Advanced Digital Communications (EQ2410)

Lecture 11, Period 3, 2016

Task 1 Consider a MIMO system with the following channel matrix:

$$\boldsymbol{H} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(a) Calculate $\boldsymbol{W} = \boldsymbol{H}^H \boldsymbol{H}$.

(b) Calculate the eigenvalues of $\boldsymbol{W}$.

(c) Calculate the eigenvectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ of $\boldsymbol{W}$ such that $\boldsymbol{V}^H \boldsymbol{V} = \boldsymbol{I}$ with $\boldsymbol{V} = (\boldsymbol{v}_1, \boldsymbol{v}_2)$.

(d) Calculate $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ and verify that $\boldsymbol{u}_i^H \boldsymbol{u}_j = \delta_{ij}$

(e) Verify now that $\boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^H = \boldsymbol{H}$.

(f) For a power constraint $E[\|\hat{\boldsymbol{x}}\|^2] = 2/3$ and $2\sigma^2 = 1$, calculate the optimal power allocation that maximizes the capacity.

How much do we gain compared to a uniform power allocation?