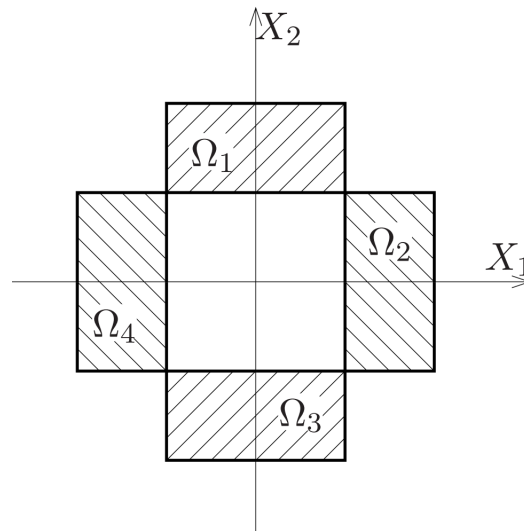


Tutorial #7: Vector Quantization

Problem 1

Consider a random source for which pairs of samples are distributed according to the pdf below. All the rectangular areas, i.e. $\Omega_1, \Omega_2, \Omega_3, \Omega_4$, have the same dimensions: width of 1 and height of 2. The joint pdf, $f_{X_1, X_2}(x_1, x_2)$, is $1/8$ for all the shaded areas, and zero everywhere else.



1. Derive the marginal density. Recall that the marginal density is defined as $f_{X_1}(x_1) = \int f_{X_1, X_2}(x_1, x_2) dx_2$
2. From the marginal pdf, design an optimal scalar quantizer operating at 1 bit/sample. Calculate the MSE of the quantizer.
3. Design a 2-dimensional VQ with a rate of 1 bit/sample, with outperforms the scalar one. Show that your VQ is superior by computing the distortion and comparing it 2.

Problem 2

Analyze the complexity of 2-stage VQ. Assume the first stage has K -bits, and the second stage has $24 - K$ bits. Measure the computational complexity in terms of the total number of codevectors that need to be tested during encoding.

1. Sketch the complexity as a function of K . Use a logarithmic scale for the complexity.
2. What value of K minimizes the computational complexity ?
3. What value of K minimizes the expected distortion ?

Problem 3

Consider split VQ of LP parameter vectors. We wish to encode 10-dimensional vectors of line spectral frequencies. We use a 2-split, where the first split is 6-dimensional, and the second is 4-dimensional. A total of 24 bits is used to quantize a whole vector.

1. How should the bits be allocated between the splits to minimize computational complexity ?