Turbulence modelling

- Reynolds decomposition

$$\tilde{u}_i(\boldsymbol{x}, t) = U_i(\boldsymbol{x}) + u_i(\boldsymbol{x}, t)$$

where $U_i(\boldsymbol{x}) = \overline{\tilde{u}_i(\boldsymbol{x}, t)}$ and $\overline{u_i(\boldsymbol{x}, t)} = 0$

- The “mean” is time average, ensemble average or averaging in homogeneous directions. $U_i(\boldsymbol{x})$ may actually vary in time with a time scale much longer than the turbulent time scale.

- Take the mean of the Navier-Stokes equations -> RANS

$$\frac{\partial U_i}{\partial x_i} = 0$$

$$\frac{\partial U_i}{\partial t} + U_k \frac{\partial U_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_k}\left(\nu \frac{\partial U_i}{\partial x_k} - \overline{u_i u_k}\right)$$

Reynolds stresses

- Not “small”
- Significant effects on the flow
- Needs to be modelled in terms of mean flow quantities
- Reduces the problem to steady (or slowly varying)
- 2D assumptions possible

- Equation can be derived from Navier-Stokes equations
- Need modelling

Eddy-viscosity models (EVM)

- Assume: Reynolds stresses related to an "eddy viscosity", ν_T

$$\overline{u_i u_j} = -2\nu_T S_{ij} \qquad \left(+ \frac{1}{3}\overline{u_k u_k}\delta_{ij} \right)$$

$$\text{where } S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

Based on Boussinesq (1877)

- Eddy viscosity ~ turbulence velocity V and length L scales: $\nu_T \sim VL$

One-equation models

- One transport equation for K (turbulent kinetic energy) or ν_T.
- Additional information from global conditions (typically wall distance)
- Works well for attached boundary layers
- Not very general, but more than algebraic models

- Example: Spalart-Allmaras (1992)
  - reasonable and robust model for external aerodynamics
  - Boeing's ”standard model”

Two-equation models

- Two transport equations for the turbulence scales (K–ε or K–ω)
- Completely determined in terms of local quantities (except near-wall corrections which may be dependent on wall distance)
- Works well for attached boundary layers
- Somewhat more general than zero-, one-equation models
- Model transport equations loosely connected to the exact equations.

- Examples:
  - Standard K–ε model (Launder & Spalding 1974)
  - Wilcox K–ω (1988, ...) models
  - Menter (1994) SST K–ω model (performing reasonable well also in separated flows)

    Airbus' "standard model"

Eddy-viscosity models …

- Problems:
  - No dependency on rotation or curvature. Real turbulence strongly dependent.
  - Modelled production proportional to strain rate squared ~ S^2. Exact production ~ S. Results in a overestimated production of K in highly sheared flows (around stagnation points, impinging jets, pressure gradient BLs, separated flows).
- Fixes
  - Rotation & curvature corrections
  - Yap correction (limit excessive turbulent lengthscale)
  - Menter SST correction (limit excessive ν_T).

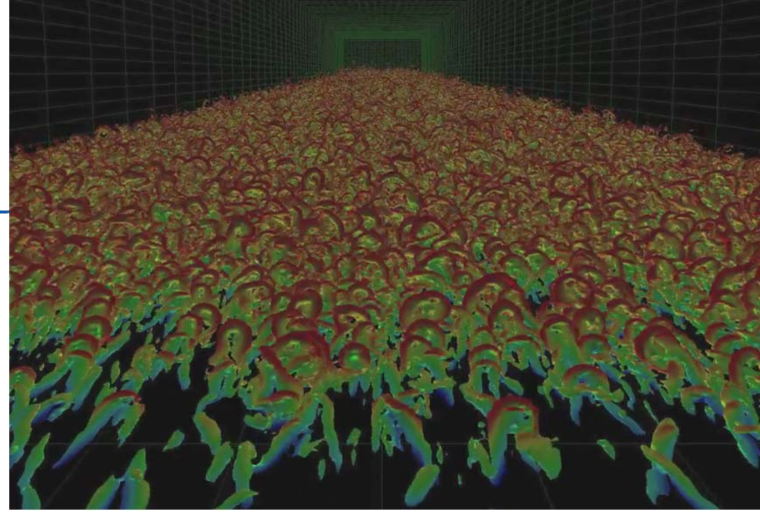
LES and LES/RANS hybrids

- Simulation of only the large scale turbulence (compare with DNS, simulation of all scales)
  - Always time dependent and 3D -> expensive
- Wall free turbulence simulations almost *Re* independent
- Wall bounded turbulence largely *Re* dependent
  - fully resolved near-wall region very expensive (almost as DNS)
  - wall-function or near-wall RANS coupling saves computational cost
  - hybrid RANS-LES (RANS in attached BLs and LES in wall-free separated regions) a very active research field, eg DES

LES and LES/RANS hybrids ...

- LES in academic research for:
  - low *Re* generic flows
  - complement to DNS for higher *Re*
  - gives detailed knowledge about turbulence
- LES in industrial use in:
  - internal flow with complex geometries
  - flows around blunt bodies (with large separated regions)
  - atmospheric boundary layers (e.g. weather forecasts)
  - combustion simulation
  - other complex flow physics at moderate *Re*
- Warning: LES is extremely expensive in high attached and slightly separated wall-bounded flows, if properly resolved.

How expensive is DNS?



- **DNS of flat plate turbulent boundary layer**
  - **Schlatter, et al., KTH, Dept. of Mechanics**
  - **APS meeting 2010: http://arxiv.org/abs/1010.4000**
  - **http://www.youtube.com/watch?v=4KeaAhVoPIw**
  - **http://www.youtube.com/watch?v=zm9-hSP4s3w**
  - $Re_\theta = 4300$
  - **8192×513×768 = 3.2×10^9 spectral modes (7.5×10^9 nodes)**
  - $\Delta x^+ = 9$, $\Delta z^+ = 4$ **–>** **box:** $L^+ = 70\,000$, $H^+ = W^+ = 3\,000$
  - **BL relations:** $Re_x = 1.4\times10^6$
  - **CPU time: 3 months @ 4000 CPU cores = 1 unit**
- **DNS of model airplane, same Reynolds number (**$Re_x = 1.4\times10^6$**)**
  - **Only a narrow stripe – wing requires about 1 000 stripes**
  - $N_{nodes} = 10^{13}$
  - **CPU =** 10^3 **units**

Empirical turbulent BL relations

- **Skin friction coefficient:** $$\frac{C_f}{2} = \frac{\tau_w}{\rho U_\infty^2} = \left(\frac{Re_\tau}{Re_\delta}\right)^2 \approx 0.0296 Re_x^{-1/5}$$
- **Boundary layer thickness:** $$\frac{\delta}{x} = \frac{Re_\delta}{Re_x} \approx 0.37 Re_x^{-1/5}$$
- **Boundary layer momentum thickness:** $$Re_\tau \approx 1.13 Re_\theta^{0.843}$$
- **Reynolds numbers:**

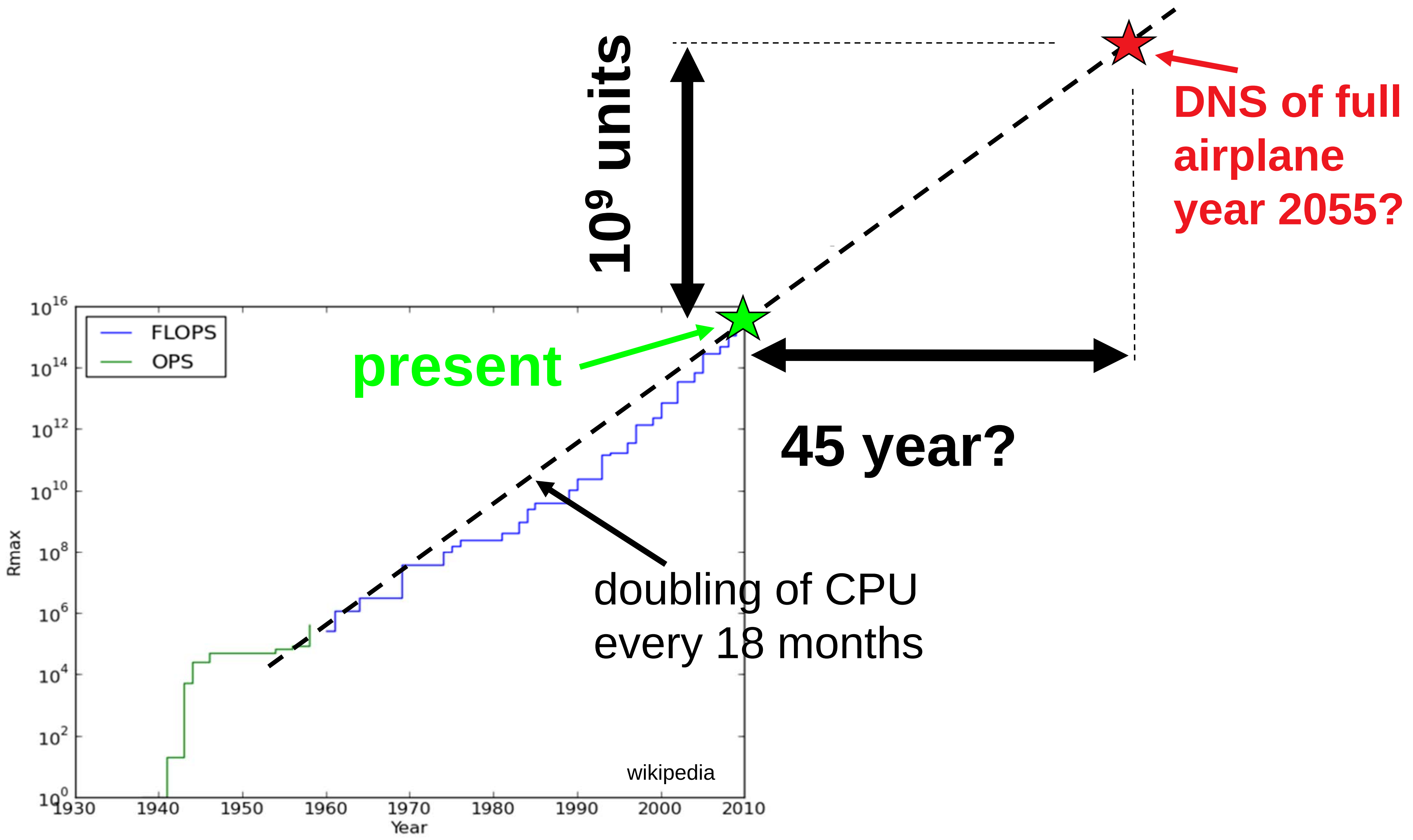
$$Re_\tau \equiv \frac{\delta u_\tau}{\nu} \qquad Re_\theta \equiv \frac{\theta U_\infty}{\nu} \qquad Re_\delta \equiv \frac{\delta U_\infty}{\nu} \qquad Re_x \equiv \frac{x U_\infty}{\nu}$$

DNS – full scale airplane

- **Re scaling – wall bounded flow**
  - **Nodes:** $N_{nodes} \sim \dfrac{L \times B \times H}{\Delta x \Delta z \Delta y} \sim L^{+2} H^{+} \sim Re_x^{5/2}$
  - **Time steps:** $N_{\Delta T} \sim \dfrac{T}{\Delta T} \sim T^{+} \sim Re_x^{4/5}$
  - **CPU time:** $N_{CPU} \sim N_{nodes} \times N_{\Delta T} \sim Re_x^{33/10}$

- **DNS of Airplane ($Re_x = 70 \times 10^6$) (factor of 50)**
  - **$N_{nodes} = 10^{17}$**
  - **CPU = 10^9 units**

Supercomputer development



Computational effort – different approaches

Name	Aim	Unsteady	*Re*-dependence	3/2D	Empiricism	Grid	Steps	Ready
2DURANS	Numerical	Yes	Weak	No	Strong	10^5	$10^{3.5}$	1980
3DRANS	Numerical	No	Weak	No	Strong	10^7	10^3	1990
3DURANS	Numerical	Yes	Weak	No	Strong	10^7	$10^{3.5}$	1995
DES	Hybrid	Yes	Weak	Yes	Strong	10^8	10^4	2000
LES	Hybrid	Yes	Weak	Yes	Weak	$10^{11.5}$	$10^{6.7}$	2045
QDNS	Physical	Yes	Strong	Yes	Weak	10^{15}	$10^{7.3}$	2070
DNS	Numerical	Yes	Strong	Yes	None	10^{16}	$10^{7.7}$	2080

From Spalart, Int. J. Heat and Fluid Flow, 2000

- **RANS: Reynolds Averaged Navier-Stokes**
- **URANS: Unsteady RANS – slowly in time**
- **DES: Detached Eddy Simulation**
- **LES: Large eddy simulation**
- **QDNS: Quasi DNS, or wall resolved LES**
- **DNS: Direct Numerical Simulation (of the Navier-Stokes eq’s)**
- **“Ready”: When first results can be expected**