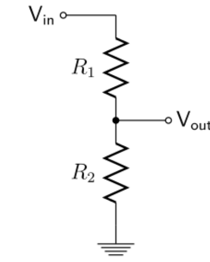
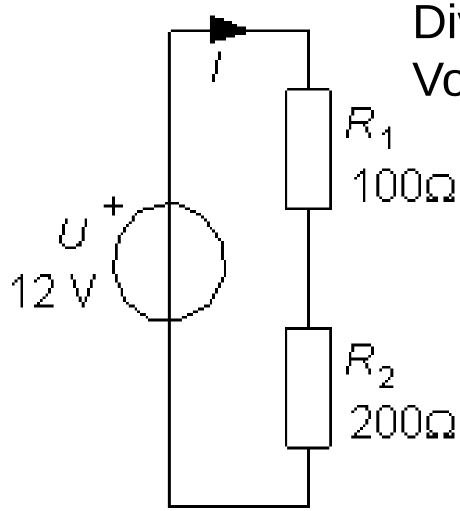
Voltage divider formula





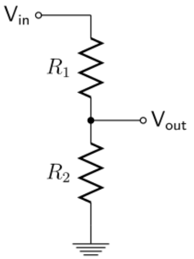
Divided Voltage | Total Voltage | Voltage division factor

$$U_1 = U \times \frac{R_1}{R_1 + R_2} = 12\,\frac{100}{100+200} = 4\text{V}$$

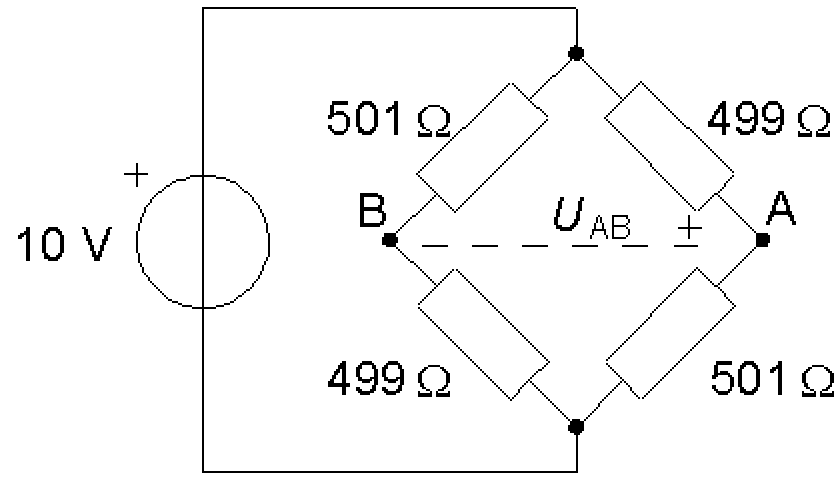
$$U_2 = U \times \frac{R_2}{R_1 + R_2} = 12\,\frac{200}{100+200} = 8\text{V}$$

According to the voltage divider formula you get a divided voltage, for example U_1 across the resistor $R_{1,}$ by multiplying the total voltage U with a voltage division factor. This voltage division factor is the resistance R_1 divided by the sum of all the resistors that are in the series connection.

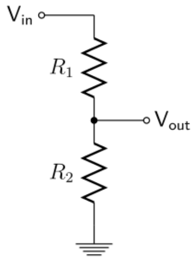
Unbalanced Wheatstone bridge



Points A and B are approximately at half the battery voltage. A is closer to "+ pole" and B is closer to "-pole." The difference U_{AB} can be measured with a sensitive millivolt meter connected between A and B.



Unbalanced Wheatstone bridge



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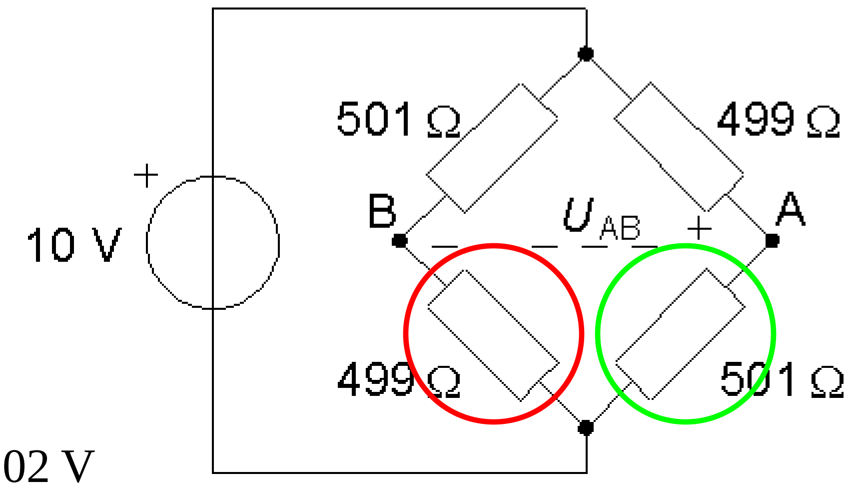
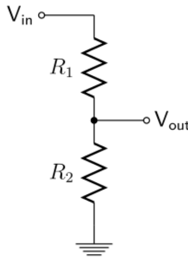


$$U_{AB} = 10\frac{501}{499+501} - 10\frac{499}{501+499} = 0{,}02\text{ V}$$

Unbalanced Wheatstone bridge



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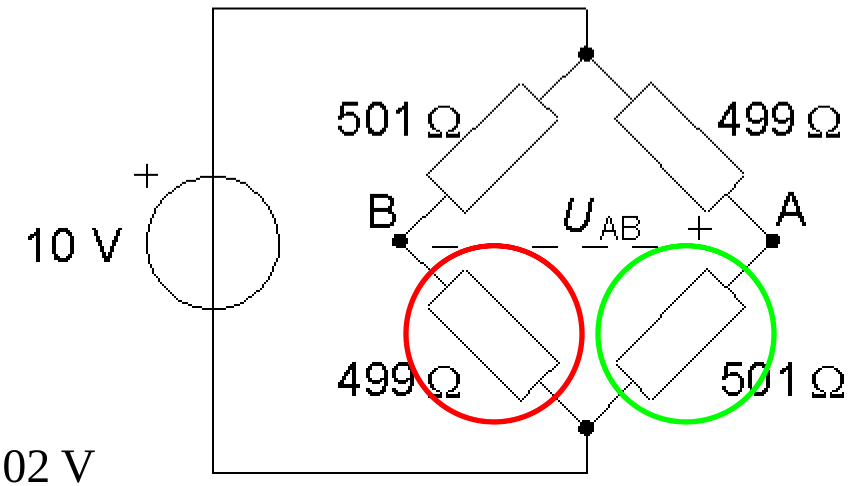
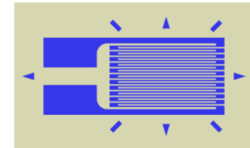


$$U_{AB} = 10\frac{501}{499+501} - 10\frac{499}{501+499} = 0{,}02 \text{ V}$$

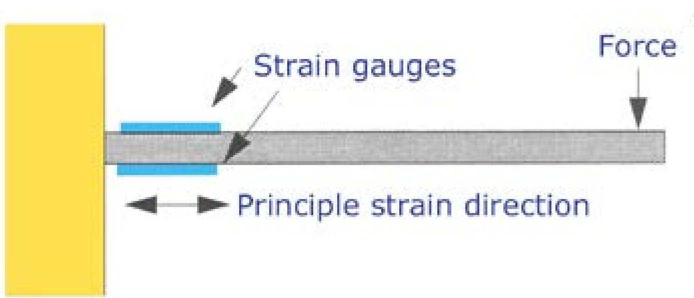
Why has the resistors values 501 and 499?

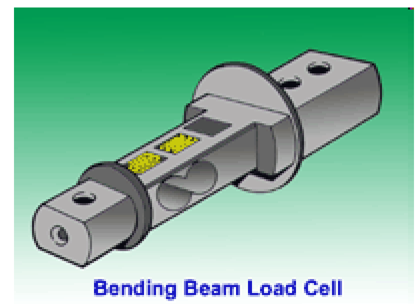
Loadcell

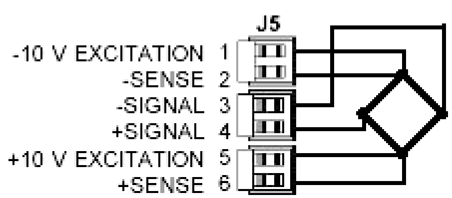


Industrial scales. Two strain gauges on the top of a beam increases from 500 to 501. Two strain gauge on the bottom of a beam decreases 500 to 499.

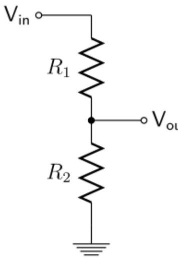
The gauges are connected as a Wheatstone bridge. The unbalance voltage is a direct measure on the force *F* (or if it's a scale $F = mg$).



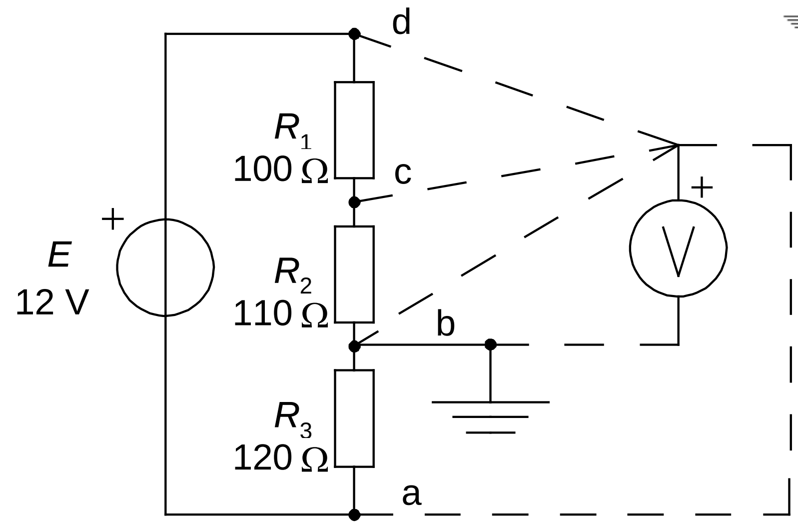




Potential (7.1)

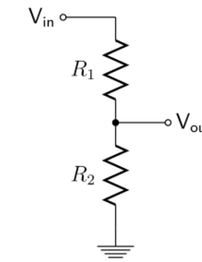


A voltage divider cosists of three resistors R_1 = 100 Ω, R_2 = 110 Ω, R_3 = 120 Ω, they are connected to a emf E = 12 V.

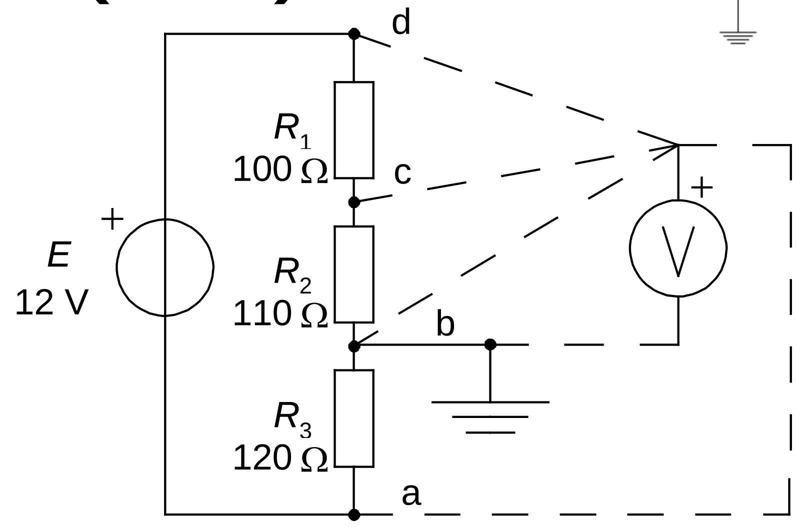


One measures the potential (voltage relative to ground) at various sockets on the the voltage divider.

Voltmeter negative terminal is all the time connected to the socket **b**, ground, while the positive terminal of the voltmeter in turn connects to the **a**, **b**, **c**, and **d**. What does the voltmeter show?



Potential (7.1)



Socket	a)	b)	c)	d)
Voltmeter [V]				

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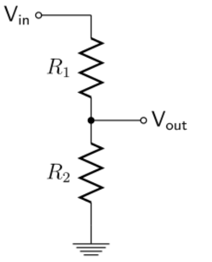
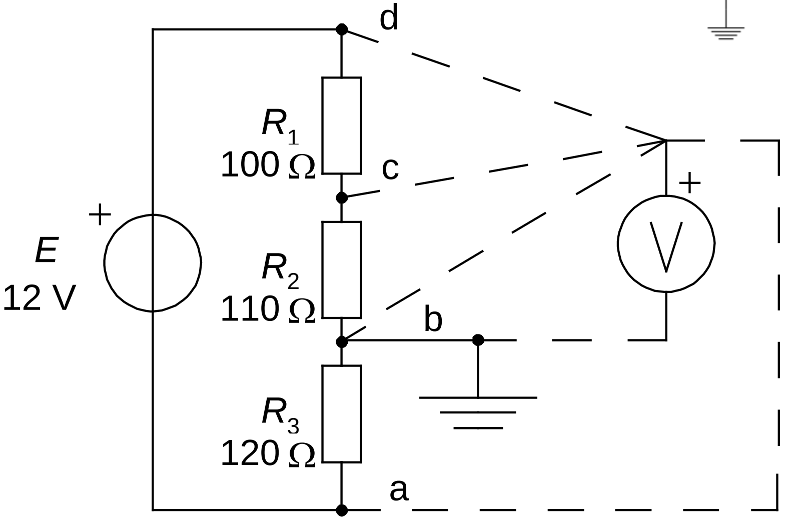
Potential (7.1)

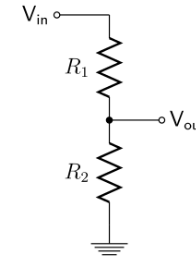


$$U_{ab} = -U_{ba} = -12\frac{120}{100+110+120} = -4{,}37$$

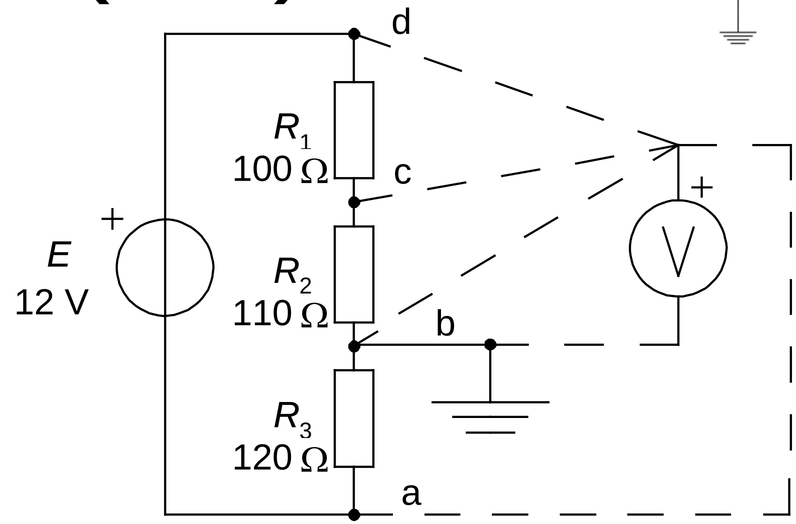


Socket	a)	b)	c)	d)
Voltmeter [V]	-4,37			

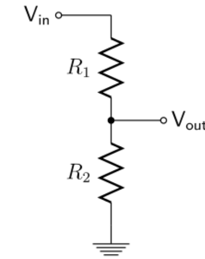
Potential (7.1)



$$U_{ab} = -U_{ba} = -12\frac{120}{100+110+120} = -4{,}37$$



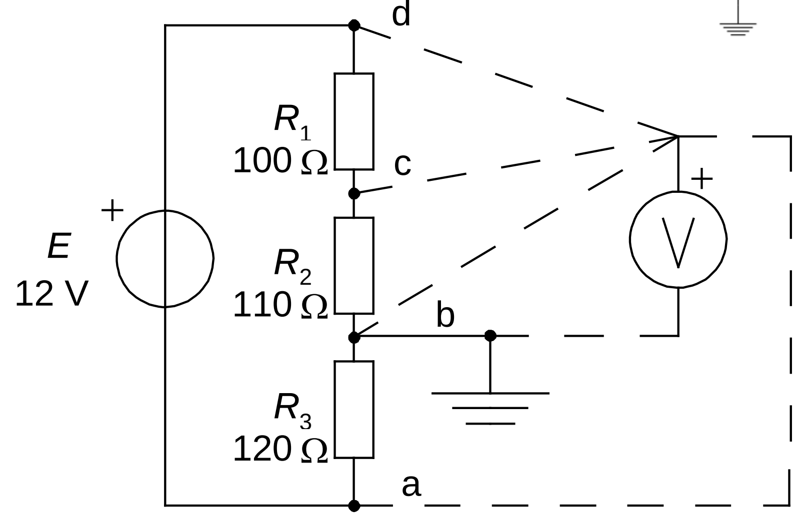
Socket	a)	b)	c)	d)
Voltmeter [V]	-4,37	0		



Potential (7.1)

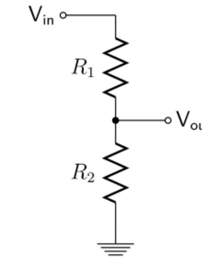
$$U_{ab} = -U_{ba} = -12\frac{120}{100+110+120} = -4{,}37$$

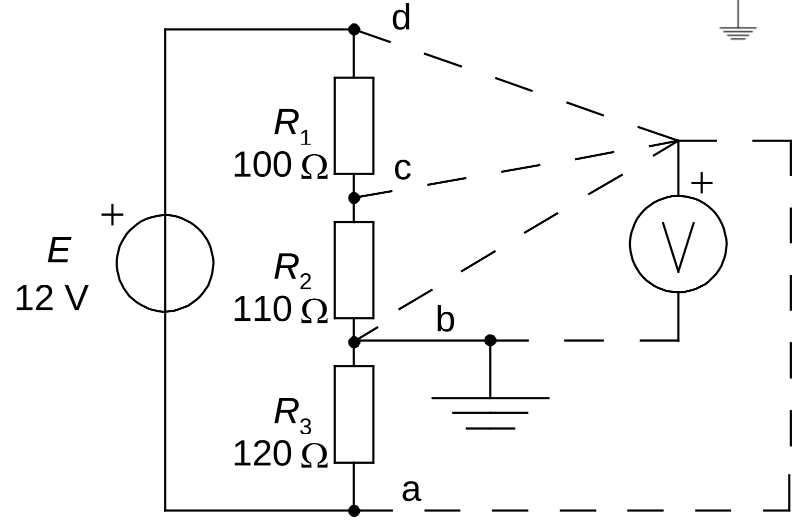
$$U_{cb} = 12\frac{110}{100+110+120} = 4$$



Socket	a)	b)	c)	d)
Voltmeter [V]	-4,37	0	4	

Potential (7.1)





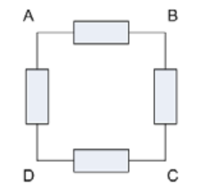
$$U_{ab} = -U_{ba} = -12\frac{120}{100+110+120} = -4{,}37$$

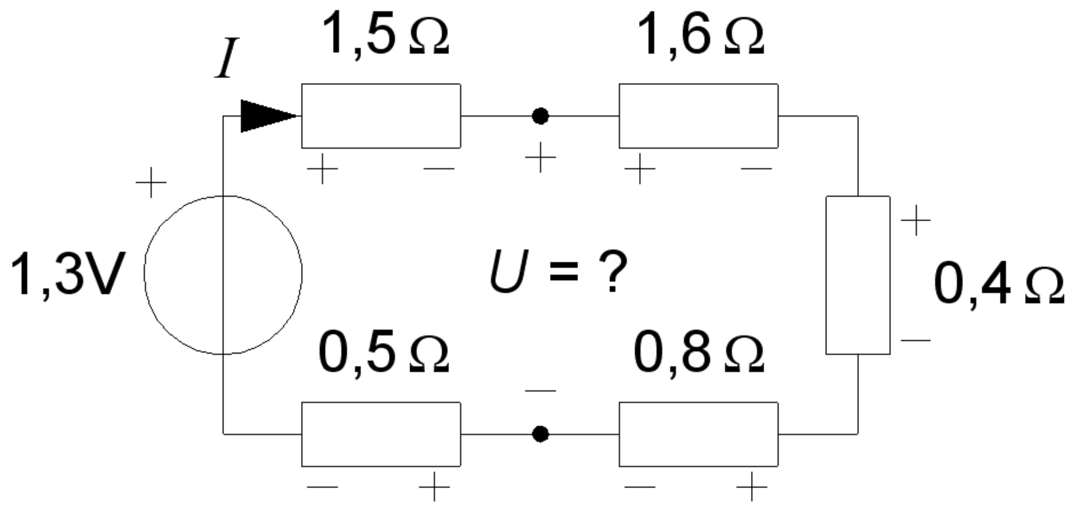
$$U_{cb} = 12\frac{110}{100+110+120} = 4$$

$$U_{db} = 12\frac{100+110}{100+110+120} = 7{,}64$$

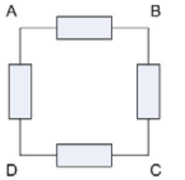
Socket	a)	b)	c)	d)
Voltmeter [V]	-4,37	0	4	7,64

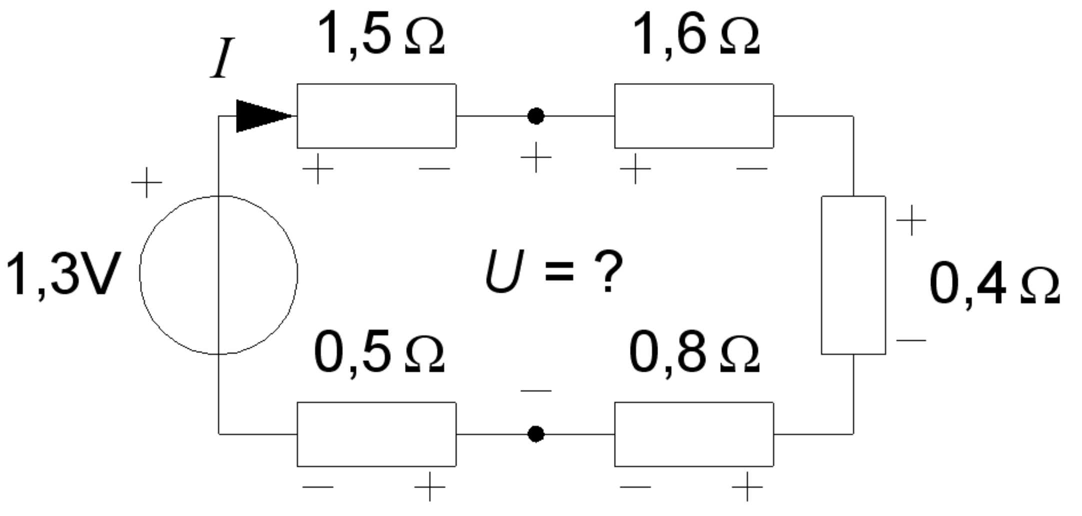
Kirchhoff's voltage law (5.3)





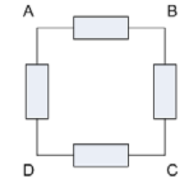
Kirchhoff's voltage law (5.3)

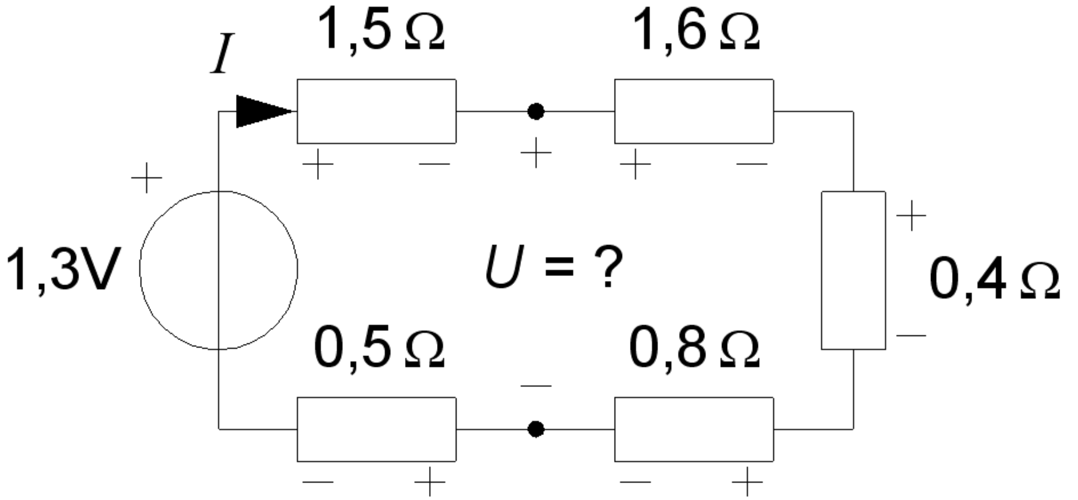




$$I = \frac{1{,}3}{1{,}5 + 1{,}6 + 0{,}4 + 0{,}8 + 0{,}5} = 0{,}27$$

Kirchhoff’s voltage law (5.3)



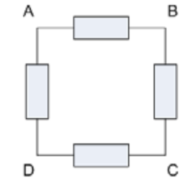


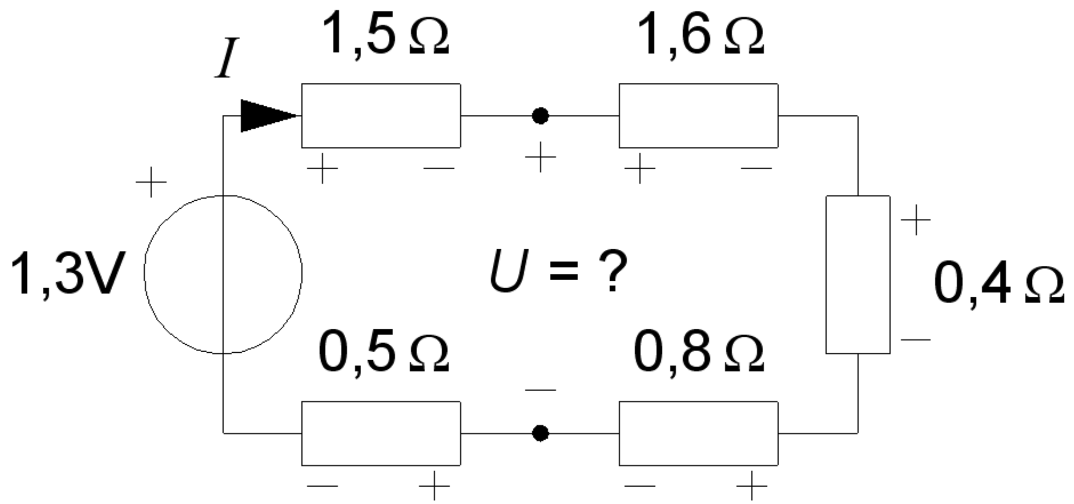
$$I = \frac{1{,}3}{1{,}5+1{,}6+0{,}4+0{,}8+0{,}5} = 0{,}27$$

$$U_{0,5} = 0{,}5 \cdot 0{,}27 = 0{,}14$$

$$U_{1,5} = 1{,}5 \cdot 0{,}27 = 0{,}41$$

Kirchhoff's voltage law (5.3)





$$I = \frac{1{,}3}{1{,}5+1{,}6+0{,}4+0{,}8+0{,}5} = 0{,}27$$

$$U_{0,5} = 0{,}5 \cdot 0{,}27 = 0{,}14$$

$$U_{1,5} = 1{,}5 \cdot 0{,}27 = 0{,}41$$

$$U = -0{,}14 + 1{,}3 - 0{,}41 = 0{,}76 \text{ V}$$

eller $U = 0{,}27 \cdot (0{,}8+0{,}4+1{,}6) = 0{,}76$ V

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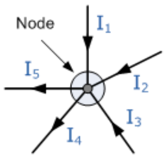
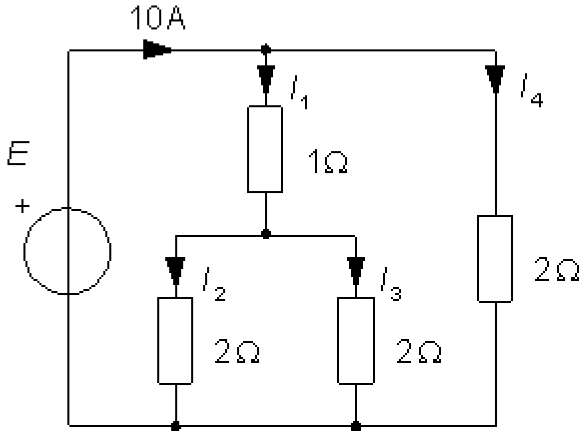
Kirchhoffs current law (5.1)



Can you guess the currents?



$I_1 =$ 5 A
$I_2 =$ 2,5 A
$I_3 =$ 2,5 A
$I_4 =$ 5 A

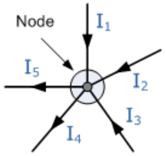
$I_1 + I_4 = 10$

$I_1 = I_2 + I_3 \quad I_2 = I_3$

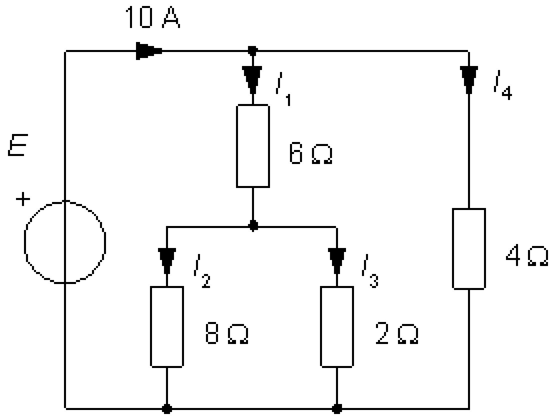
Parallel circuit, OHM's law: $I_4 \cdot 2 = I_1 \cdot (1+2//2) \Rightarrow I_4 = I_1 = 10/2 = 5$

$I_1 = I_2 + I_3 \Rightarrow I_2 = I_3 = 5/2 = 2,5$

Kirchhoffs current law (5.2)



Now we must calculate

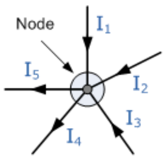


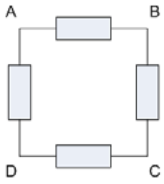
$$R_{ERS} = \frac{\left(6+\frac{8\cdot 2}{8+2}\right)\cdot 4}{\left(6+\frac{8\cdot 2}{8+2}\right)+4} = 2.62\ \Omega \quad E = R_{ERS}\cdot I = 2,62\cdot 10 = 26,2\ \text{V}$$

$$I_4 = \frac{E}{4} = \frac{26,2}{4} = 6,55\ \text{A} \qquad I_1 = I - I_4 = 10 - 6,55 = 3,45\ \text{A}$$

$$I_2 = \frac{E - 6\cdot I_1}{8} = \frac{26,2 - 3,45\cdot 6}{8} = \frac{5,5}{8} = 0,69\ \text{A} \qquad I_3 = \frac{5,5}{2} = 2,75\ \text{A}$$

Kirchhoffs laws? (6.3)

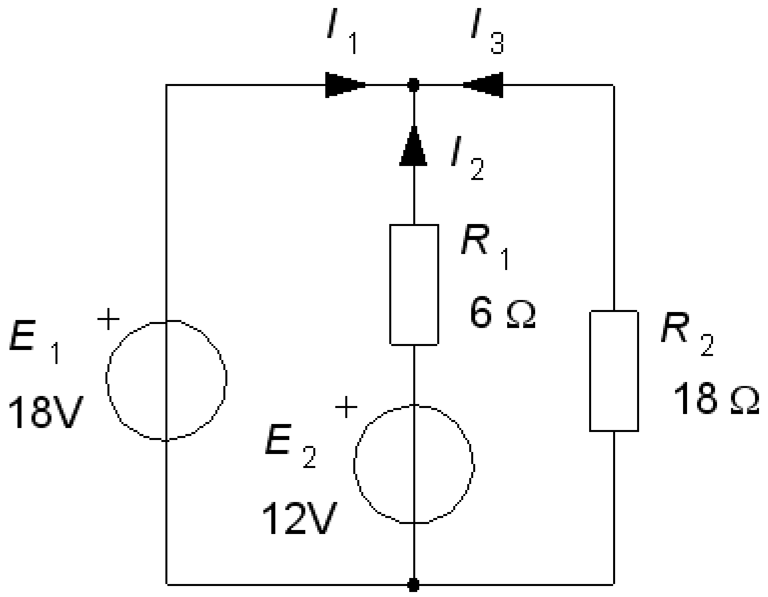




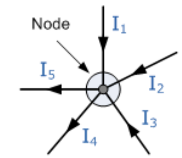
a) $U_{R2} = ?$

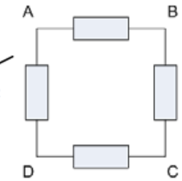
b) $I_2 = ?$

c) $I_1 = ?$



Kirchhoffs laws? (6.3)



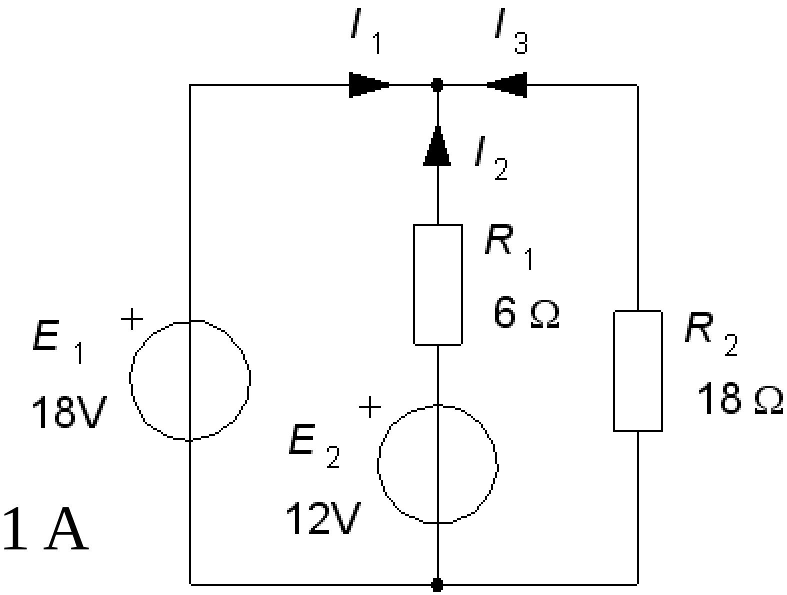


a) U_{R2} = ? $= 18$ V (E_1)

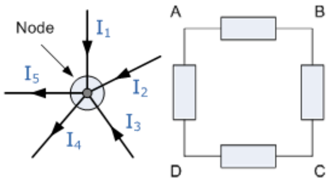
b) I_2 = ?

c) I_1 = ?

$18 + I_3 18 = 0 \quad I_3 = -18/18 = -\ 1$ A



Kirchhoffs laws? (6.3)



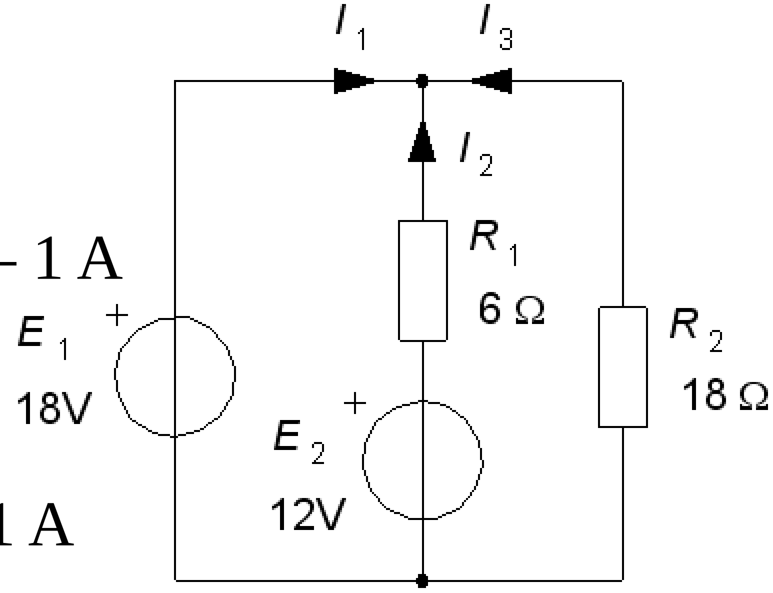
a) U_{R2} = ? = 18 V (E_1)

b) I_2 = ? $18 + 6I_2 - 12 = 0$

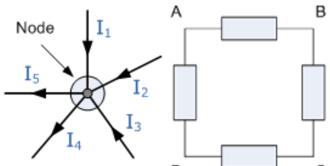
$I_2 = (12 - 18)/6 = -1$ A

c) I_1 = ?

$18 + I_3 18 = 0$ $I_3 = -18/18 = -1$ A



Kirchhoffs laws? (6.3)

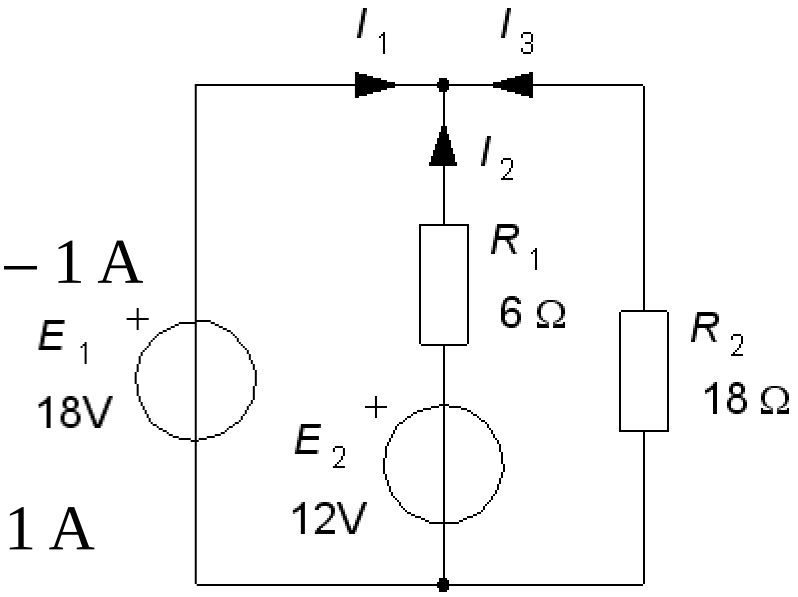


a) U_{R2} = ? $= 18\ \text{V}\ (E_1)$

b) I_2 = ? $18 + 6I_2 - 12 = 0$

$I_2 = (12 - 18)/6 = -1\ \text{A}$

c) I_1 = ?

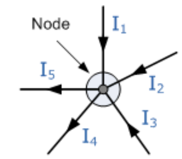


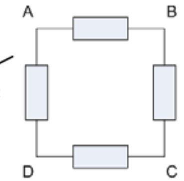
$18 + I_3 18 = 0 \quad I_3 = -18/18 = -1\ \text{A}$

$I_1 + I_2 + I_3 = 0$

$I_1 = - I_2 - I_3 = -(-1) - (-1) = 2\ \text{A}$

Kirchhoffs laws? (6.3)

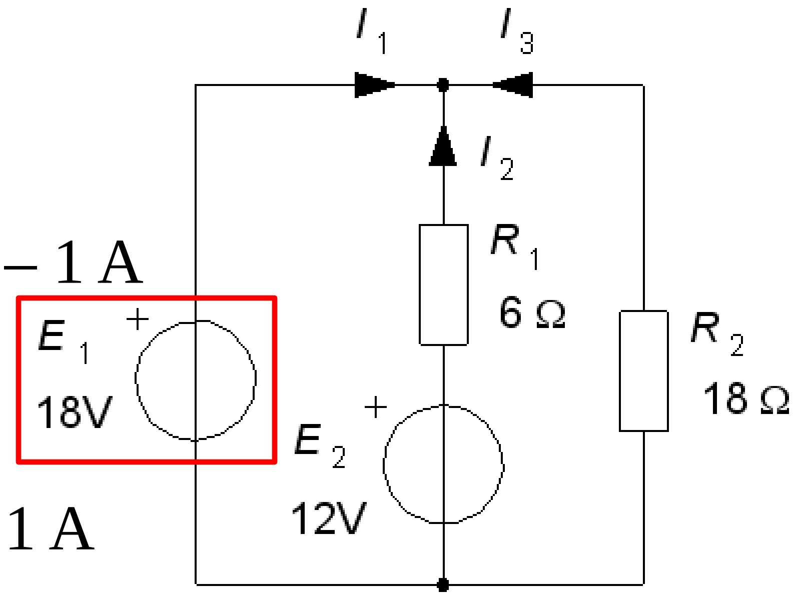




a) $U_{R2} = ?$ $= 18\ \text{V}\ (E_1)$

b) $I_2 = ?$ $18 + 6I_2 - 12 = 0$

$I_2 = (12 - 18)/6 = -1\ \text{A}$



c) $I_1 = ?$

$18 + I_3 18 = 0 \quad I_3 = -18/18 = -\ 1\ \text{A}$

$I_1 + I_2 + I_3 = 0$

$I_1 = -\ I_2 - \ I_3\ = -\ (-1) - \ (-1) = \ 2\ \text{A}$

That E_1 is an ideal emf is what simplifies the calculations!

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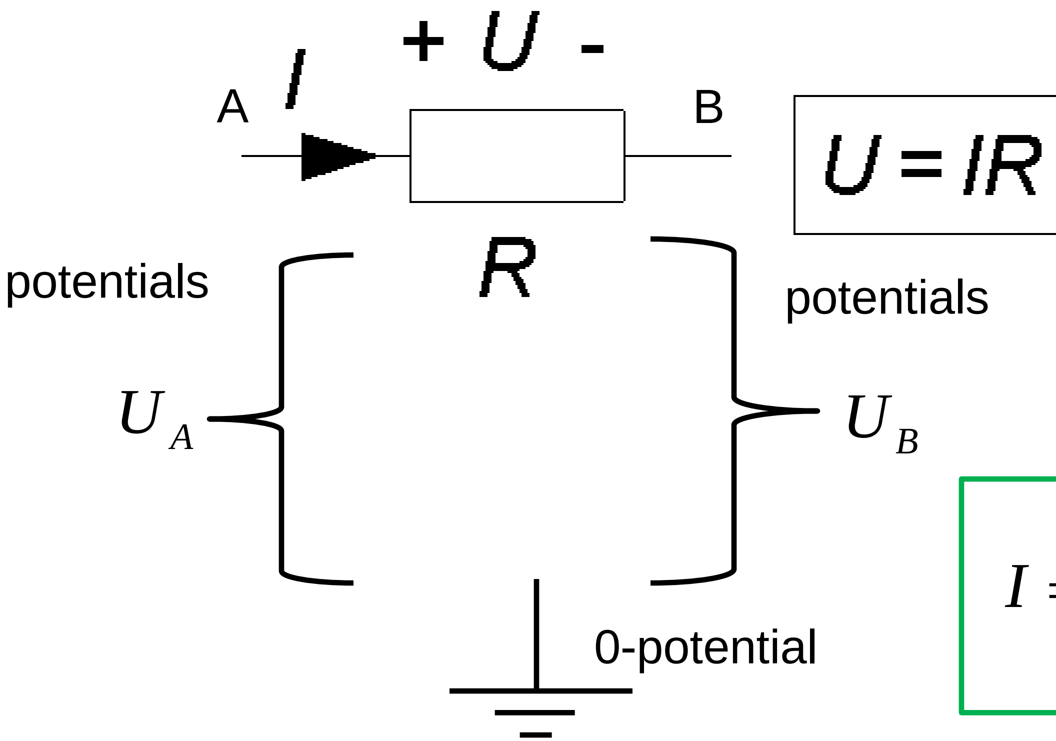
Node analysis

$$U_{AB} = U_A - U_B$$



$$I = \frac{U_{AB}}{R} = \frac{U_A - U_B}{R}$$

Or with node analysis (7.3)

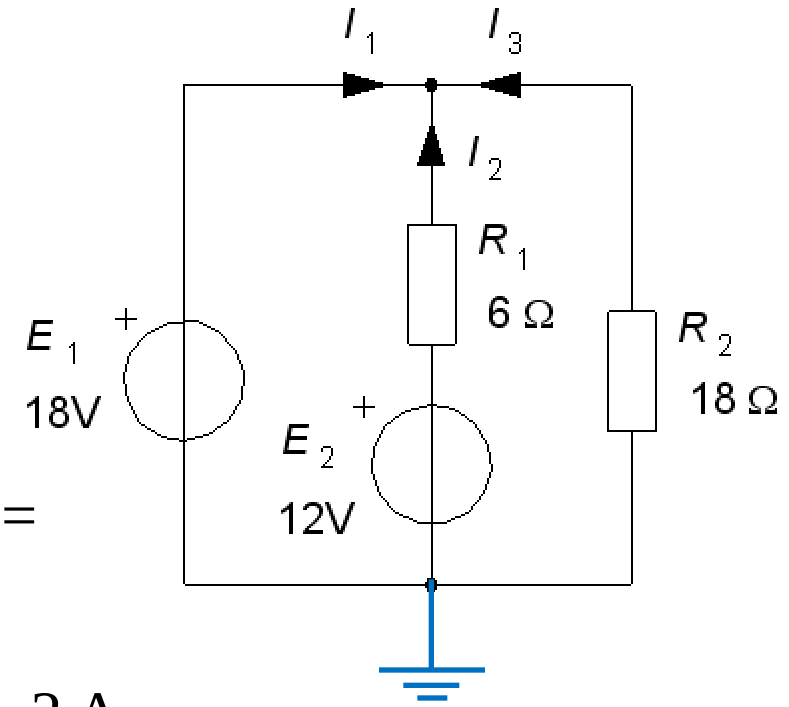
$$I_1 + I_2 + I_3 = 0 \quad I_1 = - I_2 - I_3$$

$$E_1 = 18 \text{ V}$$

$$I_3 = - (E_1 - 0)/R_2 = - 18/18$$
$$= - 1 \text{ A}$$

$$I_2 = - (E_1 - E_2)/R_1 = - (18 - 12)/6 =$$
$$= - 1\text{A}$$

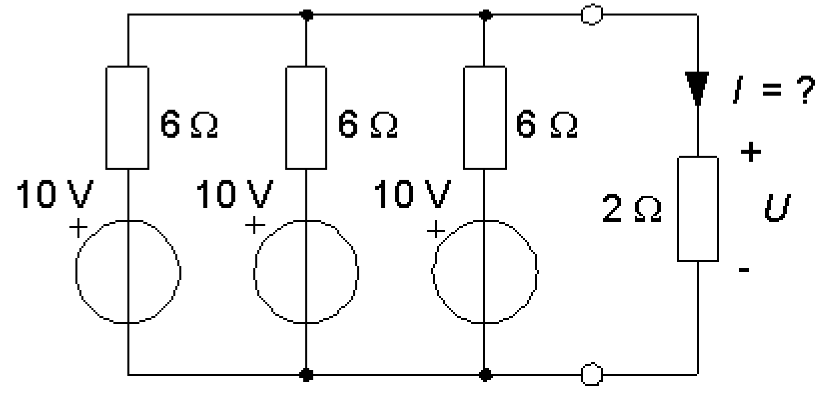
$$I_1 = - I_2 - I_3 = - (- 1) - (- 1) = 2 \text{ A}$$



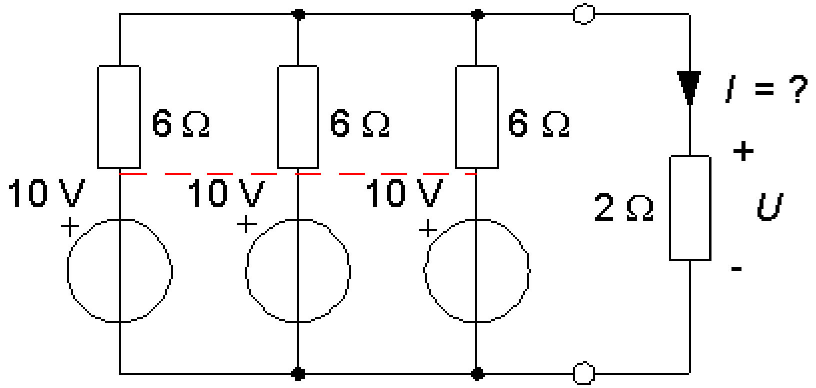
Batteries in parallel (4.4)

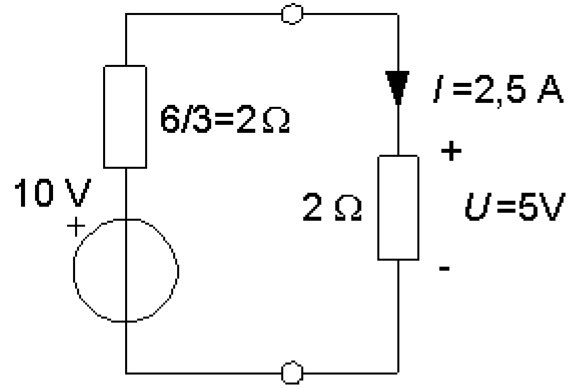
Three similar batteries E = 10 V and the internal resistance 6 Ω are parallel-connected to deliver current to a resistor with resistance 2 Ω.

a) How much will current I and terminal voltage U be?

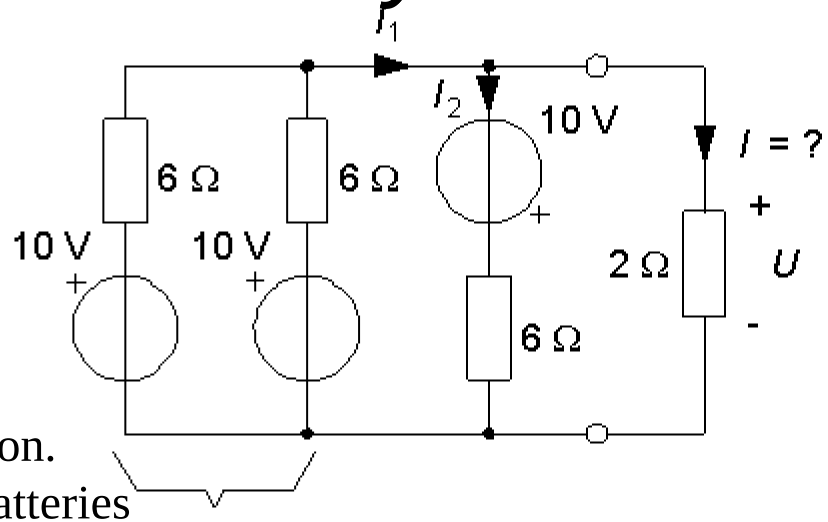


The three internal resistances 6Ω have common voltage in both ends, and is thereby effectively paralleled. R_I = 6/3 = 2Ω. I = 2,5 A och U = 5V.



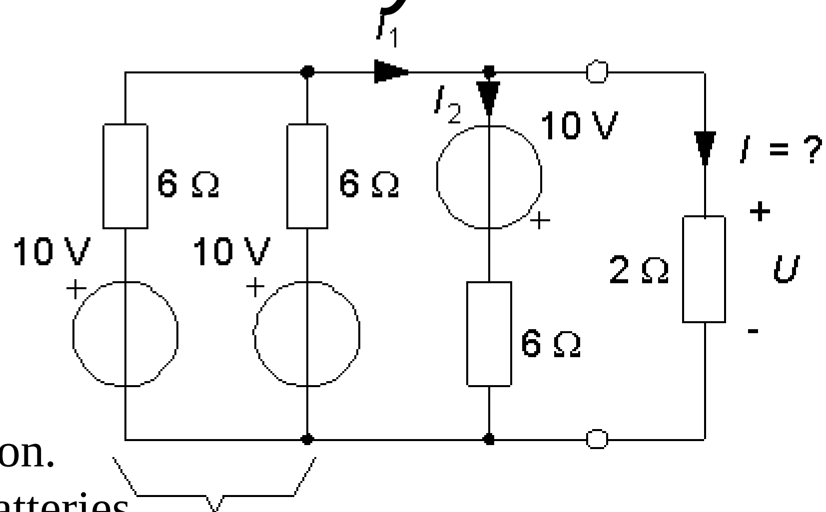


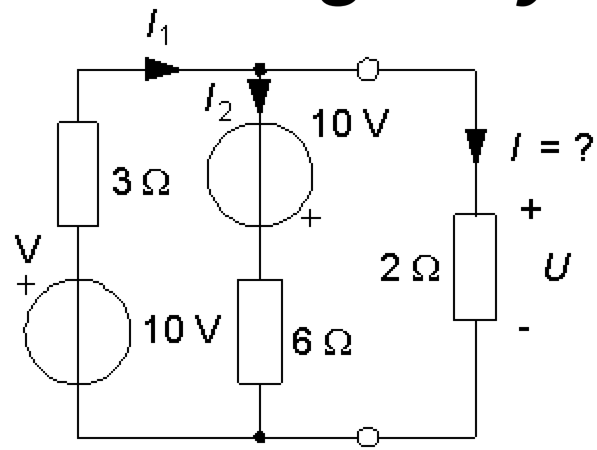
One battery inserted the wrong way!



Suggestion.
Merge batteries

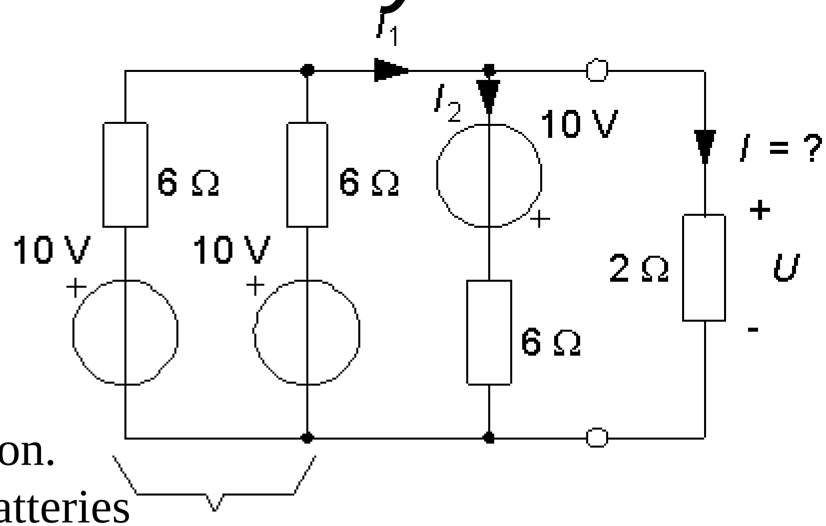
One battery inserted the wrong way!

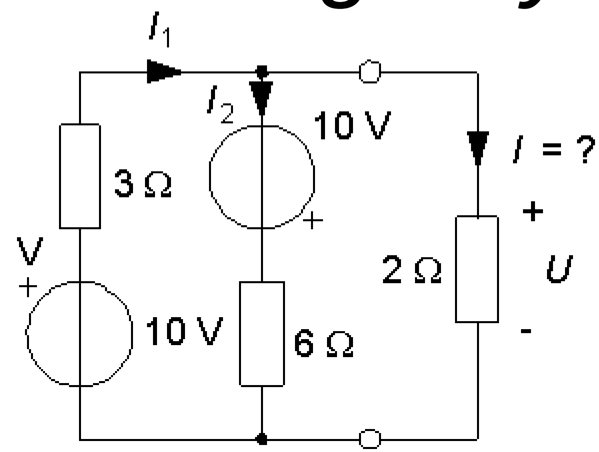




Suggestion.
Merge batteries

One battery inserted the wrong way!

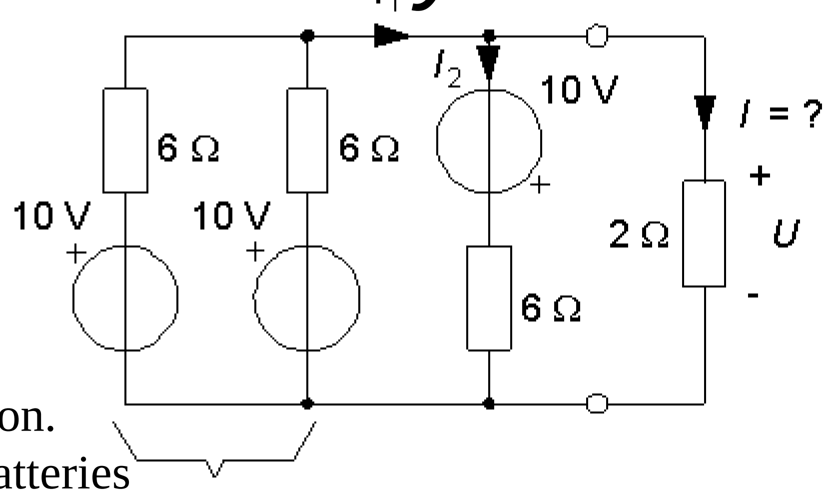


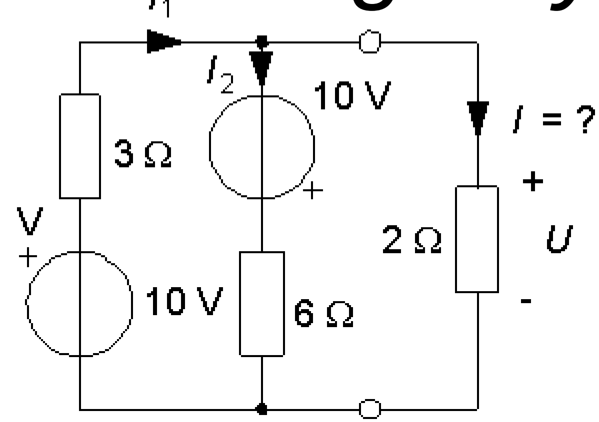


Suggestion.
Merge batteries

This is now a more complicated circuit that requires Kirchhoff's laws to be solved ...

One battery inserted the wrong way!





Suggestion.
Merge batteries

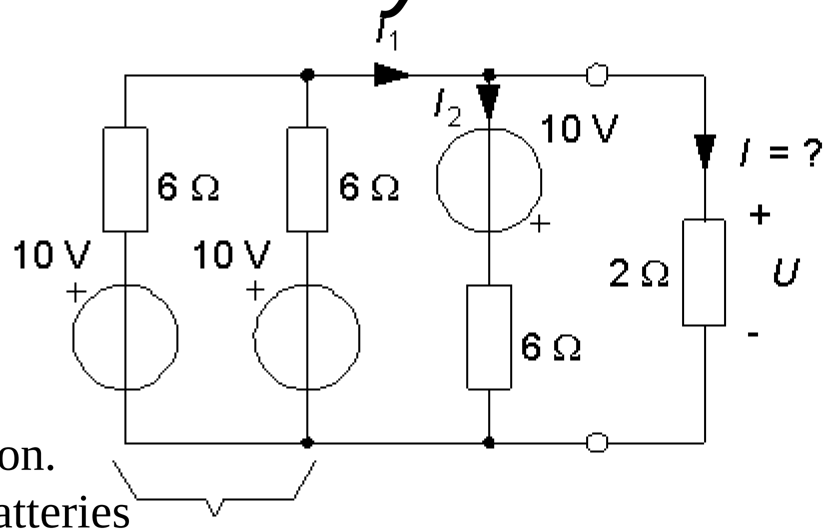
$$I_1 - I_2 - I = 0$$

$$10 - 3I_1 + 10 - 6I_2 = 0 \quad \Leftrightarrow \quad -3I_1 - 6I_2 + 0I = -20$$

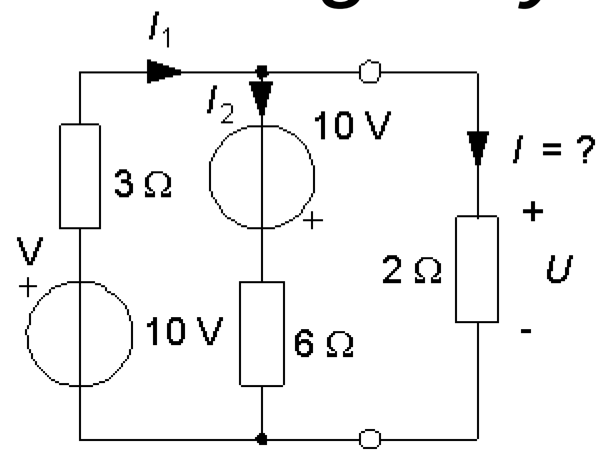
$$6I_2 - 10 - 2I = 0 \quad \Leftrightarrow \quad 0I_1 + 6I_2 - 2I = 10$$

$$\begin{pmatrix} 1 & -1 & -1 \\ -3 & -6 & 0 \\ 0 & 6 & -2 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ 10 \end{pmatrix}$$

One battery inserted the wrong way!



Suggestion.
Merge batteries



$$I_1 - I_2 - I = 0$$

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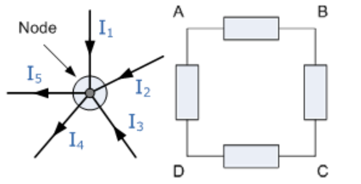
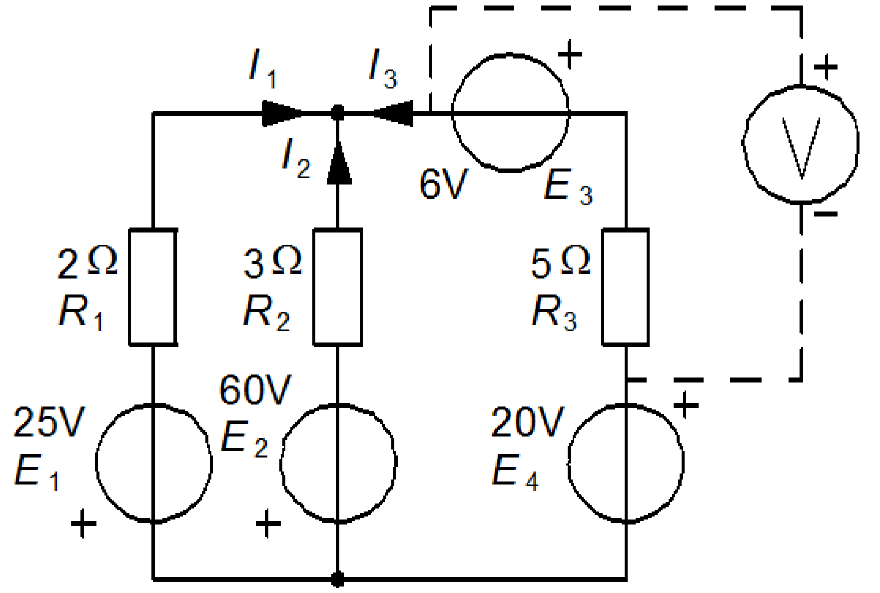
$$\begin{pmatrix} 1 & -1 & -1 \\ -3 & -6 & 0 \\ 0 & 6 & -2 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I \end{pmatrix} = \begin{pmatrix} 0 \\ -20 \\ 10 \end{pmatrix}$$

I_1 = 2,78 A
I_2 = 1,94 A
I = 0,83 A

$U = I \cdot 2 = 0{,}83 \cdot 2 = \boxed{1{,}67 \text{ V}}$

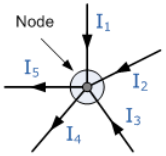
Kirchhoff law's (6.5)

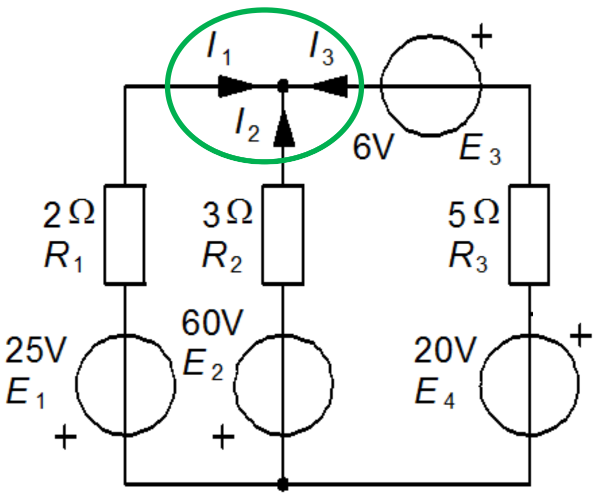




a) Use Kirchhoff's two laws to set up an equation system by which the three currents I_1 I_2 and I_3 can be calculated. (You need not solve the system of equations)

Kirchhoff law’s (6.5)

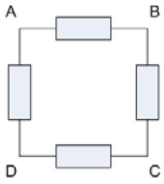


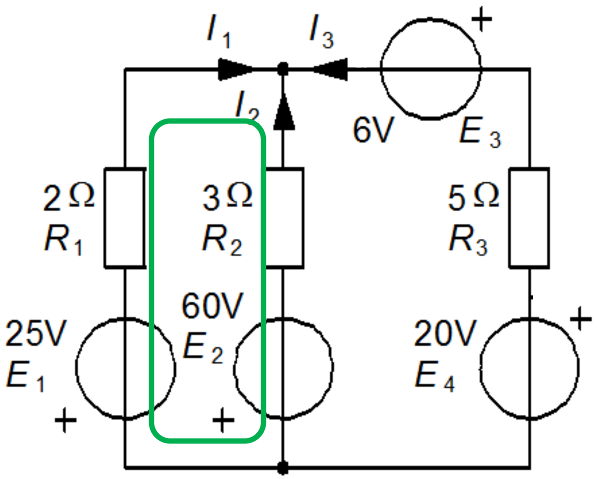


Kirchhoff current law:

$$I_1 + I_2 + I_3 = 0$$

Kirchhoff law's (6.5)





Kirchhoff current law:

$$I_1 + I_2 + I_3 = 0$$

Kirchhoff voltage law (left mesh):

$$-25 - 2 \cdot I_1 + 3 \cdot I_2 + 60 = 0$$

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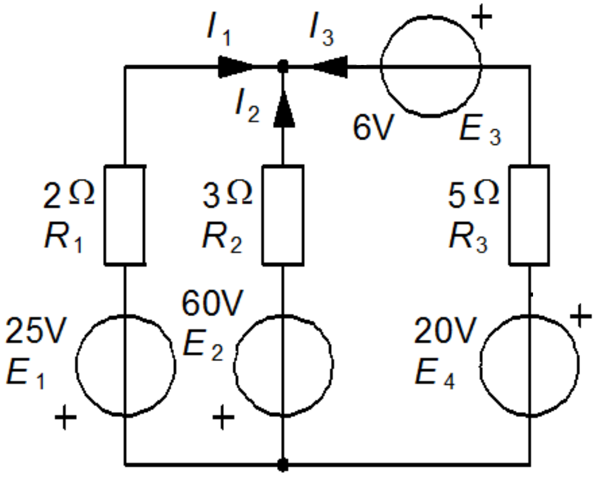
Kirchhoff law's (6.5)



Kirchhoff current law:

$$I_1 + I_2 + I_3 = 0$$

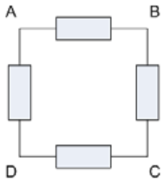
Kirchhoff voltage law (left mesh):

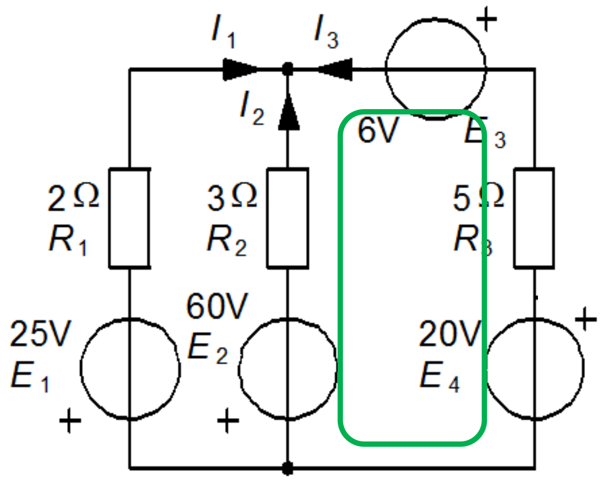
$$-25 - 2 \cdot I_1 + 3 \cdot I_2 + 60 = 0$$

Trim up:

$$-2 \cdot I_1 + 3 \cdot I_2 + 0 \cdot I_3 = -35$$

Kirchhoff law's (6.5)





Kirchhoff current law:

$$I_1 + I_2 + I_3 = 0$$

Kirchhoff voltage law (left mesh):

$$-25 - 2 \cdot I_1 + 3 \cdot I_2 + 60 = 0$$

Trim up:

$$-2 \cdot I_1 + 3 \cdot I_2 + 0 \cdot I_3 = -35$$

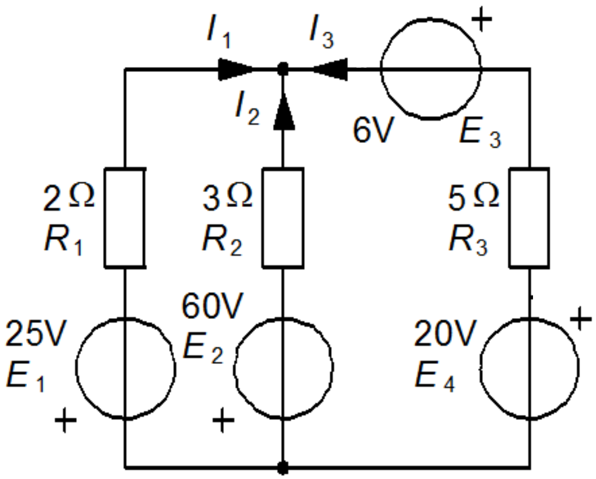
Kirchhoff voltage law (right mesh):

$$-60 - 3 \cdot I_2 + 6 + 5 \cdot I_3 - 20 = 0$$

Trim up:

$$0 \cdot I_1 - 3 \cdot I_2 + 5 \cdot I_3 = 74$$

Kirchhoff law's (6.5)



$$I_1 + I_2 + I_3 = 0$$

$$-2 \cdot I_1 + 3 \cdot I_2 + 0 \cdot I_3 = -35$$

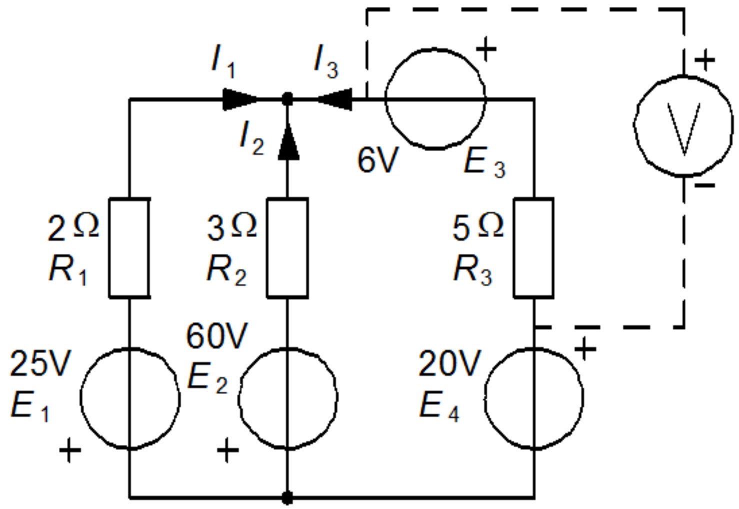
$$0 \cdot I_1 - 3 \cdot I_2 + 5 \cdot I_3 = 74$$

$$R \quad \cdot \quad I \;=\; U$$

$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & 3 & 0 \\ 0 & -3 & 5 \end{pmatrix} \bullet \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -35 \\ 74 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 1{,}87 \\ -10{,}4 \\ 8{,}55 \end{pmatrix}$$

Kirchhoff law's (6.5)



If equationsystem is solved one gets:

I_1 = 1,87 I_2 = -10,4 I_3 = 8,55 [A].

b) What does the voltmeter at the right in the figure show (give both amount and sign) [V]?

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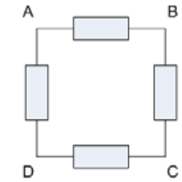
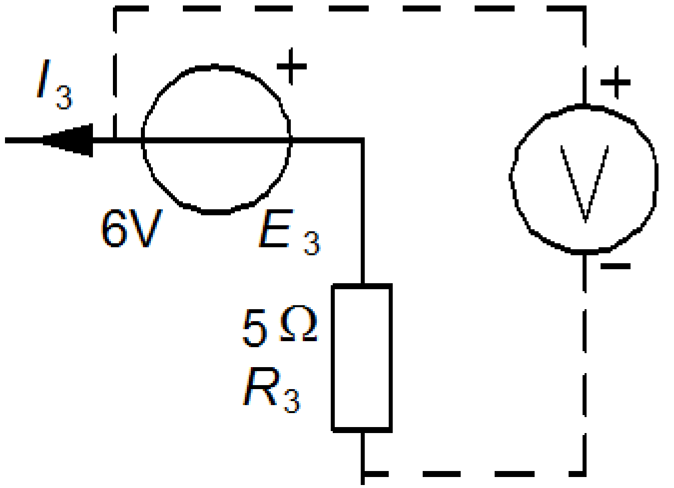
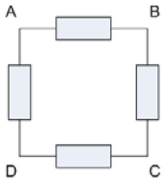
Kirchhoff law's (6.5)

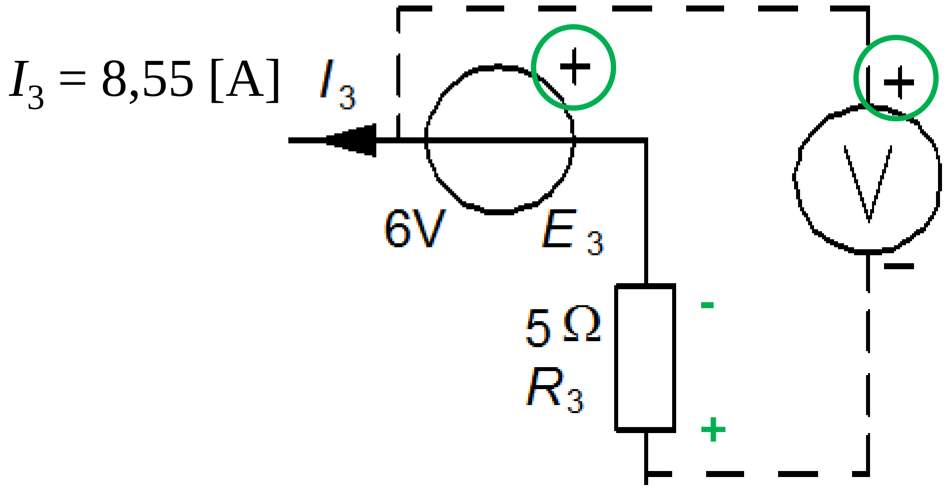


$I_3 = 8{,}55$ [A]



Kirchhoff law's (6.5)





Voltmeter (U) shows

$$U + E_3 + R_3 \cdot I_3 = 0 \Rightarrow U = -6 - 5 \cdot 8{,}55 = -48{,}75 \text{ V}$$