Lecture 10
FH-SS and CPM

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Lecture 10: Multi-User Detection, Frequency Hop Spread Spectrum, and Continuous Phase Modulation Advanced Digital Communications (EQ2410)[1]

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Thursday, Feb. 25, 2016
10:00-12:00, B23

[1] Textbook: U. Madhow, *Fundamentals of Digital Communications*, 2008

Notes

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Multi-User Detection

Asymptotic Efficiency

Frequency Hop Spread Spectrum

Coded Frequency Hop Multiple Access

Continuous Phase Modulation

Minimum Shift Keying

Gaussian MSK

Receiver Design

Overview

Lecture 9

- Direct sequence spread spectrum
- CDMA

Lecture 10: Frequency hop spread spectrum and continuous phase modulation

Notes

Multi-User Detection

- Synchronous CDMA, N-dimensional received vector

$$\mathbf{r} = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{W} \;=\; \mathbf{U}\mathbf{b} + \mathbf{W} \;=\; \mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{W} \;=\; \mathbf{s}_\mathbf{b} + \mathbf{W}$$

  with
  - the AWGN $\mathbf{W} \sim N(0, \sigma^2 \mathbf{I})$,
  - the symbol vector $\mathbf{b} = (b_1, \ldots, b_K)^T$,
  - the matrix of channel gains $\mathbf{A} = \mathrm{diag}(A_k)$
  - the matrix of spreading sequences $\mathbf{S} = (\mathbf{s}_1, \ldots, \mathbf{s}_K)$,
  - and $\mathbf{U} = (A_1\mathbf{s}_1, \ldots, A_K\mathbf{s}_K)$ and $\mathbf{s}_\mathbf{b} = \mathbf{U}\mathbf{b}$.

- Conventional detector/matched filtering can be expressed as

$$\mathbf{z}^{MF} = \mathbf{S}^H \mathbf{r} = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{S}^H \mathbf{W}$$

  with the vector of MF decision statistics $\mathbf{z}^{MF}$ and the correlation matrix $\mathbf{R} = \mathbf{S}^H\mathbf{S}$.

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Multi-User Detection

- Maximum-likelihood detection: choose $\mathbf{b}$ that maximizes

$$\lambda(\mathbf{b}) = \langle \mathbf{r}, \mathbf{s}_\mathbf{b} \rangle - \frac{1}{2}\|\mathbf{s}_\mathbf{b}\|^2$$

  with

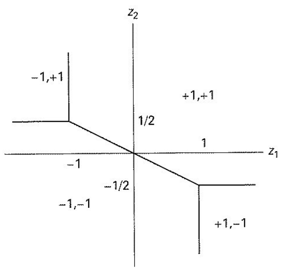
$$\begin{aligned} \langle \mathbf{r}, \mathbf{s}_\mathbf{b} \rangle &= \mathbf{s}_\mathbf{b}^H \mathbf{r} = \mathbf{b}^H\mathbf{A}^H\mathbf{S}^H\mathbf{r} = \mathbf{b}^H\mathbf{A}^H\mathbf{z}^{MF} \quad \text{and} \\ \|\mathbf{s}_\mathbf{b}\|^2 &= \mathbf{b}^H\mathbf{A}^H\mathbf{S}^H\mathbf{S}\mathbf{A}\mathbf{b} = \mathbf{b}^H\mathbf{A}^H\mathbf{R}\mathbf{A}\mathbf{b} \end{aligned}$$

Example: 2 users with spreading vectors $\mathbf{s}_1$, $\mathbf{s}_2$ and $\langle \mathbf{s}_1, \mathbf{s}_2 \rangle = \rho$

- Let

$$\begin{aligned} z_1 = \langle \mathbf{r}, \mathbf{s}_1 \rangle &= A_1 b_1 + \rho A_2 b_2 + N_1 \\ z_2 = \langle \mathbf{r}, \mathbf{s}_2 \rangle &= \rho A_1 b_1 + A_2 b_2 + N_2 \end{aligned}$$

- $\langle \mathbf{r}, \mathbf{s}_\mathbf{b} \rangle = A_1 b_1 z_1 + A_2 b_2 z_2$ with the matched-filter outputs z_1, z_2
- Decision regions for $A_1 = 1$, $A_2 = 2$ and $\rho = -0.5$



Notes

Multi-User Detection

- Linear zero-forcing (or decorrelating) detector for user k

$$z_k^{ZF} = \langle \mathbf{r}, \mathbf{c}_{ZF} \rangle = \mathbf{c}_{ZF}^H \mathbf{r} \quad \text{with} \quad \mathbf{c}_{ZF} = \mathbf{S}(\mathbf{S}^H\mathbf{S})^{-1}\mathbf{e}_k = \mathbf{S}\mathbf{R}^{-1}\mathbf{e}_k$$

- ZF detector expressed in matrix form for the vector $\mathbf{z}^{ZF} = (Z_1^{ZF}, \dots, Z_K^{ZF})^T$

$$\mathbf{z}^{ZF} = \mathbf{C}_{ZF}^H \mathbf{r} = \mathbf{R}^{-1}\mathbf{S}^H\mathbf{r} = \mathbf{R}^{-1}\mathbf{z}^{MF}.$$

- Linear MMSE detector for the k-th user by minimizing the

$$MSE = \mathrm{E}[\| \langle \mathbf{r}, \mathbf{c} \rangle - b_k \|^2]$$

$$z_k^{MMSE} = \langle \mathbf{r}, \mathbf{c}_{MMSE} \rangle = \mathbf{c}_{MMSE}^H \mathbf{r} \quad \text{with} \quad \mathbf{c}_{MMSE} = \mathrm{E}[\mathbf{r}\mathbf{r}^H]^{-1} \cdot \mathrm{E}[b_k \mathbf{r}]$$

with

$$\begin{aligned} \mathrm{E}[\mathbf{r}\mathbf{r}^H] &= E_s \cdot \mathbf{S}\mathbf{A}\mathbf{A}^H\mathbf{S}^H + \sigma^2\mathbf{I} \\ \mathrm{E}[b_k\mathbf{r}] &= E_s \cdot \mathbf{S}\mathbf{A}\mathbf{e}_k \end{aligned}$$

- MMSE detector expressed in matrix form with $\mathbf{z}^{MMSE} = (Z_1^{MMSE}, \dots, Z_K^{MMSE})^T$

$$\mathbf{z}^{MMSE} = \mathbf{A}^H\mathbf{S}^H(\mathbf{S}\mathbf{A}\mathbf{A}^H\mathbf{S}^H + \sigma^2/E_s\mathbf{I})^{-1}\mathbf{r}$$

- Decision feedback methods: successive interference cancellation.

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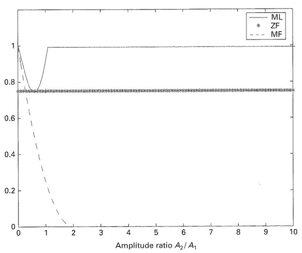
Asymptotic Efficiency



- Error probability for MUD P_e, error probability in the single-user case $P_{e,su}$
- Asymptotic efficiency

$$\eta = \lim_{\sigma^2 \to 0} \frac{\log P_e}{\log P_{e,su}}$$

- Example: $P_e = e^{-a/\sigma^2}$ and $P_{e,su} = e^{-b/\sigma^2}$

$$\Rightarrow \eta = a/b$$

- Asymptotic efficiency for ML detection: $\eta_{ML} = 1 - |\rho|^2$
- Asymptotic efficiency for ZF/MMSE detection:

$$\eta_{ZF} = \|\mathbf{P}_I^{\perp}\mathbf{s}_1\|^2 / \|\mathbf{s}_1\|^2 = \|\mathbf{P}_I^{\perp}\mathbf{s}_1\|^2$$

(→independent of interference amplitudes)

- Asymptotic efficiency for matched filter (MF): constant error probability (i.e., $a = 0$) as A_k/A_1 becomes large.
- Near/far resistance: minimum of the asymptotic efficiency

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Frequency Hop Spread Spectrum

- Direct sequence spread spectrum
  - The bandwidth of a narrowband signal is spread to a large bandwidth.
  - All the bandwidth is used all the time.
- Frequency hop spread spectrum
  - Narrowband signaling with varying carrier frequency
  - Hopping pattern specifies how the carrier frequency is changed.
  - → On average the whole bandwidth is used.
  - Slow FH: multiple symbols per hop; fast FH: one symbol per hop.
  - Frequency diversity (GSM standard)
  - Randomized multiple access (Bluetooth)
  - Phase synchronization is difficult
    → non-coherent or differential coherent modulation

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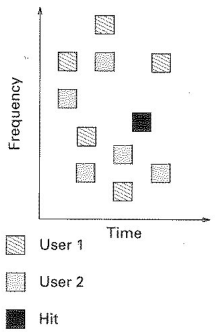
Coded Frequency Hop Multiple Access

- K users share q frequencies; each user has its own hopping pattern; users are synchronized;



- Probability that a collision occurs ("hit")

$$p_{hit} = 1 - \left(\frac{q-1}{q}\right)^{K-1}$$

- Collision leads to an erasure of the symbols of that hop; p_{hit} gives the erasure probability.
- Coded system: use an erasure-correcting code across the hops.
- Example: a (n, k) Reed-Solomon code can correct $(n-k)$ erasures; probability that a frame gets lost:

$$P_F = \sum_{h=n-k+1}^{n} \binom{n}{h} p_{hit}^{h} (1 - p_{hit})^{n-h}$$

Notes

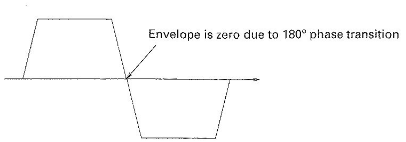
Continuous Phase Modulation

- Signal of the form $e^{j\theta(t)}$, where $\theta(t)$ is a continuous function of t carrying the data.
- Passband signal: $u_p(t) = \cos(2\pi f_c t + \theta(t))$
- Constant envelope: signal can be recovered even if it passes through severe nonlinearities.
- Example: $u_p(t)$ through a limiter function *sign*()

$$\begin{aligned} v_p(t) &= sign(u_p(t)) = sign(\cos(2\pi f_c t + \theta(t))) \\ &\approx a_1 \cos(2\pi f_c t + \theta(t)) + a_3 \cos(3(2\pi f_c t + \theta(t))) + \dots \end{aligned}$$

$\to u_p(t)$ can be recovered by low-pass filtering!

- Counterexample: QPSK with a non-ideal pulse is not CPM; it has as well the form $e^{j\theta(t)}$ but the envelope can become zero, e.g., for transition from -1 to +1 .



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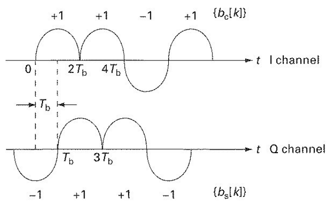
Minimum Shift Keying

- Minimum shift keying (MSK, both linear modulation and CPM)
- Transmission of 2 bits $b_c[k], b_s[k] \in \{\pm 1\}$

$$u(t) = \sum_k (b_c[k]p(t - 2kT_b) + jb_s[k]p(t - 2kT_b - T_b))$$

with $p(t) = \sin(\pi/T_s t) I_{[0,2T_b]}$ and $T_s = 2T_b$



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Minimum Shift Keying

- Alternative formulation (restricting to the interval $[0, 2T_b]$)

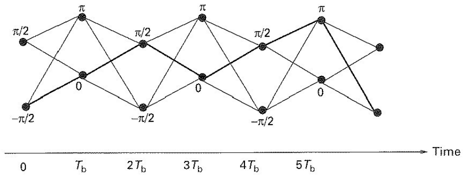
$$0 \le t \le T_b: \quad u(t) = b_c[0]\sin\frac{\pi t}{2T_b} + jb_s[-1]\cos\frac{\pi t}{2T_b} = e^{j(a[0]\frac{\pi t}{2T_b} + \frac{\pi}{2}b_s[-1])} = e^{jb_s[-1](\frac{\pi}{2} - b_c[0]\frac{\pi t}{2T_b})}$$

with $a[0] = -b_c[0]/b_s[-1] = -b_c[0] \cdot b_s[-1]$

$$T_b \le t \le 2T_b: \quad u(t) = b_c[0]\sin\frac{\pi t}{2T_b} - jb_s[1]\cos\frac{\pi t}{2T_b} = e^{j(a[1]\frac{\pi t}{2T_b} - \frac{\pi}{2}b_s[1])} = e^{jb_s[1](b_c[0]\frac{\pi t}{2T_b} - \frac{\pi}{2})}$$

with $a[1] = b_c[0]/b_s[1] = b_c[0] \cdot b_s[1]$.

- Observation: $\theta(t)$ is piecewise linear



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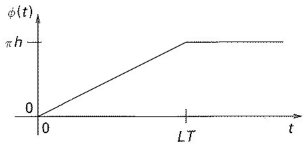
Minimum Shift Keying

- With the phase pulse

$$\phi(t) = \begin{cases} 0, & t < 0, \\ \frac{\pi t}{2T_b}, & 0 \le t \le T_b, \\ \frac{\pi}{2}, & t \ge T_b. \end{cases}$$



the the phase signal is given as

$$\theta(t) = \sum_n a[n]\phi(t - nT_b)$$

- Instantaneous frequency

$$f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} = \sum_n a[n]g(t - nT_b)$$

with the MSK frequency pulse

$$g(t) = \frac{1}{4T_b} I_{[0,T_b]}(t)$$

→ instantaneous frequency in MSK is modulated by a rectangular pulse.

Notes

Gaussian MSK

- Special case of MSK: the frequency pulse $g(t)$ is smoothed by a Gaussian filter (impulse response $h(t)$, frequency response $H(f)$)

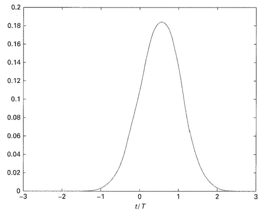
$$h(t) = \frac{1}{2\pi v^2} e^{-\frac{t^2}{2v^2}} \quad \text{and} \quad H(f) = \exp\left(-\left(\frac{f}{B}\right)^2 \frac{\log 2}{2}\right)$$

with $v^2 = \ln 2/(4\pi^2 B^2)$.

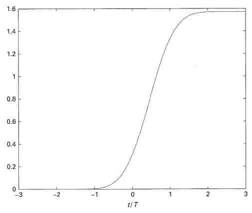
- G-MSK frequency pulse

$$g_{GMSK}(t) = \frac{1}{4T}\left[Q\left(\frac{t-T}{v}\right) - Q\left(\frac{t}{v}\right)\right]$$

Frequency pulse



Phase pulse



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Receiver Design

- Suboptimal approach: since the frequency is modulated by the symbols $a[n]$, FM demodulation to get $f(t)$ and linear demodulation of $f(t)$. $\rightarrow$ poor performance for dispersive channels!
- Maximum likelihood: Viterbi on the phase trellis (there are only 4 distinct values possible at integer multiples of T_b
  $\rightarrow$ high complexity for dispersive channels!
- Laurent approximation
  - CPM signals can be well approximated by linear modulation

$$u(t) = \exp\left(\sum_n a[n]\phi(t-nT)\right) \approx \sum_n B[n]s(t-nT)$$

  with pseudo symbols $B[n]$.

  $\rightarrow$ Modulation pulse is not a Nyquist pulse
  - For transmission over ISI channels with $g_c(t)$, detection of $B[n]$ using channel equalization techniques (e.g., MLSE, linear ZF/MMSE, DFE).
  - See example in the book!

Notes