Department of Mathematics
SF2729 Groups and Rings
Period 3, 2016

HOMEWORK 12

Submission. The solutions should be typed and converted to .pdf. Deadline for submission is Monday March 7, 14.00. Either hand in the solutions in class, in the black mailbox for homework outside the math student office at Lindstedtsvägen 25, or by email to `skjelnes@kth.se`.

Score. For each set of homework problems, the maximal score is 3 points, and calulated as $\min\{3, \Sigma/2\}$, where Σ is the score obtained on the homework. The total score from all twelve homeworks will be divided by four when counted towards the first part of the final exam.

Problem 1. Let $R = \mathbb{Z}[y]/(y^2 - 2)$, which is a UFD. Show that $f(x) = x^2 - \sqrt{2} \in R[x]$ is irreducible.

(3 p)

Problem 2. Let p be an odd prime, and let $F = \mathbb{Z}/(p)$. Show that the polynomial $f(x) = x^2 + 1$ in $F[x]$ is reducible if and only if $p \equiv 1 (\mathrm{mod}\, 4)$. **(3 p)**

Problem 3. Let R be the ring of polynomials in $\cos(t)$ and $\sin(t)$, with real coefficients.

(a) Show that R is isomorphic to $\mathbb{R}[x, y]/(x^2 + y^2 - 1)$. **(1 p)**
(b) Show that R is not a unique factorization domain. **(1 p)**
(c) Show that $\mathbb{C}[x, y]/(x^2 + y^2 - 1)$ is a PID, and hence a UFD. **(1 p)**