EP2200 2016 Exam

(1) $C = 8$ M.bits/s
$\lambda = 5000$ packets/s
$L \sim Exp$, $\bar{L} = 100B$

$\Big\}$ M/M/1

a) $x \sim Exp(\mu)$

$\bar{x} = \frac{\bar{L}}{C} = \frac{800}{8} \cdot 10^{-6} = 10^{-4}s$, $\mu = 10^4$

$\rho = \lambda \bar{x} = 5 \cdot 10^3 \cdot 10^{-4} = \underline{\underline{0.5}}$

M/M/1
$p_i = (1-\rho)\rho^i = \left(\frac{1}{2}\right)^{i+1}$

$$P(N_q > k) = \sum_{i=k+1}^{\infty} p_i = \sum_{i=k+1}^{\infty} \left(\frac{1}{2}\right)^{i+1} = \frac{(1/2)^{k+2}}{1 - 1/2} = \left(\frac{1}{2}\right)^{k+1}$$

$$P(N_q > 5) = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

b) $W_5 = \sum_{i=1}^{6} x_i$

$E[W_5] = 6 \cdot E[x_i] = 6 \cdot 10^{-4}s$

$V[W_5] = 6 \cdot V[x_i] = 6 \cdot 10^{-8}s^2$

$P(W_5 > 10^{-3}s) = P($less than 6 services in $10^{-3}s) =$

$$= \sum_{i=0}^{5} \frac{(\mu t)^i}{i!} e^{-\mu t} = \sum_{i=0}^{5} \frac{(10^4 \cdot 10^{-3})^i}{i!} e^{-(10^4 \cdot 10^{-3})} = \dots \approx 0.067$$

c) M/M/1/k+1

State prob. expressed from normalizing M/M/1 results:

$$p_i = \frac{(1-\rho)\rho^i}{\sum_{j=1}^{k+1}(1-\rho)\rho^j} = \frac{\rho^i}{\sum_{j=1}^{k+1}\rho^j} = \frac{(1-\rho)\rho^i}{1-\rho^{k+2}}, \quad P_{k+1}\Big|_{k=5} = \frac{\frac{1}{2} \cdot \frac{1}{2}^6}{1 - \frac{1}{2}^7} = \frac{\left(\frac{1}{2}\right)^7}{1-\left(\frac{1}{2}\right)^7}$$

d) $\tau_{blocking} \sim Exp(\mu)$ [End of a service]

$$P(\text{no blocking}) = P(\text{no new packets in } \tau) = \int_0^\infty P(\text{no new} \mid \tau = t) P(\tau = t) dt =$$

$$= \int_0^\infty e^{-\lambda t} \cdot \mu t e^{-\mu t} dt = \frac{\mu}{\lambda+\mu} \underbrace{\int_0^\infty (\lambda+\mu) t e^{-(\lambda+\mu)t} dt}_{1} = \frac{\mu}{\lambda+\mu} = \frac{10^4}{1.5 \cdot 10^4} = \underline{\underline{\frac{2}{3}}}$$

(2) M/M/3

$\bar{x} = 15\text{min} = \frac{1}{4}$ hour

$\lambda = 5$ arrivals/hour

$\Big\} \quad a = \lambda \bar{x} = \frac{5}{4} = 1.25 \qquad \mu = 4$ [service/hour]

a) $N = a + \frac{\lambda}{m \cdot \mu - \lambda} P(\text{wait}) = 1.36$

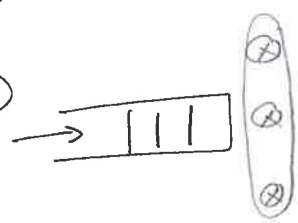
$W = \frac{1}{m\mu - \lambda} P(\text{wait}) = 0.022$

$P(\text{wait}) = \frac{m E_m(a)}{m - a(1 - E_m(a))} = \frac{3 \cdot 0.1}{3 - 1.25 \cdot 0.9} = 0.155\ldots$

$E_3(1.25) = 0.096 \approx 0.1$

b) $P(\text{service without waiting}) = 1 - P(\text{wait}) = 0.84$

c)



4 customers need to be served to get the arriving one in the server.

Time between services: $\tau = \text{Exp}(3\mu)$

$\bar{W} = 4 \cdot \frac{1}{3\mu} = \frac{1}{3}$ hour $= 20$ minutes

d) $\text{Utilization} = \frac{\lambda \bar{x}}{m} = \frac{5}{12}$

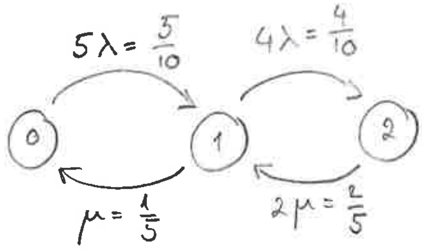
$\text{Free time} = (1 - \text{utilization})T = \frac{14}{3}$ hours $\quad (T = 8)$

(3) M/M/2/2/5

$\bar{\tau} = 10\ \text{min} \rightarrow \lambda = \frac{1}{10}$ [1/min]

$\bar{x} = 5\ \text{min} \qquad \mu = \frac{1}{5}$

a)



b)

$p_0 \cdot 5\lambda = p_1 \cdot \mu \qquad p_0 \cdot \frac{5}{10} = p_1 \cdot \frac{1}{5} \qquad p_1 = \frac{5}{2} p_0$

$p_1 \cdot 4\lambda = p_2 \cdot 2\mu \qquad p_1 \cdot \frac{4}{10} = p_2 \cdot \frac{2}{5} \qquad p_2 = p_1 = \frac{5}{2} p_0$

$p_0 + p_1 + p_2 = 1$

$p_0 \left(1 + \frac{5}{2} + \frac{5}{2}\right) = 1 \qquad p_0 = \frac{1}{6}, \quad p_1 = \frac{5}{12}, \quad p_2 = \frac{5}{12}$

$P(\text{2 servers busy}) = \frac{5}{12}$

$P(\text{arriving request blocked}) = \dfrac{p_2 \cdot 3\lambda}{p_0 \cdot 5\lambda + p_1 \cdot 4\lambda + p_2 \cdot 3\lambda} = \dfrac{3 \cdot \frac{5}{12}}{5 \cdot \frac{1}{6} + 4 \cdot \frac{5}{12} + 3 \cdot \frac{5}{12}} = \frac{15}{45} = \frac{1}{3}$

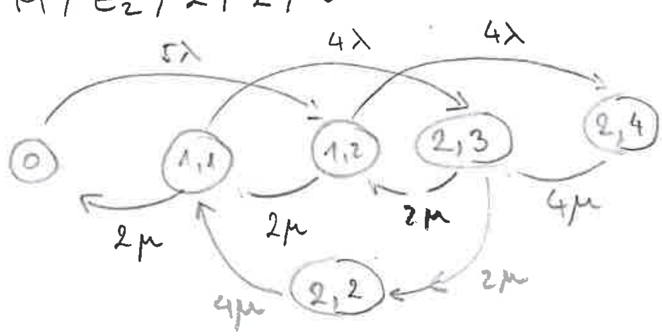
c)

$\lambda_{eff} = p_0 \cdot 5\lambda + p_1 \cdot 4\lambda = \frac{1}{4}$ [1/min]

$S = \lambda_{eff} \cdot T = \frac{1}{4} \cdot 60 = 12$

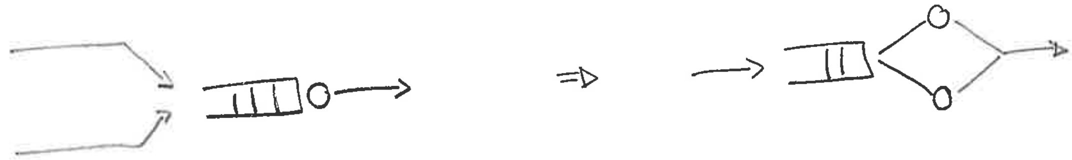
d) M / E_2 / 2 / 2 / 5

$E[x] = \frac{1}{\mu}$



state: {servers used, all stages left}

(4)



Type 1: $\lambda_1 = 2$ jobs/s $\quad x_1 = 0.2$ s $\quad \overline{x_1^2} = 0.04\,s^2$

Type 2: $\lambda_2 = 1$ job/s $\quad x_2 \sim Exp$, $\bar{x}_2 = 0.3$ s, $\overline{x_2^2} = 2 \cdot \bar{x}_2 = 0.18\,s^2$

a) M/G/1 (or M/E+D/1)

$U = \lambda_1 x_1 + \lambda_2 \bar{x}_2 = 0.4 + 0.3 = 0.7$

$T_{idle} \sim Exp(\lambda_1 + \lambda_2) \quad \bar{T}_{idle} = \frac{1}{\lambda_1 + \lambda_2} = \frac{1}{3}$ sec.

b) $\bar{W} = \frac{\lambda \overline{x^2}}{2(1-\rho)} = \frac{13}{30}\,s = 0.433\,s$

$\bar{x} = \frac{\lambda_1}{\lambda_1+\lambda_2} x_1 + \frac{\lambda_2}{\lambda_1+\lambda_2} \bar{x}_2 = \frac{7}{30}\,s$

$\overline{x^2} = \frac{\lambda_1}{\lambda_1+\lambda_2} x_1^2 + \frac{\lambda_2}{\lambda_1+\lambda_2} \overline{x_2^2} = \frac{13}{150}\,s^2 = 0.087$

$\lambda = \lambda_1 + \lambda_2 = 3$ jobs/s

$\bar{W}_1 = \bar{W}_2 = \bar{W}$

$\bar{T}_1 = \bar{W} + x_1 = \frac{19}{30}\,s$

$\bar{T}_2 = \bar{W} + \bar{x}_2 = \frac{22}{30}\,s = \frac{11}{15}\,s$

c) Type 1 high priority $\Rightarrow$ M/D/1 with λ_1, x_1

$\bar{W}_1 = \frac{\lambda_1 x_1^2}{2(1-\lambda_1 x_1)} = \frac{1}{15}\,s \approx 0.067\,s$

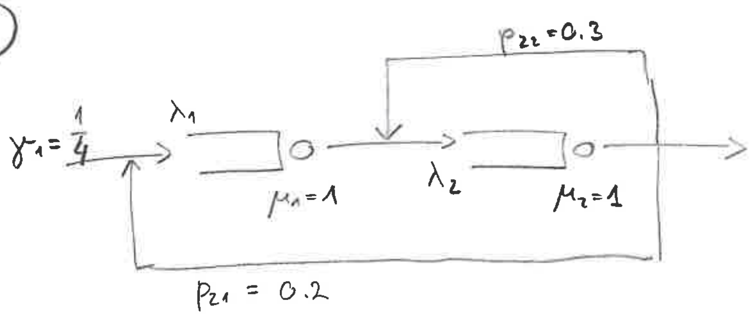
$W_2 = \frac{\frac{1}{2}(\lambda_1 x_1^2 + \lambda_2 \overline{x_2^2})}{(1-\lambda_1 x_1)(1-(\lambda_1 x_1 + \lambda_2 \bar{x}_2))} = \frac{13}{18}\,s \approx 0.722\,s$

d) $\bar{x}_1' = \bar{x}_1 = x_1$

$\bar{x}_2' = \frac{\bar{x}_2}{1 - \lambda_1 x_1} = \frac{1}{2}\,s$

5

a)



b)

$$\lambda_1 = \gamma_1 + \lambda_2 \cdot 0.2$$

$$\lambda_2 = \lambda_1 + \lambda_2 \cdot 0.3$$

$$\lambda_2 = \gamma_1 + 0.2\lambda_2 + 0.3 \cdot \lambda_2$$

$$\lambda_2 = \frac{\gamma_1}{0.5} = 2 \cdot \gamma_1 = \underline{\underline{\frac{1}{2}}}$$

$$\lambda_1 = \frac{1}{4} + \frac{1}{10} = \underline{\underline{\frac{7}{20}}}$$

P(empty network) =

P(node 1 empty) P(node 2 empty) =

$$(1-\rho_1)(1-\rho_2) = \frac{13}{20} \cdot \frac{1}{2} = \frac{13}{40}$$

$$\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{7}{20}$$

$$\rho_2 = \frac{\lambda_2}{\mu_2} = \frac{1}{2}$$

c)

$$N_1 = \frac{\rho_1}{1-\rho_1} = \frac{7}{13}, \quad N_2 = \frac{\rho_2}{1-\rho_2} = 1 \qquad N = N_1 + N_2 = \frac{20}{13}$$

$$T = \frac{N}{\gamma_1} = \frac{20/13}{1/4} = \frac{80}{13} \text{ time units}$$

d)

$$V = \frac{\lambda_1 + \lambda_2}{\gamma_1} = \left(\frac{1}{2} + \frac{7}{20}\right) / \frac{1}{4} = \frac{17}{5}$$

e)

$$\lambda_2 > \lambda_1 \Rightarrow$$

$$\lambda_2 = \frac{\gamma_1}{0.5} < 1$$

$$\underline{\underline{\gamma_1 < 0.5}}$$