

Tutorial #6: Scalar and Vector Quantization

Problem 1

Assume speech is modeled as a first order AR process

$$x(n) = ax(n-1) + u(n)$$

where $u(n)$ is a white, zero-mean, unit variance process. To make the model more realistic, we assume that a is time varying. Assume that if we extract a frame of length N , then for 50% of the frames, $a = .95$, and for the other 50% of the frames, $a = 0$ (unvoiced). Assume that N is large such that time averages can be replaced by expectations. , e.g., the short-term ACF (normalized by the frame length) can be replaced by the stochastic ACF.

1. Design a 6-bit uniform quantizer (PCM). To ensure negligible overload distortion, set $x_{\max} = 8\sigma_X$, where σ_X^2 is the long-term variance of speech.
2. Calculate the ACF measure over a long time, i.e. over a lot frames.
3. Design a first order predictor that minimizes the variance of prediction error
4. Design a 6-bit per sample DPCM system based on predictor from that last part. To ensure negligible overload, set $x_{\max} = 8\sigma_E$, where σ_E^2 is the prediction error variance. Assume the rate to be high enough, such that quantization errors can be neglected. Calculate the SNR and predication gain G_p
5. Design a 6-bit per sample ADPCM system, i.e. a DPCM system with a forward adaptive first order predictor. Neglect side information for the predictor coefficient. Calculate the SNR
6. If the frame length is 240 samples, and we use 5 bits for the predictor coefficient, what is the total rate of the system (in bits/sample) ?

Problem 2

Ten samples are collected from the following random process

$$x(n) = \rho x(n-1) + \sqrt{1 - \rho^2} u(n),$$

where $u(n)$ is a white stationary Gaussian sequence,

$$\mathbf{x} = \{.94, .89, -.24, -.44, -1.04, -.95, -.12, .22, 1.12, .12\}$$

Two different quantization schemes will be used for this sequence.

1. A scalar pdf-optimized scalar quantizer, with two levels, $y_0 = -0.56$ and $y_1 = +0.56$. Determine the quantized output sequence $\hat{\mathbf{x}}$, and the quantization error energy.
2. We divide the sequence into blocks of 2 samples each, and employ a pdf-optimized vector quantizer, with levels $y_0 = [-.31, .31], y_1 = [.3, -.32], y_2 = [-.8, -.79], y_3 = [.8, .79]$. Plot the vectors to be quantized, in the 2D space, and determine the quantized output sequence $\hat{\mathbf{x}}$. Note that this vectors quantized needs the same about of bits/sample, as the scalar quantizer.
3. Compare the quantization errors in 1. and 2. for this sequence. Is the comparison representation for the process in general ? Why is that ?