EP2200 Queuing Theory and Teletraffic Systems

Wednesday, March 23$^{\mathrm{rd}}$, 2016
Available teacher: Viktoria Fodor
Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas, Laplace transforms and Erlang tables.

1. Consider a transmission link with transmission capacity $C = 8$ Mbits/s. Packets arrive to the output buffer at the link according to a Poisson process, with a rate of 5000 packets per second. The packet sizes are Exponentially distributed with a mean of 100 Bytes. First consider that the output buffer can store infinite number of packets.
a) Express the probability that there are more than k packets waiting in the buffer (and an additional one is under retransmission). Give the probability that there are more than 5 packets waiting. (2p)
b) Consider the packets that arrive when exactly 5 other packets are already waiting. Give the mean and the variance of the waiting time of these packet. Give the probability that such a packet needs to wait more than one millisecond. (3p)
Consider now the case when the output buffer size is limited, and it can store a maximum of k packets.
c) Derive the expression of the probability that an arriving packet is dropped because the buffer is full. Give the blocking probability for $k = 5$. (3p)
d) Give the distribution of the time interval when the system is in blocking state (that is, the buffer is full). Give the probability that no new packets arrive, and therefore no packets are dropped while the buffer is full. (2p)

2. There are three clerks working at your local bank. For each of them, it takes on average 15 minutes to serve a customer, with an Exponentially distributed time. Customers arrive according to a Poisson process, 5 customers per hours on average.
a) What is the average number of customers in the bank? What is the expected waiting time of an arbitrary customer? (3p)
b) What is the probability that you arrive to the bank and you do not need to wait for service? (Assume, the system reached steady state when you arrive to the bank.) (2p)
c) Assume there are 3 customers already waiting when you arrive. What is your expected waiting time? (3p)
d) How much free time has a clerk during the 8 working hours? (2p)

3. Consider a finite population system with 5 customers. Customers are served with two servers. After service, a customer remains idle for an Exponentially distributed time, with an average of 10 minutes, and then generates a new request. If both of the servers are busy, the customer starts a new idle period. If the request is accepted, the customer receives a service for an Exponentially distributed time, with an average of 5 minutes.
a) Give the Kendall notation of the system, and draw the Markov chain of the system. (2p)
b) Consider the system in steady state. Calculate the probability that both of the servers are busy and the probability that an arriving request needs to be blocked. (3p)
c) Calculate the average number of performed services per hour. (2p)
d) Assume now that the service time distribution is Erlang-2, with the same average of 5 minutes. Give the Kendall notation, define the states, and draw the Markov chain of the system. (3p)

4. Two kinds of jobs are served by a single processor. Jobs of Type 1 arrive according to a Poisson process with a rate of 2 jobs per second. These jobs require a fixed service time of 0.2 seconds. Type 2 jobs arrive according to a Poisson process with a rate of 1 job per second. Their service time is Exponentially distributed with a mean of 0.3 seconds. Jobs that can not access the server immediately wait in an infinite buffer and are served in FIFO order.
a) Give the Kendall notation of the system. Give the utilization of the server, and the average length of the idle periods. (2p)
b) Calculate the mean waiting and system times of Type 1 as well as of Type 2 jobs. (3p)
Let us now introduce preemptive resume priority service, where Type 1 jobs are served with high priority.

c) Calculate the mean waiting time of Type 1 and Type 2 jobs in this case, that is the average time from job arrival, until the first time the job enters the server. (3p)
d) Calculate the expected time from the start of service until completed service, for Type 1 as well as for Type 2 jobs. (2p)

5. Consider a queuing network consisting of two single server, infinite buffer queuing systems. The service times are exponentially distributed, with parameters $\mu_1 = \mu_2 = 1$ service per time unit. Jobs enter at the first node with intensity $\gamma_1 = 1/4$. Let us denote by $p_{i,j}$ the probability that a job is forwarded from node i to node j, and by $p_{i,0}$ the probability that a job leaves the network after service at node i. Let $p_{1,2} = 1$, $p_{2,1} = 0.2$, $p_{2,2} = 0.3$, $p_{2,0} = 0.5$
a) Draw the queuing network with all the parameters. (1p)
b) Calculate the arrival rates at the nodes, and the probability that the queuing network is empty. (3p)
c) Calculate the average number of jobs in the queuing network, and the average time a job spends in the network. (3p)
d) Calculate the average number of services jobs receive before they leave the queuing network. (2p)
e) Give the maximum γ_1, such that the queuing network remains stable. (1p)