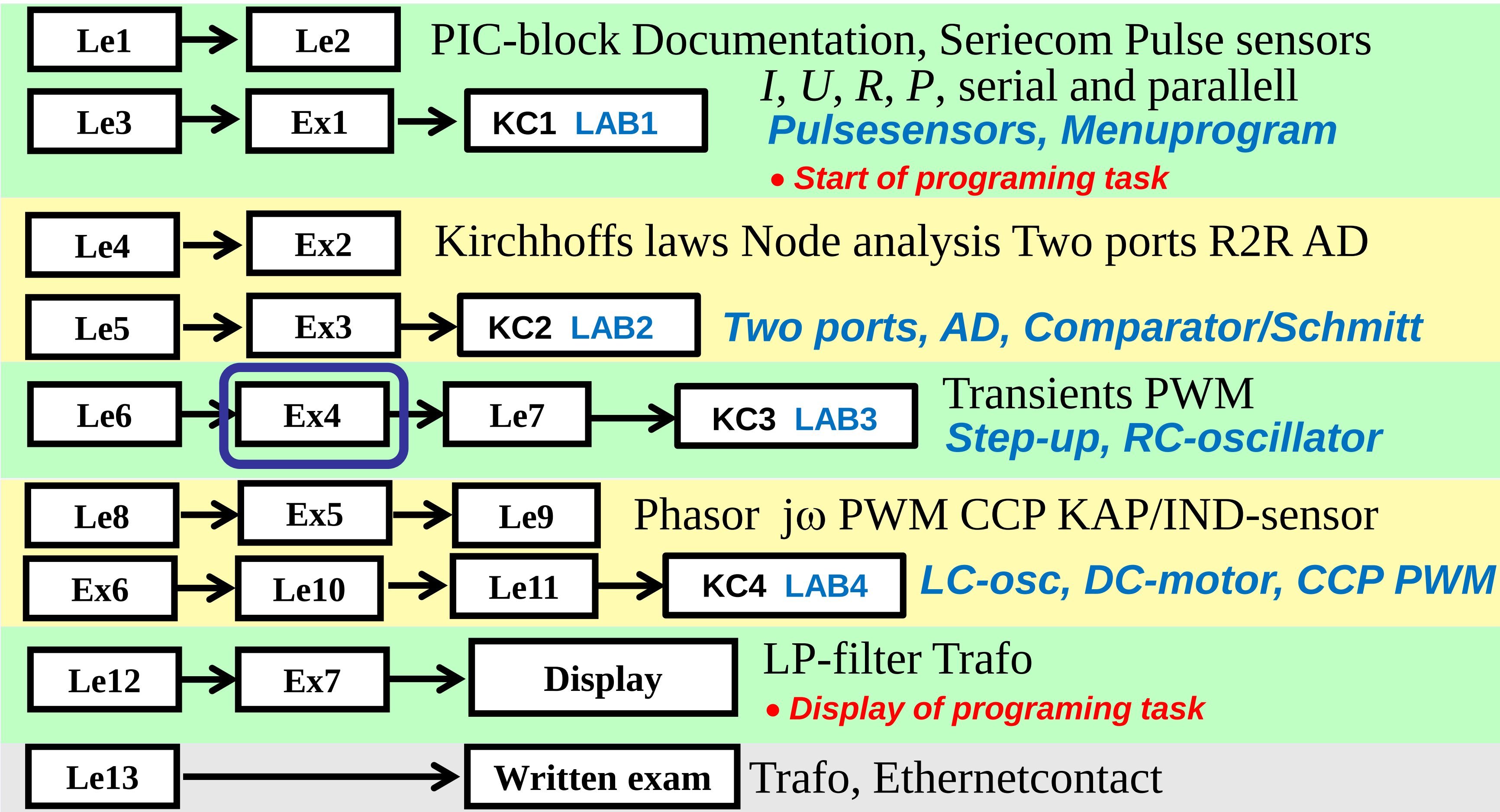
IE1206 Embedded Electronics



William Sandqvist william@kth.se

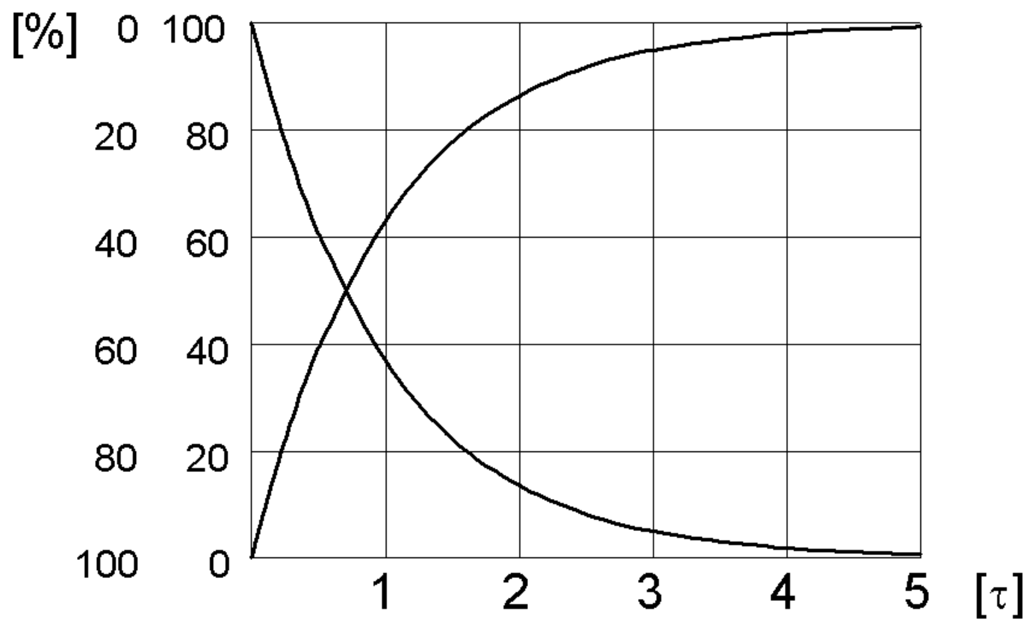
Quick Formula for exponential

- Rising process

$$x(t) = 1 - e^{-\frac{t}{\tau}}$$

- Falling process

$$x(t) = e^{-\frac{t}{\tau}}$$

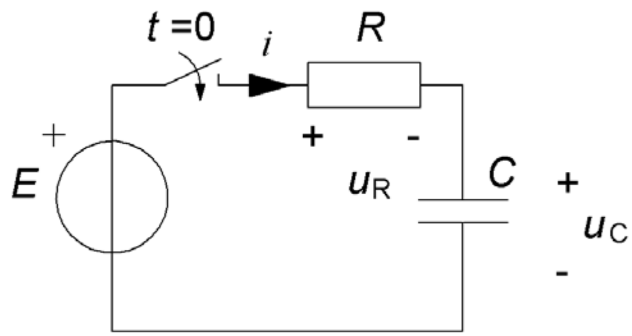


The Quick Formula directly provides the equation for a rising/falling exponential process:

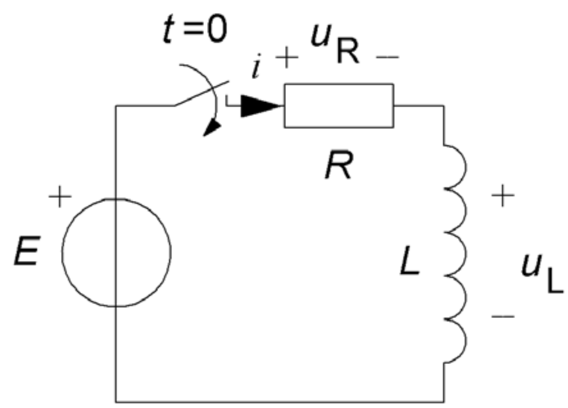
x_0 = process start value
x_∞ = process end value
τ = process time constant

$$x(t) = x_\infty - (x_\infty - x_0)e^{-\frac{t}{\tau}}$$

Time constants



$$\tau = R \cdot C$$

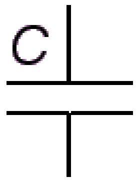


$$\tau = \frac{L}{R}$$

• More complex circuits one simplifies with equivalent circuits to one of these elementary shapes. (If this is not possible advanced courses will have a transform method available).

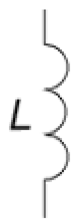
Continuity requirements

Summary



The Capacitor has voltage inertia

In a capacitor, charging is always continuous
The capacitor **voltage is always continuous**.

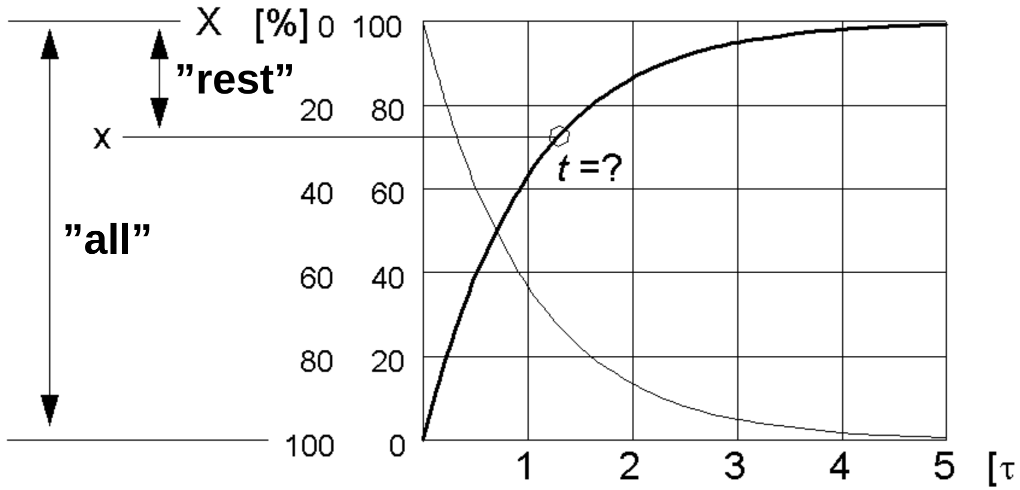


The Inductor has current inertia

In an inductor the magnetic flux is always continuous
In an inductor **current is always continuous**.

William Sandqvist william@kth.se

”All” by ”the rest”



$$x = X(1-e^{-\frac{t}{\tau}}) \quad \Rightarrow \quad \frac{x}{X} = 1-e^{-\frac{t}{\tau}} \quad \Rightarrow \quad \ln\left(1-\frac{x}{X}\right) = -\frac{t}{\tau} \quad \Rightarrow \quad t = -\tau \cdot \ln\frac{X-x}{X}$$

$$\boxed{t} = \tau \cdot \ln\frac{X}{X-x} = \boxed{\tau \cdot \ln\frac{\text{"all"}}{\text{"rest"}}}$$

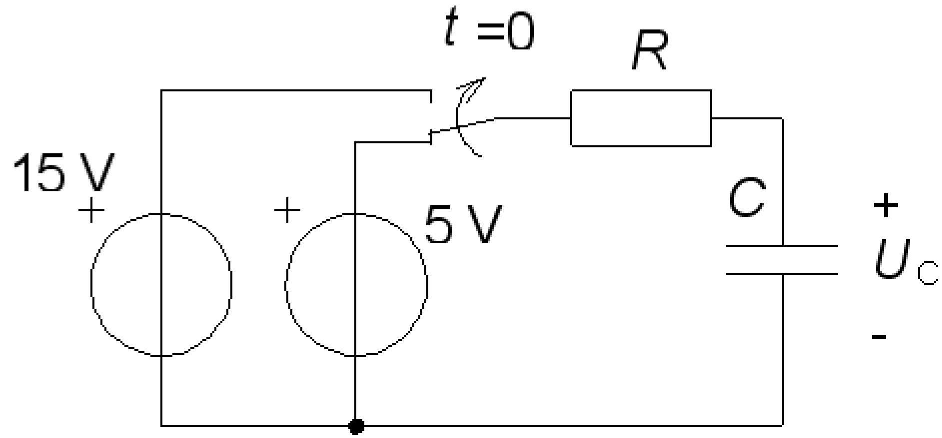
Capacitor charging (10.5)

R = 2000 Ω and C = 1000 μF

Obtain an expression for $u_C(t)$

Draw function $u_C(t)$

Calculate how long it takes for u_C to reach +10V?



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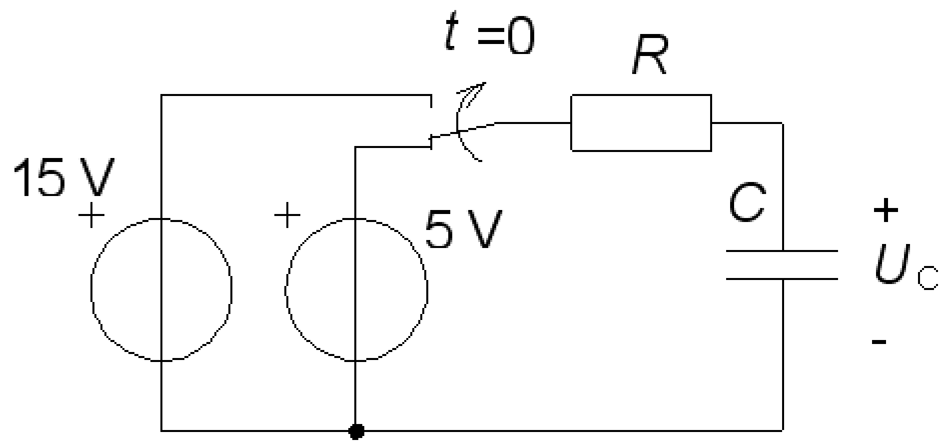

$u_{C0} = 5$ V
$u_{C\infty} = 15$ V
$\tau = 2000 \cdot 1000 \cdot 10^{-6} = 2$ s

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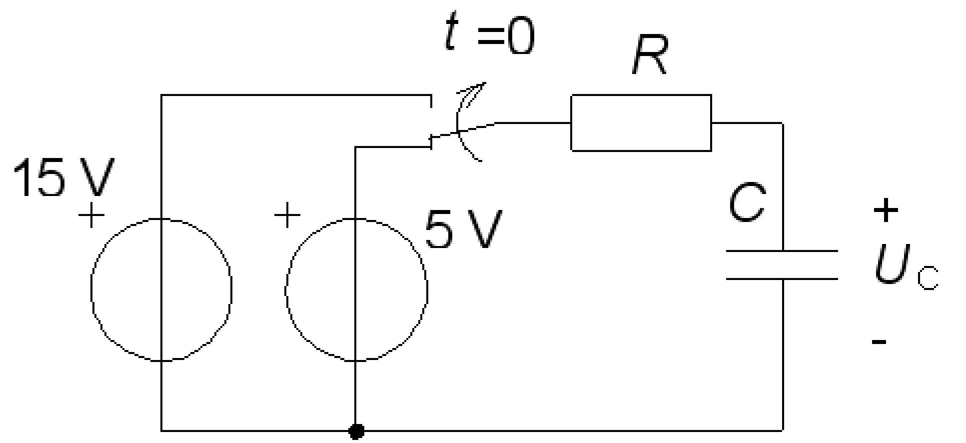

u_{C0} = 5 V
$u_{C\infty}$ = 15 V
τ = 2000·1000·10^{-6} = 2 s

$$x(t) = x_\infty - (x_\infty - x_0)\cdot e^{-\frac{t}{\tau}}$$

$$u_C(t) = 15 - (15-5)\cdot e^{-\frac{t}{2}} = 15 - 10\cdot e^{-0,5\cdot t}$$

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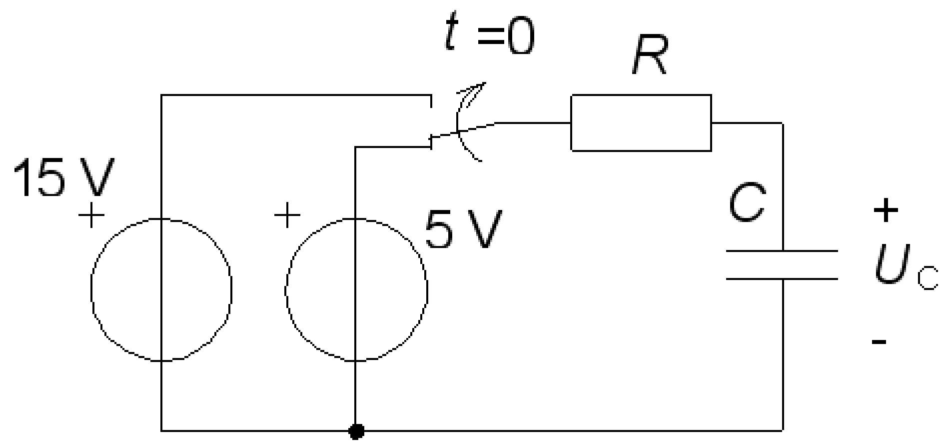

u_{C0} = 5 V
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$$x(t) = x_\infty - (x_\infty - x_0) \cdot e^{-\frac{t}{\tau}}$$

$$u_C(t) = 15 - (15 - 5) \cdot e^{-\frac{t}{2}} = 15 - 10 \cdot e^{-0,5 \cdot t}$$

Note: Capacitor voltage is continuous – If you put a voltage across a capacitor it can not charge instantaneously (would require infinite current). The voltage will not change at once.

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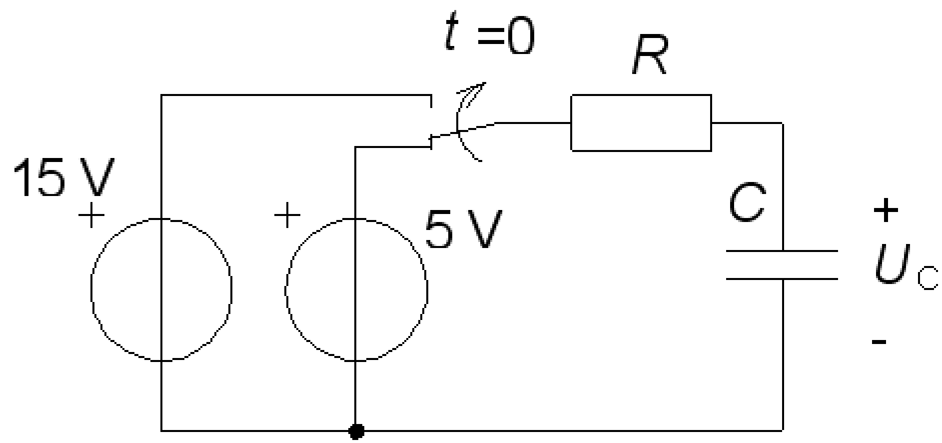

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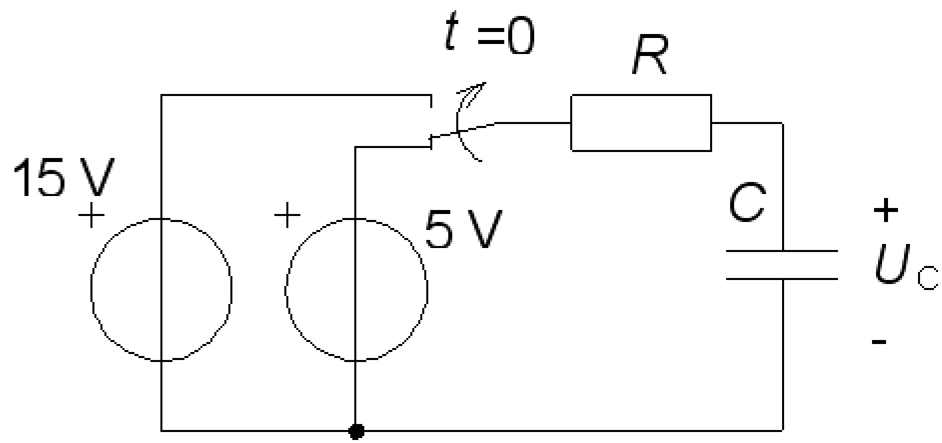

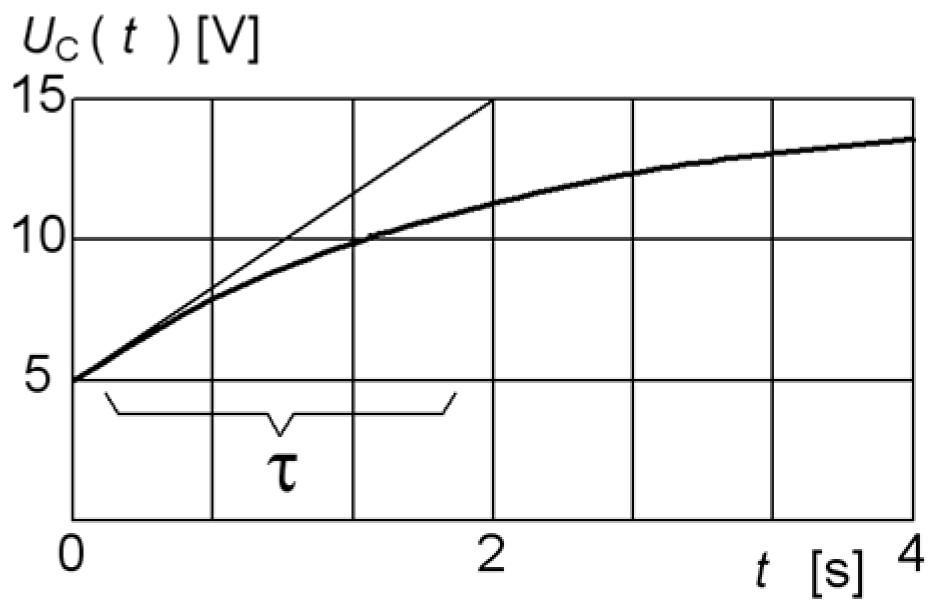

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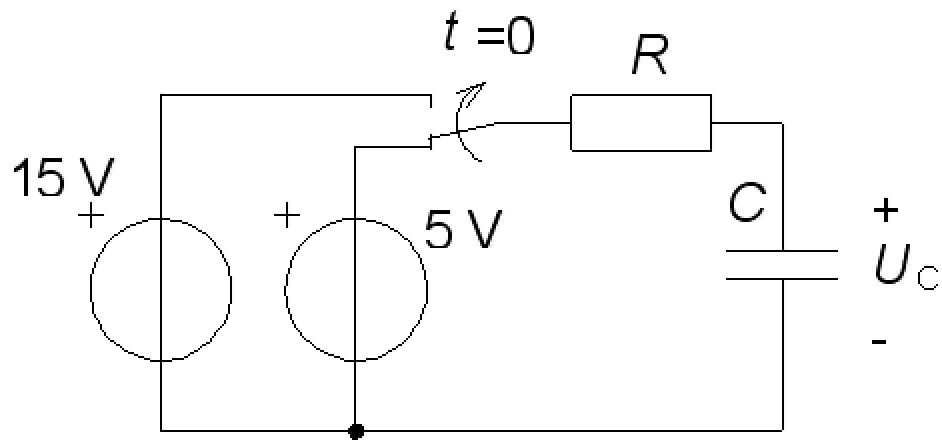

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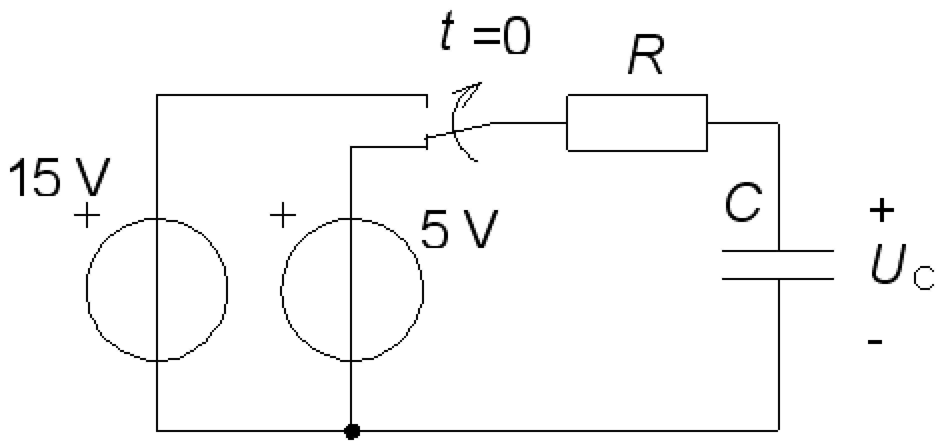

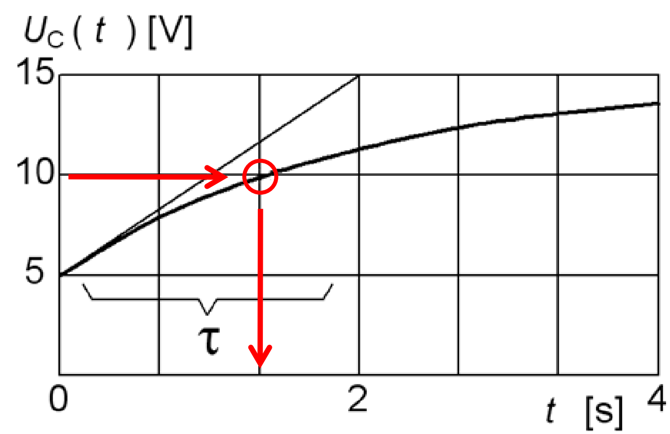

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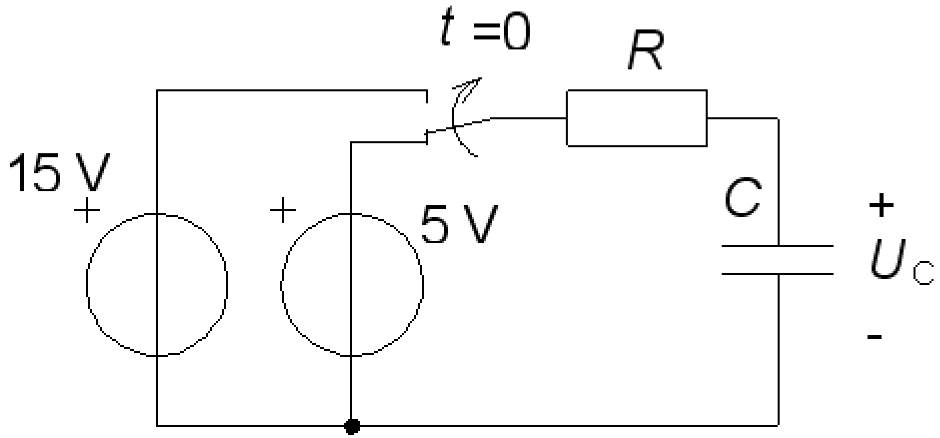
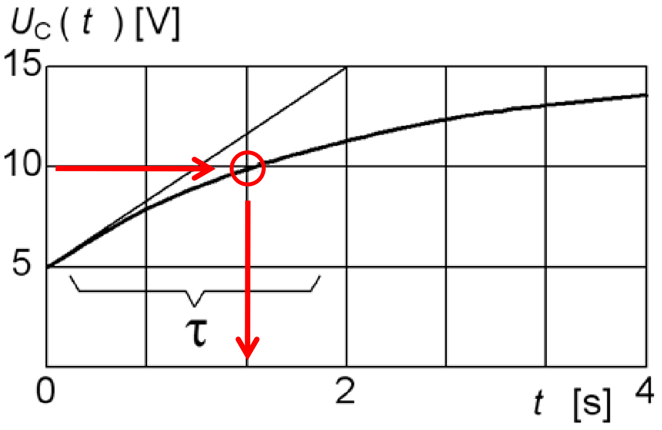
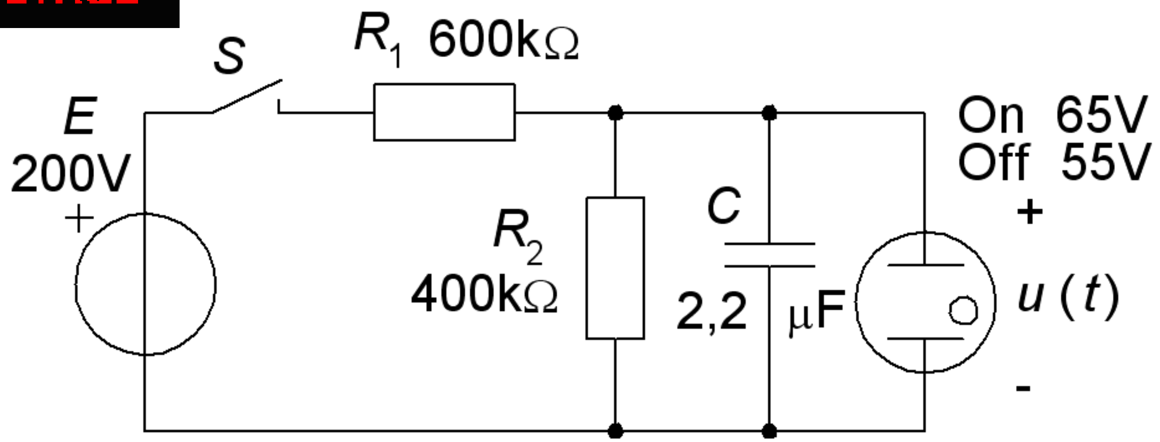


$$t = \tau \cdot \ln \frac{\text{"all"}}{\text{"rest"}} = 2 \cdot \ln \frac{15-5}{15-10} =$$

$$= 2 \cdot 0{,}695 = 1{,}39 \text{ s}$$



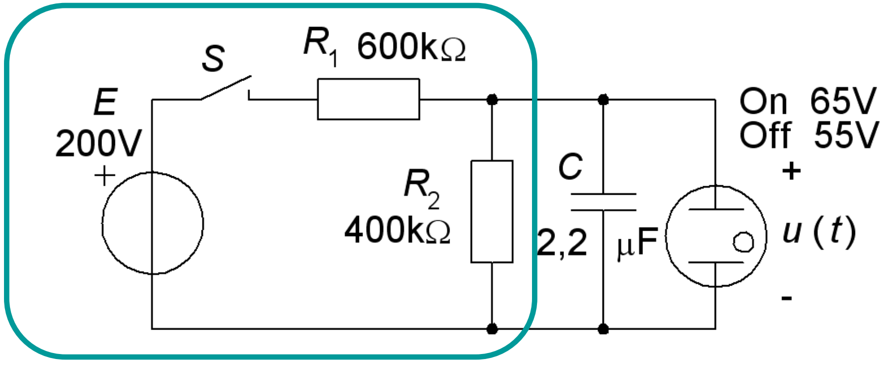
Neon lamp (10.9)



Flash-circuit with neon lamp.

Neon lamp (10.9)

a) *When will the first flashing light be?*



The circuit's Thevenin equivalent:
R_I = 600||400 = 240 kΩ
E_0 = 200·400/1000 = 80V

The capacitor is charged from 0V up to 80V at 65V the neon lamp lights up (and discharges the capacitor to 55V when it goes off).

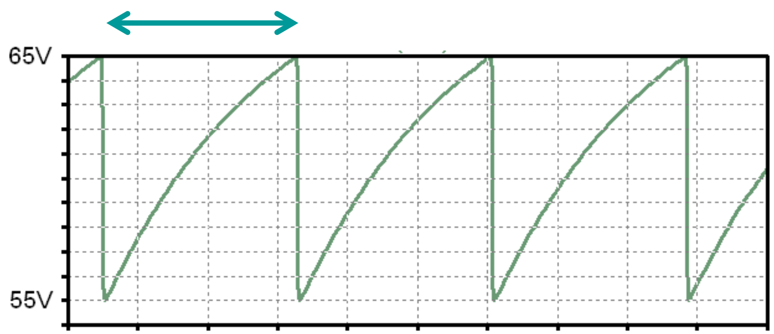
$$\tau = R_I \cdot C = 240 \cdot 10^3 \cdot 2{,}2 \cdot 10^{-6} = 0{,}528$$

$$t = \tau \cdot \ln \frac{all}{rest} = 0{,}528 \cdot \ln \frac{80-0}{80-65} = 0{,}88\ s$$

Neon lamp (10.9)

b) *How long will it take until the next blink?*

The capacitor is now charging from 55V up to 80V at 65V when the neon lamp lights up (and discharges the capacitor to 55V, then it goes off).



$$\tau = 0{,}528$$

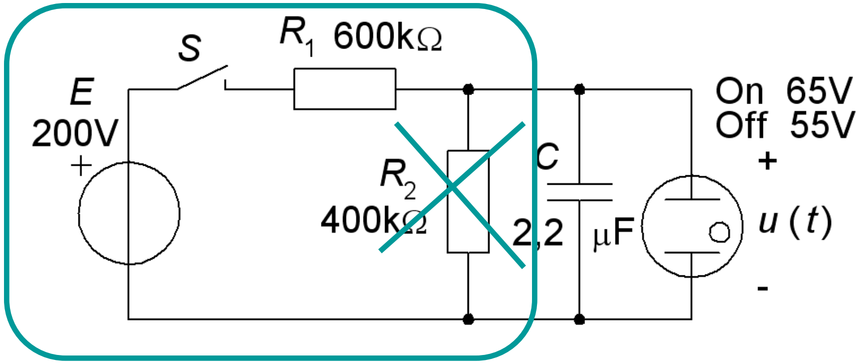
$$t = \tau \cdot \ln \frac{all}{rest} = 0{,}528 \cdot \ln \frac{80-55}{80-65} = 0{,}27\ s$$

Flash frequency:

$$f = \frac{1}{T} = \frac{1}{0{,}27} = 3{,}7\ \text{Hz}$$

Neon lamp (10.9)

c) *If* R_2 *is removed, how long does it then between flashes?*



If R_2 is removed E will not be votage divded.
E = 200.
Timeconstant will be changed.

The capacitor is charging now from 55V up to 200V at 65V when the neon lamp lights up (and discharges the capacitor to 55V when it goes off).

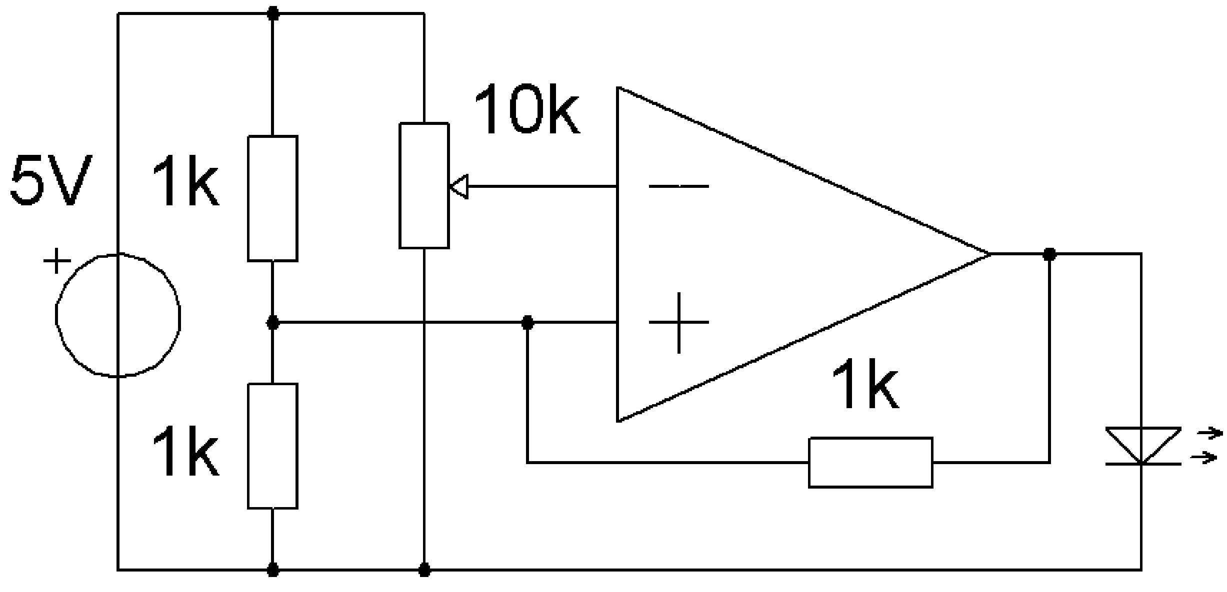
$$\tau = R_1 \cdot C = 600 \cdot 10^3 \cdot 2{,}2 \cdot 10^{-6} = 1{,}32$$

$$t = \tau \cdot \ln \frac{all}{rest} = 1{,}32 \cdot \ln \frac{200-55}{200-65} = 0{,}094\ s$$

Flash frequency:

$$f = \frac{1}{T} = \frac{1}{0{,}094} = 11\,\text{Hz}$$

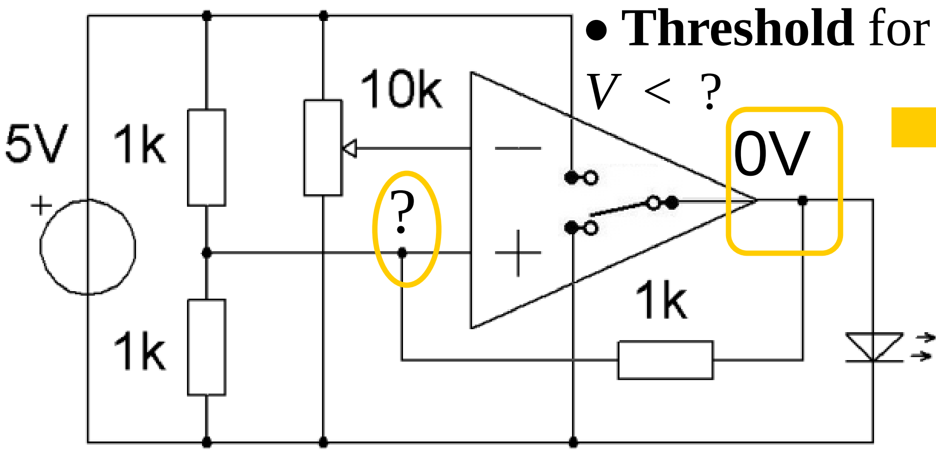
Schmitt-trigger (10.10)

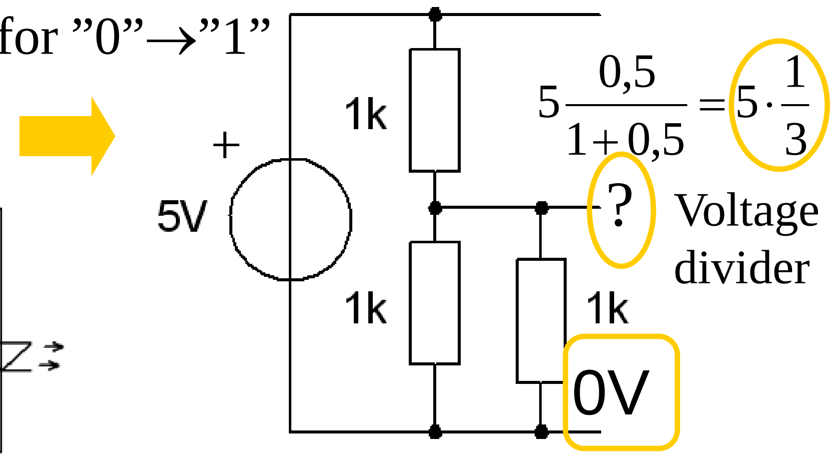


Trigger levels? (10.10)

- **Threshold** for ”0”→”1”

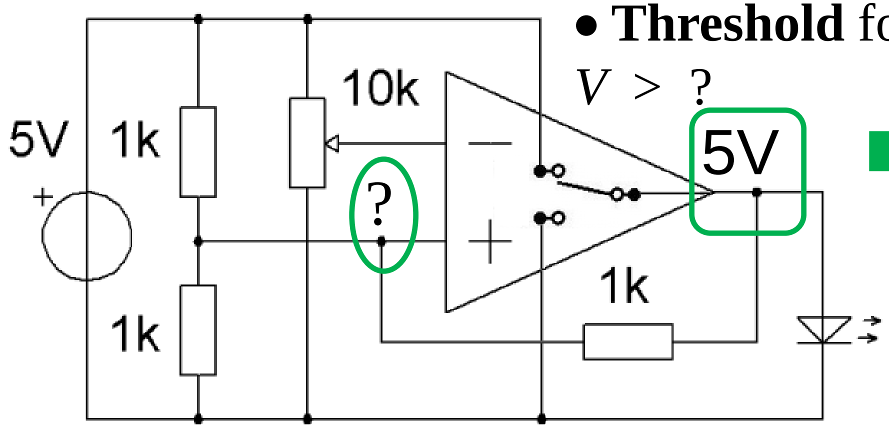
$V < ?$

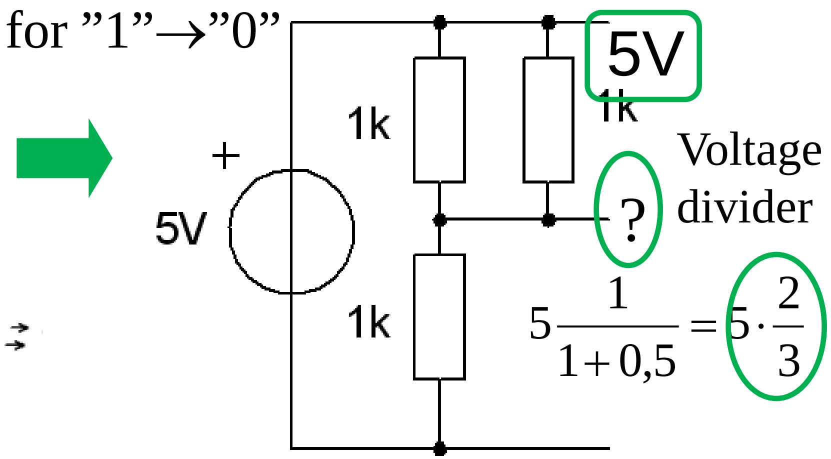




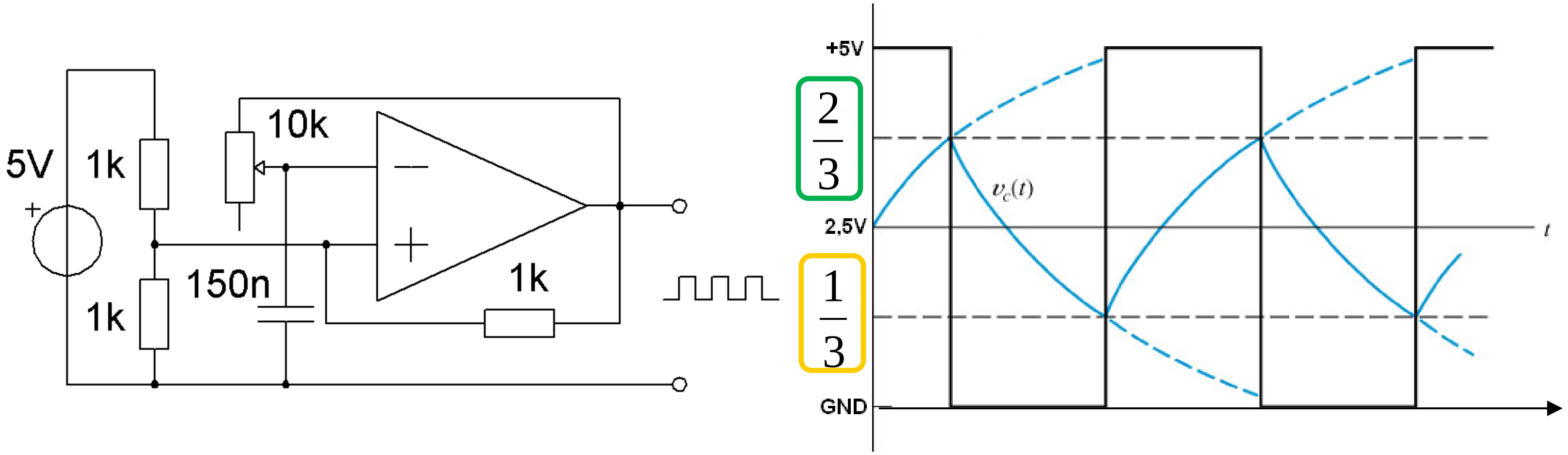
- **Threshold** for ”1”→”0”

$V > ?$



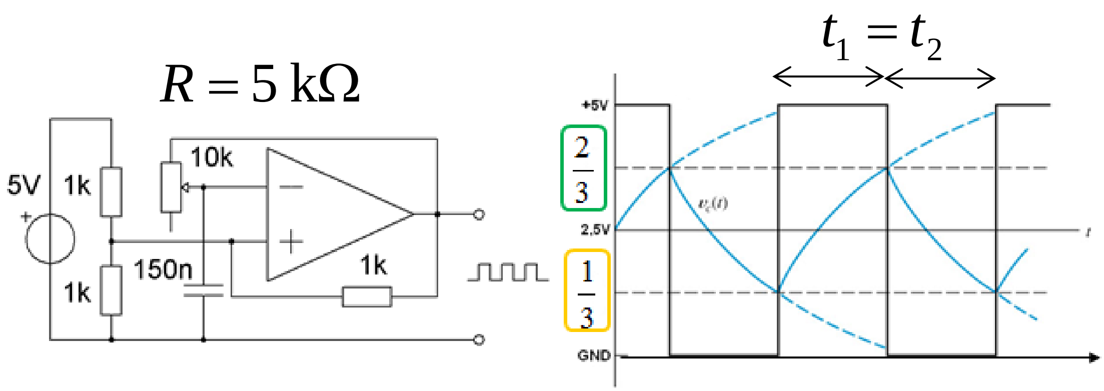


RC-oscillator (10.10)



The comparator charges the capacitor to the upper trigger level, then it turns the output on and discharges the capacitor to the lower trigger level. The frequency of the output of the comparator depends on the product $R \cdot C$. Since C is constant so will the R controls the frequency.

RC-oscillator frequency (10.10)



$$\tau = R \cdot C = 5 \cdot 10^{3} \cdot 150 \cdot 10^{-9} = 0{,}75 \cdot 10^{-3}$$

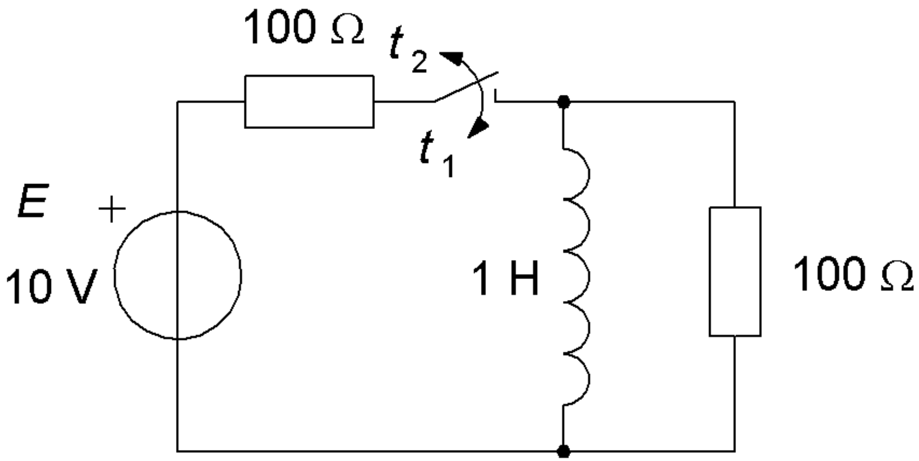
$$t_1 = \tau \cdot \ln \frac{all}{rest} = 0{,}75 \cdot 10^{-3} \cdot \ln \frac{5 - \frac{1}{3} \cdot 5}{\frac{1}{3} \cdot 5} = 0{,}75 \cdot 10^{-3} \cdot \ln 2 = 5{,}2\ \text{ms}$$

$$t_2 = t_1 \quad T = 2 \cdot t_1 = 2 \cdot 5{,}2 \cdot 10^{-3} = 10{,}4\ \text{ms} \quad f = \frac{1}{T} = \frac{1}{10{,}4 \cdot 10^{-3}} = 962\ \text{Hz}$$

The supply voltage 5V went shorten away. The frequency is thus independent of changes in the supply voltage!

Inductor **connection** and disconnection (10.8)

E is a DC source. At the time t_1 the switch is closed.



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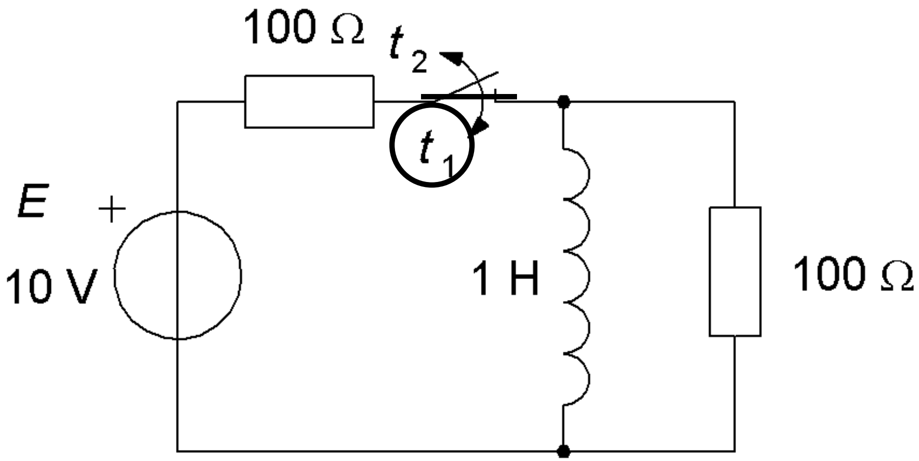

a) How large is the current through the coil in the first moment?

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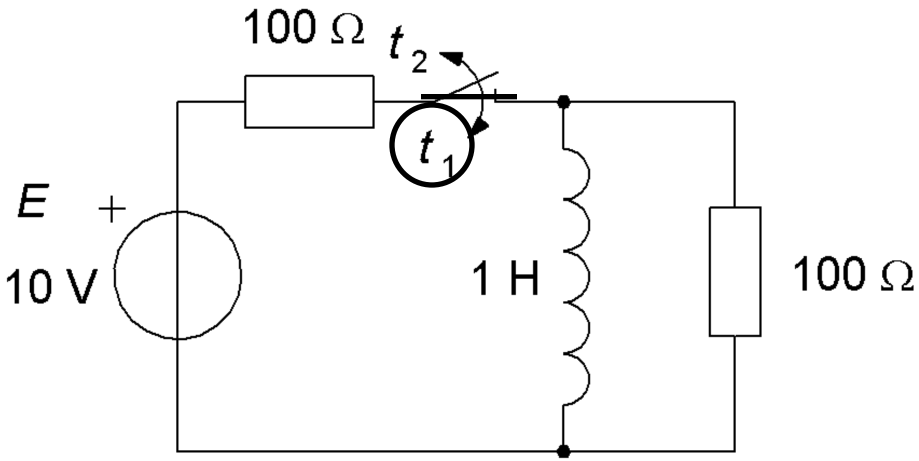
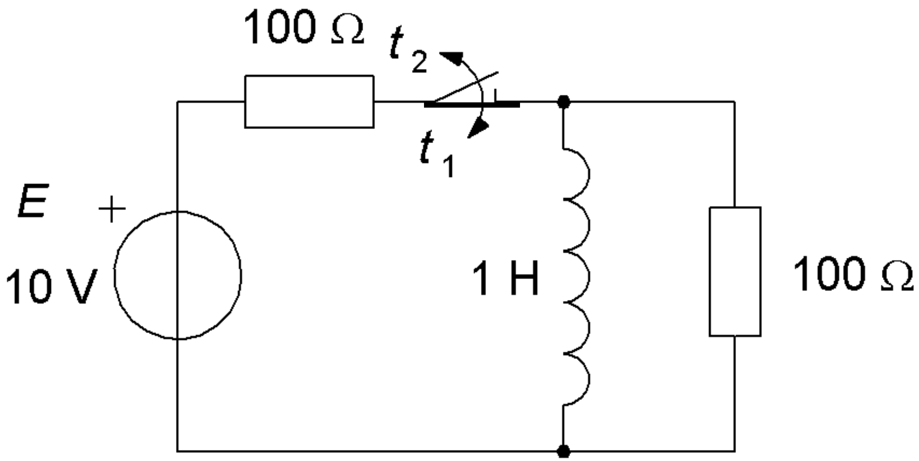


a) How large is the current through the coil in the first moment?

Answer: The inductor has has ”current inertia”. The first moment (t_1) the current will be the ”same” $i = 0$.

Inductor **connection** and disconnection (10.8)

b) How large is the current through the inductor after a long time interval?



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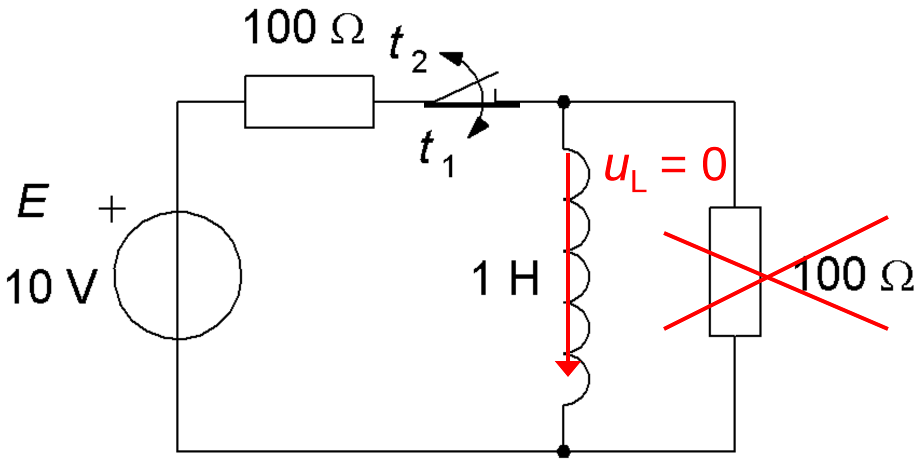

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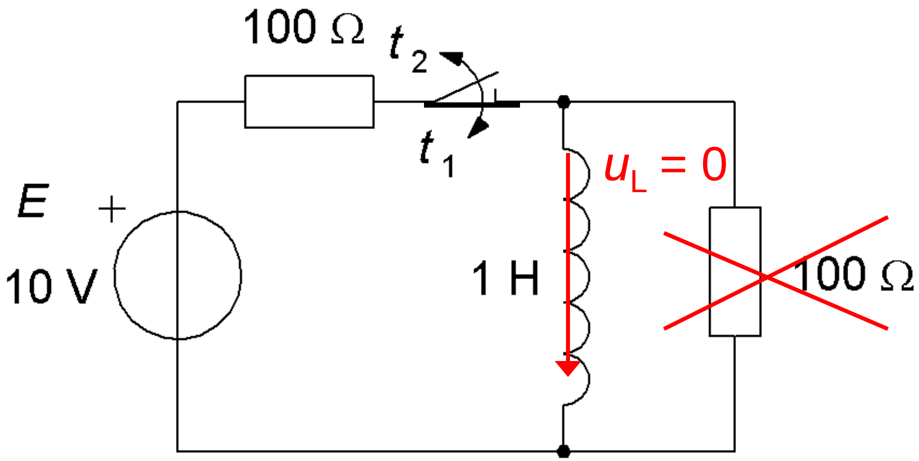

Answer: After a long time, the changes have faded away. The voltage across the inductor (is due to changes) then is 0, the inductor is "shorting" the 100 Ω parallel resistor. The 100 Ω series resistor limits the current from the voltage source. i = 10V/100Ω = 0,1 A.

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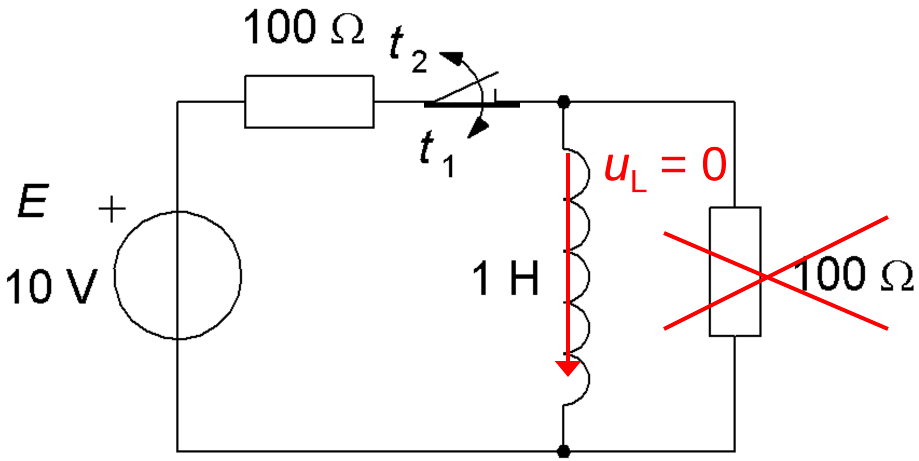

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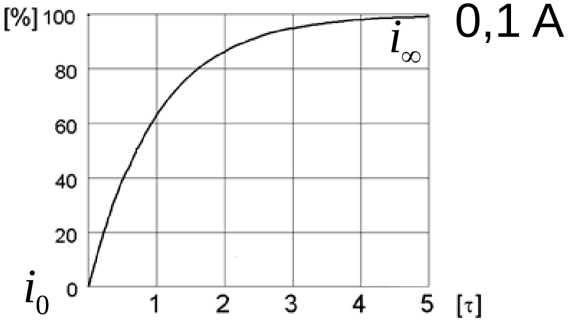
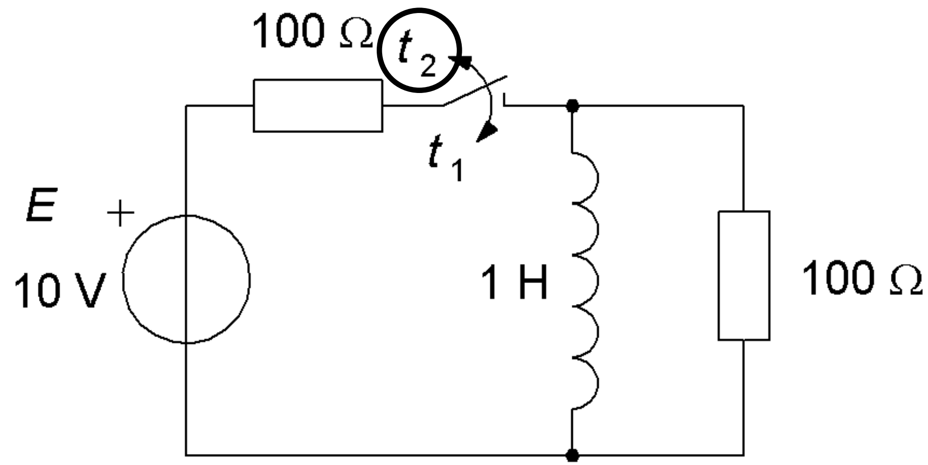


William Sandqvist william@kth.se

Inductor connection and **disconnection** (10.8)

c) Later at time t_2 the switch is ***opened***.

Now set up an expression of current through the coil as a function of time t for the time after t_2. Let t_2 be a new starting time $t = t_2 = 0$.



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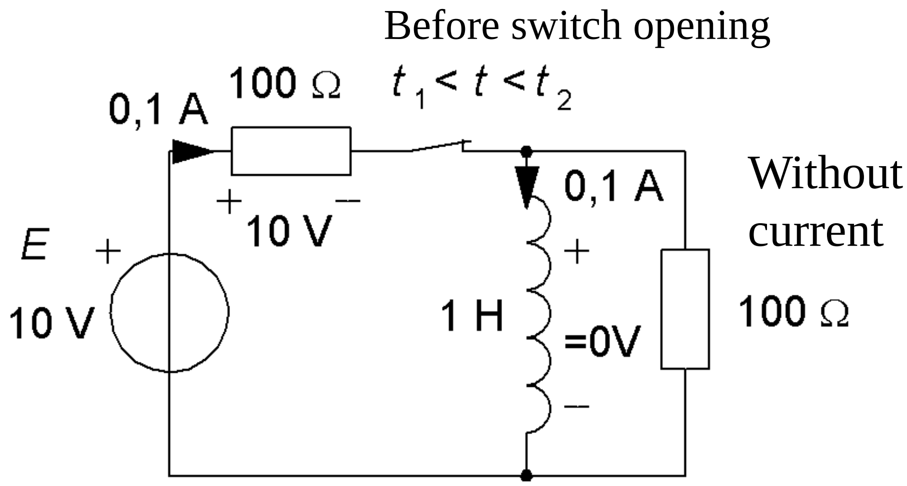
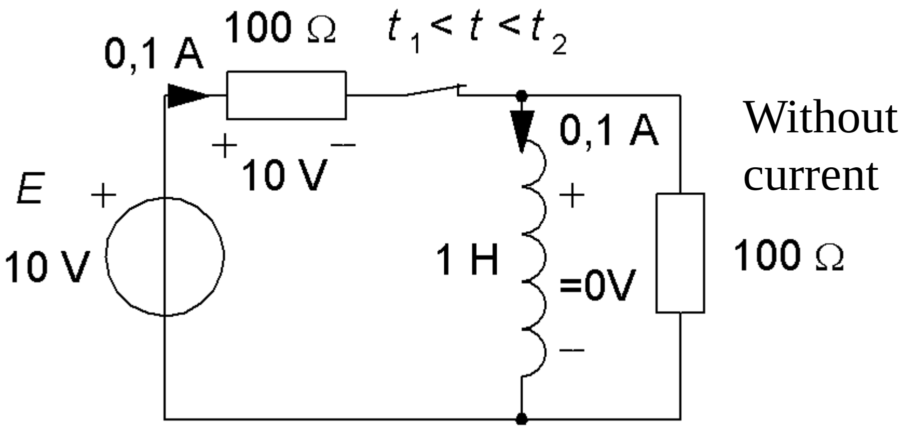
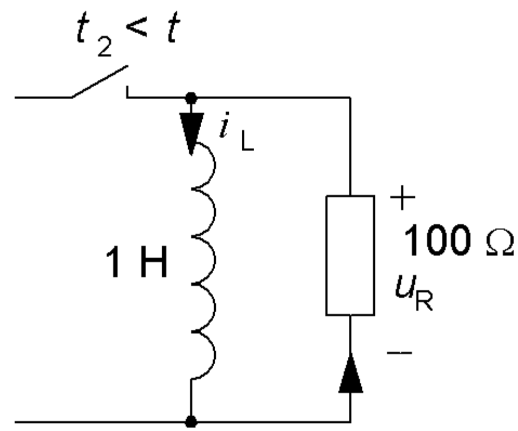


Inductor connection and **disconnection** (10.8)

Before switch opening



After switch opening



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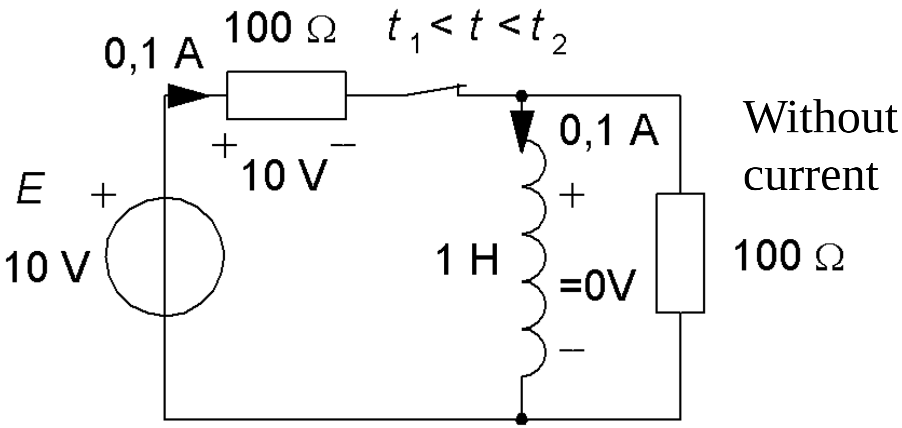
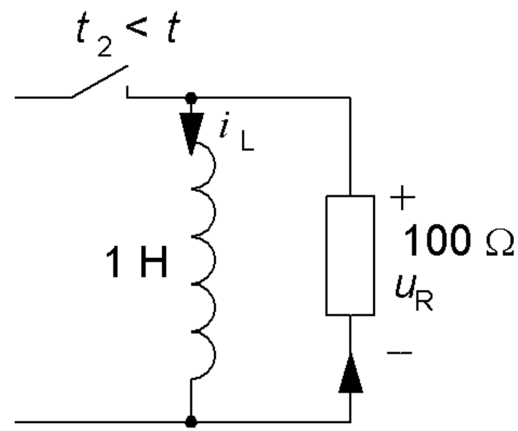


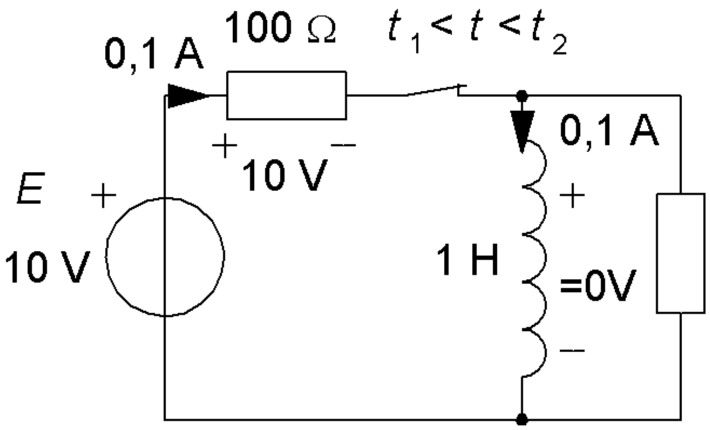
After switch opening



After t_2 the current starts from the ”same value” 0,1 A (i_0) as before the switch opening, and then the current will decrease down to 0 (i_∞).
Time constant will be $\tau = L/R = 1/100 = 0{,}01$ s.

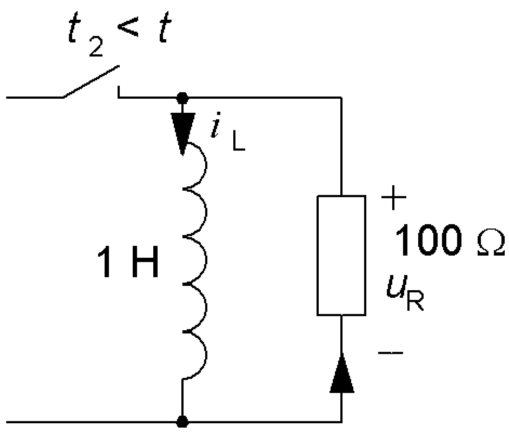
Inductor connection and **disconnection** (10.8)

Before switch opening



Without current

After switch opening



After t_2 the current starts from the ”same value” 0,1 A (i_0) as before the switch opening, and then the current will decrease down to 0 (i_∞). Time constant will be $\tau = L/R = 1/100 = 0,01$ s.

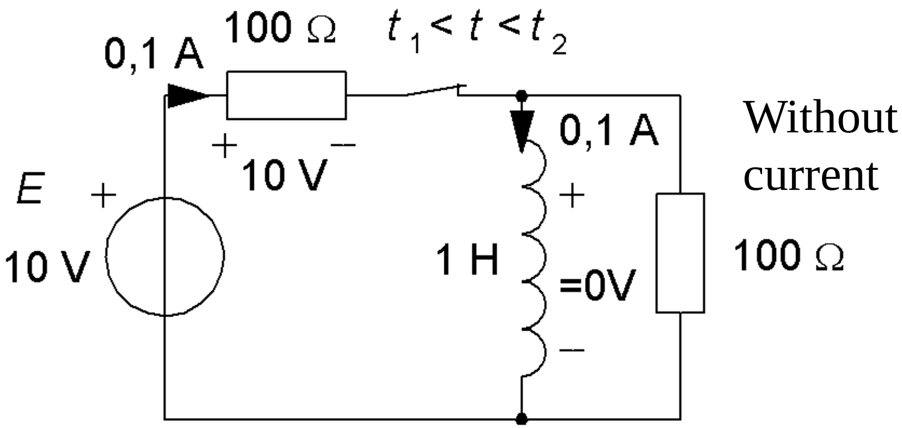
Quick formula: $x(t) = x_\infty - (x_\infty - x_0)e^{-\frac{t}{\tau}}$

$$\overset{i_{L\infty}\downarrow}{} \quad \overset{i_{L\infty}\downarrow}{} \quad \overset{i_{L0}\downarrow}{}$$

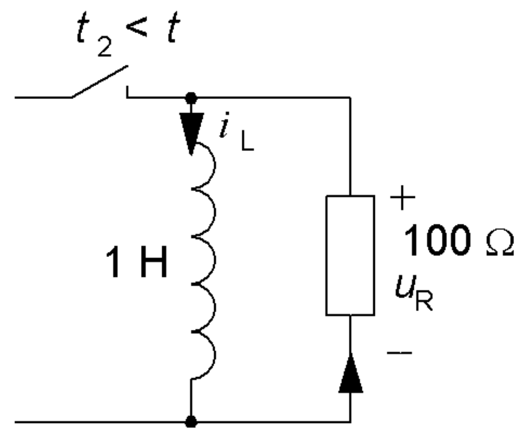
$$i_\mathrm{L}(t) = 0 - (0 - 0{,}1)\cdot e^{-\frac{t}{0{,}01}} \quad \Leftrightarrow \quad i_\mathrm{L}(t) = 0{,}1\cdot e^{-\frac{t}{0{,}01}} = 0{,}1\cdot e^{-100\cdot t}$$

Inductor connection and **disconnection** (10.8)

Before switch opening



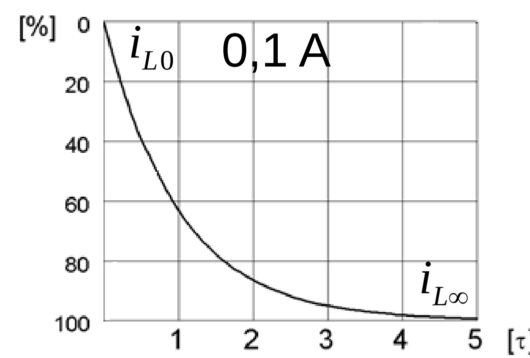
After switch opening



After t_2 the current starts from the ”same value” 0,1 A (i_0) as before the switch opening, and then the current will decrease down to 0 (i_∞). Time constant will be $\tau = L/R = 1/100 = 0{,}01$ s.

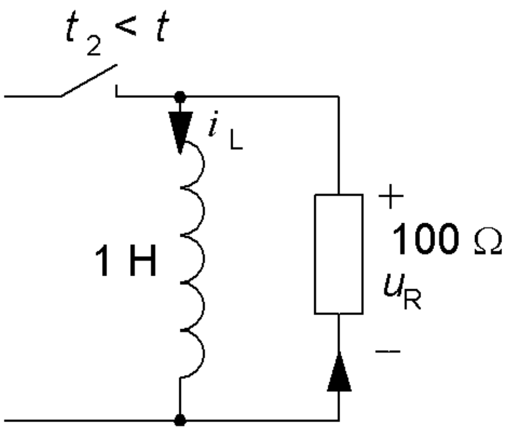
Snabbformeln: $x(t) = x_\infty - (x_\infty - x_0)e^{-\frac{t}{\tau}}$

$$i_L(t) = 0 - (0 - 0{,}1) \cdot e^{-\frac{t}{0{,}01}} \Leftrightarrow \boxed{i_L(t)} = 0{,}1 \cdot e^{-\frac{t}{0{,}01}} = \boxed{0{,}1 \cdot e^{-100 \cdot t}}$$



Inductor connection and **disconnection** (10.8)

When the voltage source 10 V is disconnected, the current is driven by the inductor. The voltage drop over the 100 Ω resistor U_R at first is -100·0,1 = -10 V. The minus sign comes from the fact that the current is entering the resistor in the part of the resistor we defined negative.



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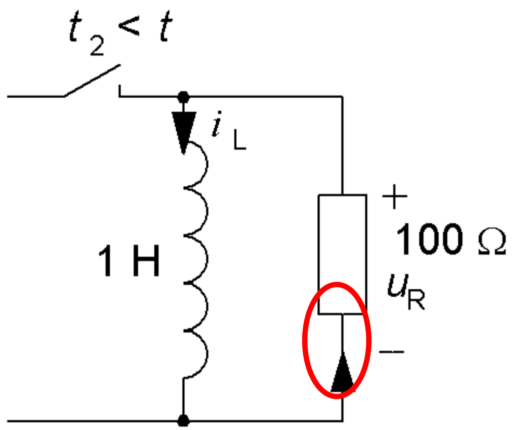

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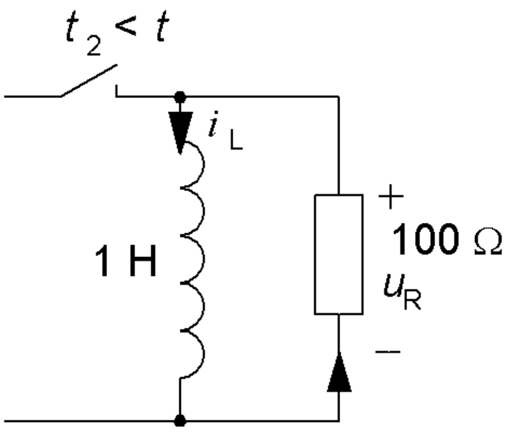

- Suppose the resistor is 1000 Ω. Then u_R at first moment had been -100 V !

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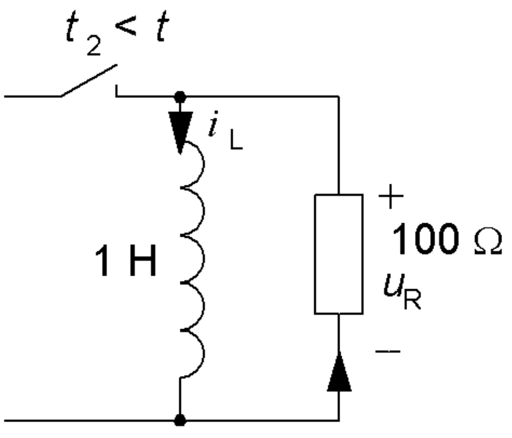

- Suppose the resistor is 1000 Ω. Then u_R at first moment had been -100 V !
- Suppose the resistor is 10000 Ω then the voltage had been -1000V !

Inductor connection and **disconnection** (10.8)

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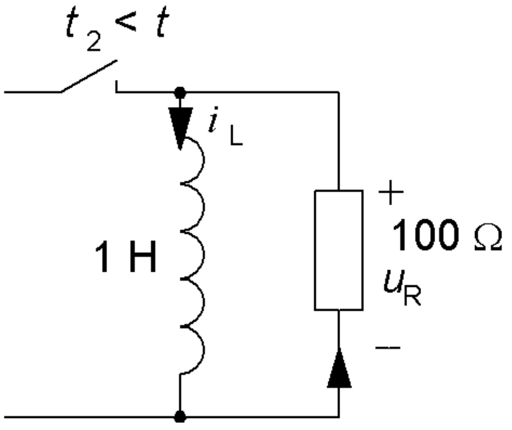
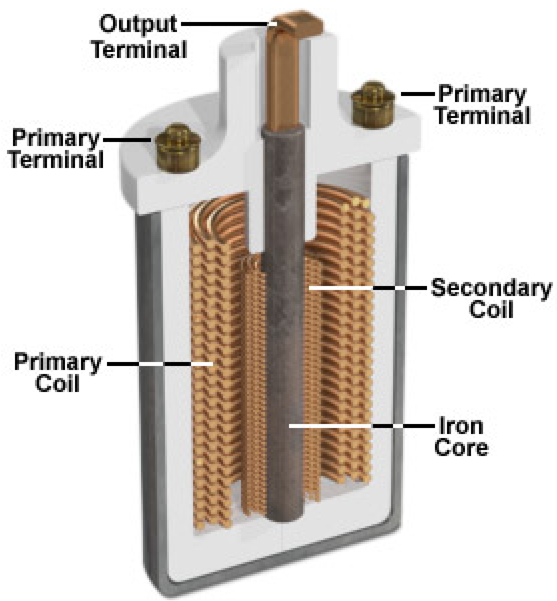
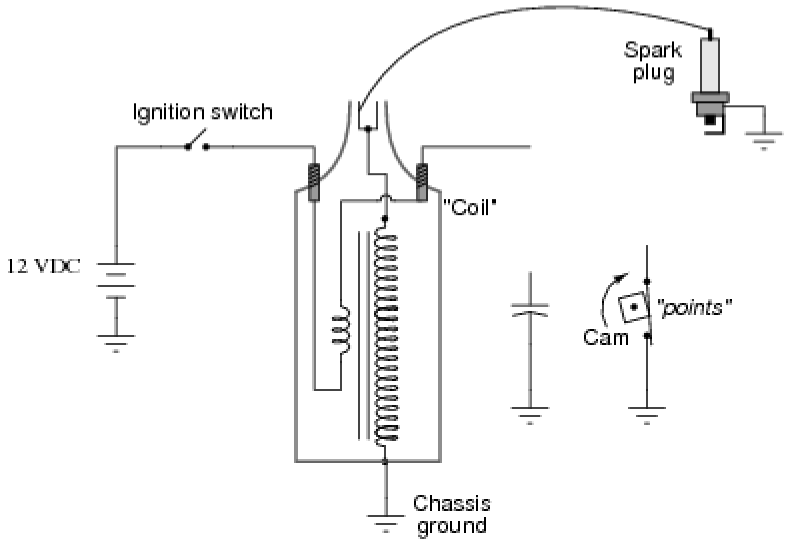


- Suppose the resistor is 1000 Ω. Then u_R at first moment had been -100 V !
- Suppose the resistor is 10000 Ω then the voltage had been -1000V !

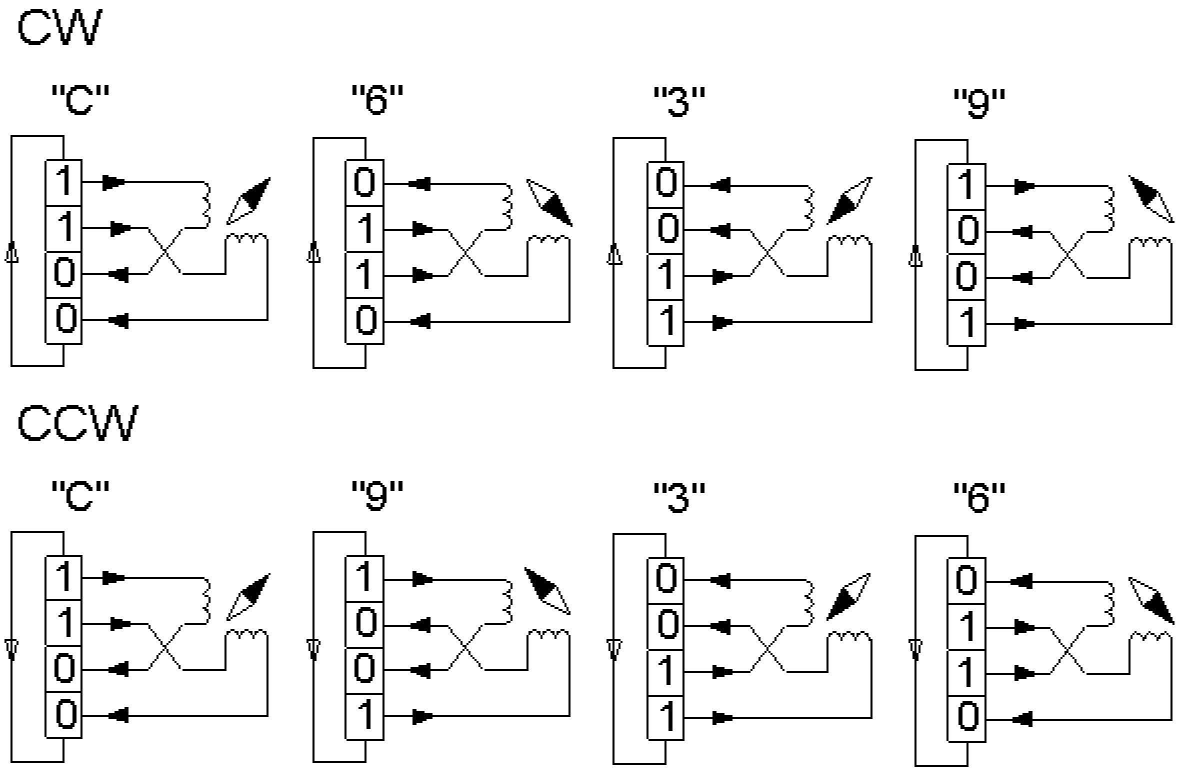
- When the circuit is broken the inductor tries to ”keep” the current, until all the magnetic energy has been consumed. If you omit the resistor from the circuit, ie, $R = \infty$ there will be a very high voltage.

Ex. To break the current to a coil will produce a high voltage



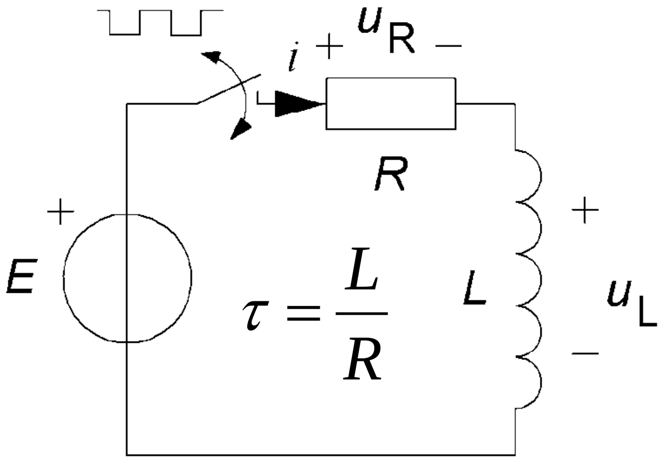


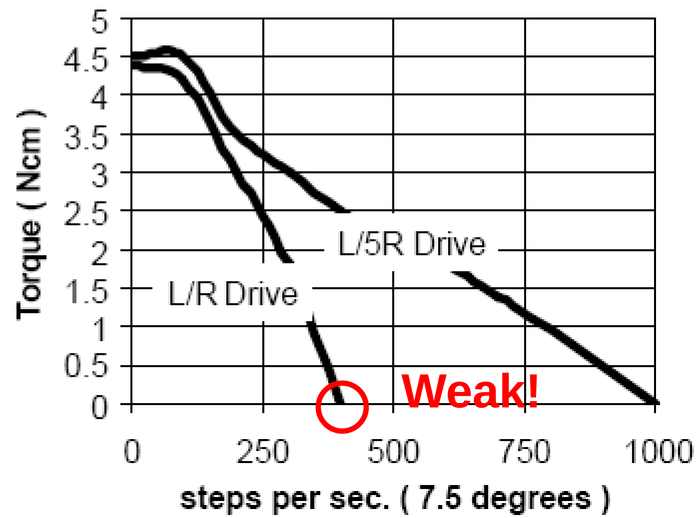
(Steppermotor the digital motor)



William Sandqvist william@kth.se

How fast can it run?



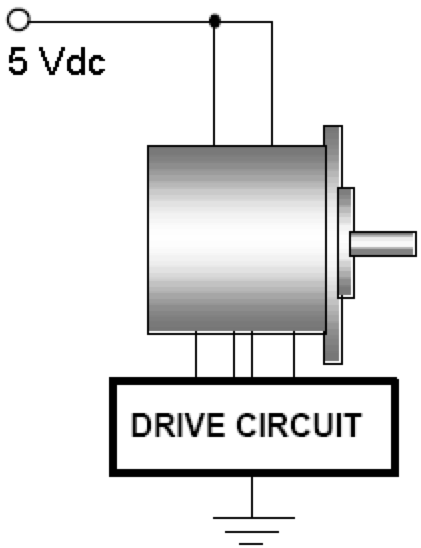


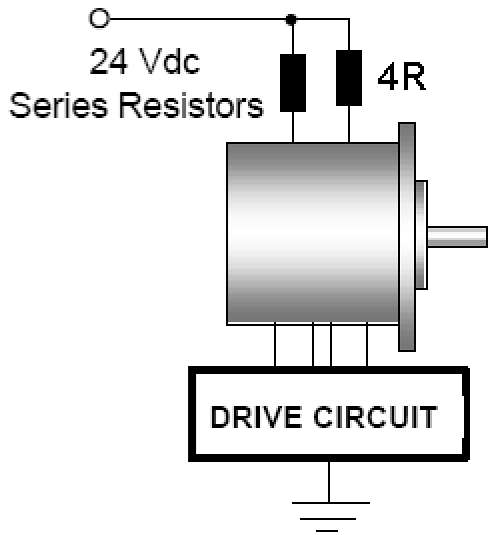
- The motor takes one step every pulse. 

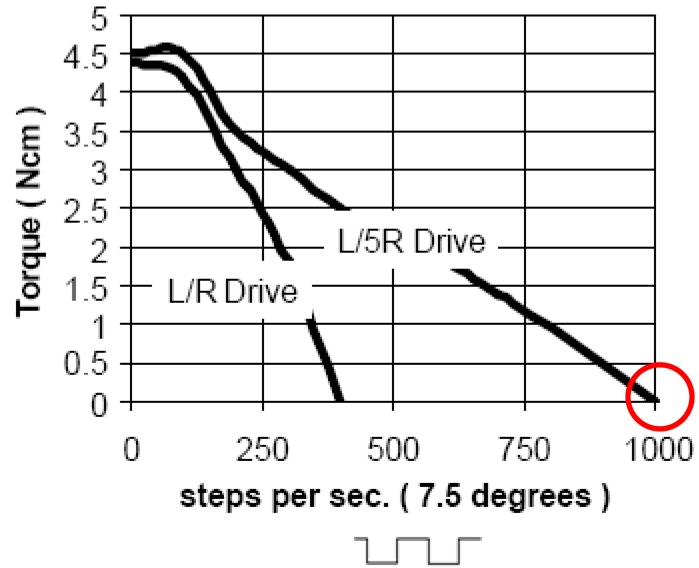
The faster you drive, the shorter the pulses. Due to the **time constant** τ do not have time to reach the peak current in the windings and the motor becomes weak.

But there is a trick …

L/5R is faster – Who could have guessed?







$$\tau = \frac{L}{R + 4 \cdot R} = \frac{L}{5 \cdot R}$$

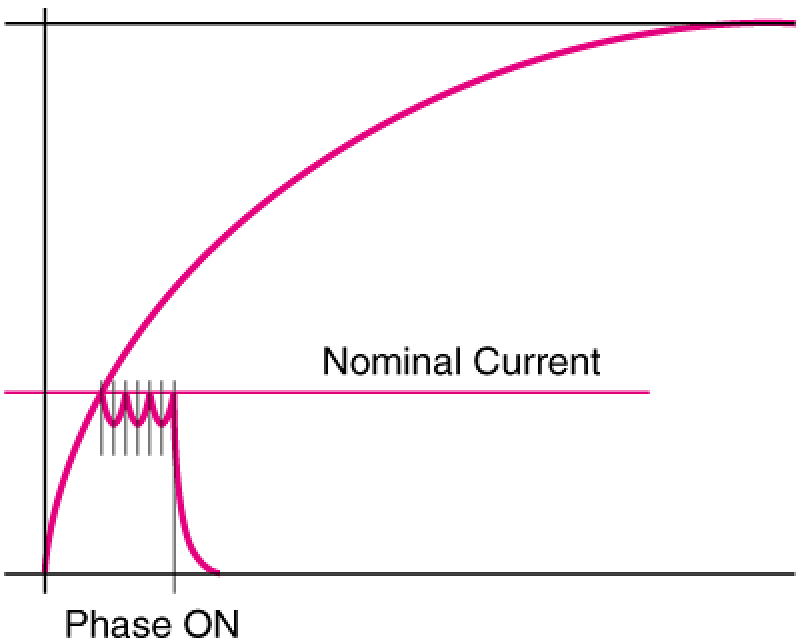
One introduces series resistors. At the same time you raise the voltage to maintain the current. Now the engine can run much faster!

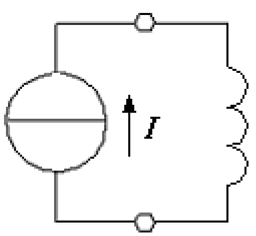
William Sandqvist william@kth.se

Fastest?

If the stepper motor is driven from a **current source** then this will have a high internal resistance ($R_I = \infty$). Time constant will be close to 0 and the stepper motor will have torque at higher pulse frequencies.

A driver with constant current are called a ”chopper”.
(One disadvantage of a chopper is that it generates a lot of interference).





$$\tau = \frac{L}{R} = \frac{L}{\infty} = 0$$