Advanced Digital Communications (EQ2410)

Lecture 10, Period 3, 2016

Task 1 Verify the derivation of the decision regions shown on slide 4 which are based on the following example:

The ML rule must maximize the log likelihood ratio, which is proportional to

$$\Lambda(\mathbf{b}) = \langle \mathbf{r}, \mathbf{s_b} \rangle - \frac{1}{2}||\mathbf{s_b}||^2. \tag{8.78}$$

Since

$$\begin{aligned} \langle \mathbf{r}, \mathbf{s_b} \rangle &= A_1 b_1 \langle \mathbf{r}, \mathbf{s}_1 \rangle + A_2 b_2 \langle \mathbf{r}, \mathbf{s}_2 \rangle \\ &= A_1 b_1 z_1 + A_2 b_2 z_2, \end{aligned} \tag{8.79}$$

we realize that, in order to compute $\Lambda(\mathbf{b})$ for all possible $\mathbf{b}$ (four possible values in our case), it is necessary and sufficient to compute the matched filter statistics z_1 and z_2. The problem with conventional reception is that these statistics are being used separately as in (8.76), rather than jointly according to the ML rule

$$\hat{\mathbf{b}}_{\mathrm{ML}} = \arg \max_{\mathbf{b}} \langle \mathbf{r}, \mathbf{s_b} \rangle - \frac{1}{2}||\mathbf{s_b}||^2. \tag{8.80}$$

Let us get the ML rule into a more explicit form. Consider the second term above:

$$\begin{aligned} ||\mathbf{s_b}||^2 &= ||A_1 b_1 \mathbf{s}_1 + A_2 b_2 \mathbf{s}_2||^2 \\ &= A_1^2 b_1^2 ||\mathbf{s}_1||^2 + A_2^2 b_2^2 ||\mathbf{s}_2||^2 + 2A_1 A_2 b_1 b_2 \langle \mathbf{s}_1, \mathbf{s}_2 \rangle \\ &= A_1^2 + A_2^2 + 2A_1 A_2 b_1 b_2 \rho. \end{aligned} \tag{8.81}$$

Throwing away terms independent of $\mathbf{b}$, we obtain, using (8.79) and (8.81) in (8.80), that

$$(\hat{b}_1, \hat{b}_2)_{\mathrm{ML}} = \arg \max_{b_1, b_2} A_1 b_1 z_1 + A_2 b_2 z_2 - A_1 A_2 b_1 b_2 \rho. \tag{8.82}$$

By writing out and comparing the terms above for the four possible values of $(b_1, b_2) = (\pm 1, \pm 1)$, we get the ML decision regions as a function of z_1, z_2. For example, consider $A_1 = 1$, $A_2 = 2$ and $\rho = -1/2$. The ML decision is $(+1, +1)$ if the following three inequalities hold:

$$\begin{aligned} z_1 + 2z_2 + 1 &> z_1 - 2z_2 - 1, \\ z_1 + 2z_2 + 1 &> -z_1 - 2z_2 + 1, \\ z_1 + 2z_2 + 1 &> -z_1 + 2z_2 - 1, \end{aligned}$$

which reduces to $z_2 > -1/2$, $z_1 + 2z_2 > 0$ and $z_1 > -1$. By doing this for all possible values of (b_1, b_2), we obtain the decision regions shown in Figure 8.9. In contrast, the MF decision regions are simply the four quadrants in the figure, since they do not account for the correlation between the spreading waveforms. The relative performance of the MF and ML rules is discussed in Example 8.4.3.

Task 2 Discuss how successive interference cancellation ("decision feedback multi-user detection") can be realized.

Task 3

(a) Show that for $0 \le t \le T_b$ and with $a[0] = -b_c[0]/b_s[-1] = -b_c[0] \cdot b_s[-1]$ we have

$$\begin{aligned} u(t) &= e^{j\theta(t)} \\ &= b_c[0] \sin \frac{\pi t}{2T_b} + j b_s[-1] \cos \frac{\pi t}{2T_b} \\ &= e^{j(a[0]\frac{\pi t}{2T_b} + \frac{\pi}{2} b_s[-1])} \\ &= e^{j b_s[-1](\frac{\pi}{2} - b_c[0]\frac{\pi t}{2T_b})}. \end{aligned}$$

(Hint: Show that for $a \in \{\pm 1\}$ $a \cdot \sin(x) = \sin(ax)$ and $\cos(ax) = \cos(x)$.)

(b) In a similar way we can show that for $T_b \le t \le 2T_b$ and with $a[1] = b_c[0]/b_s[1] = b_c[0] \cdot b_s[1]$ we get

$$u(t) = e^{j\theta(t)} = e^{j b_s[1](b_c[0]\frac{\pi t}{2T_b} - \frac{\pi}{2})}.$$

Show that for integer multiples of T_b the phase $\theta(t)$ only takes on values $\theta(kT_b) \in \{-\pi/2, 0, \pi/2, \pi\}$.

(c) Verify the construction of the phase trellis. Is there a connection between the symbols and the states?