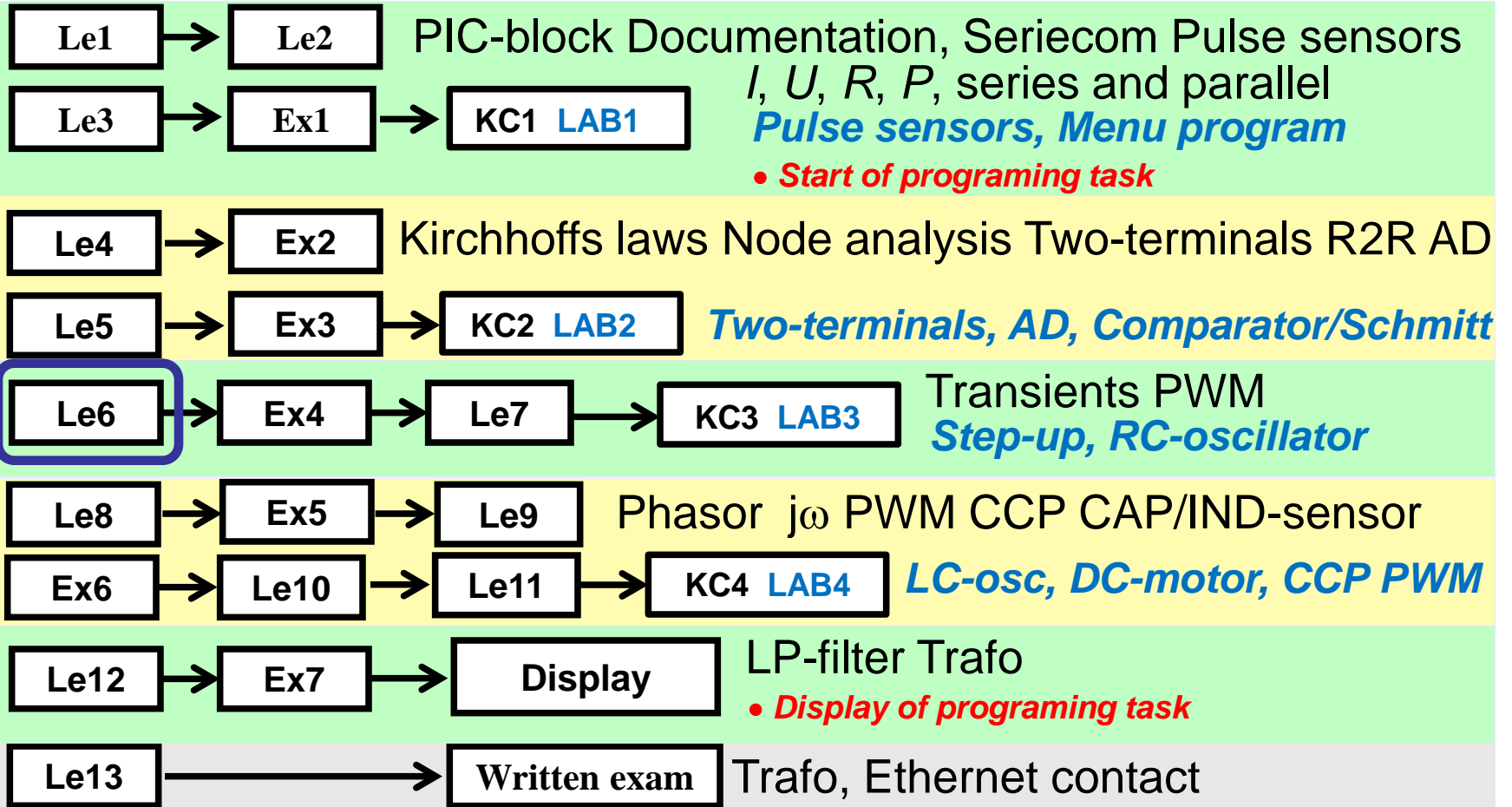


IE1206 Embedded Electronics



electric fields



The force between charges can be calculated using Coulomb's Law. The force between like charges is repulsive, between different charges attractive.

The electric field E at a point charge Q_1 can be seen as the force on a "test charge", a "unit charge" ($Q_2 = +1$).

The electric lines of force are starting from a positive charge and end on a negative charge.

The force lines may not cross each other.

$$F = k \cdot \frac{Q_1 \cdot Q_2}{r^2} \quad \bar{E} = k \cdot \frac{Q_1 \cdot 1}{r^2} \quad k = \frac{1}{4\pi \cdot \epsilon_0} = 9 \cdot 10^9 \text{ Nm}^2/\text{C}^2$$

The constant k has a very big value, **the electrical forces are strong.**

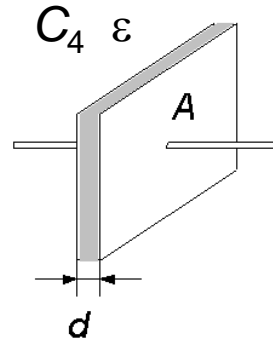
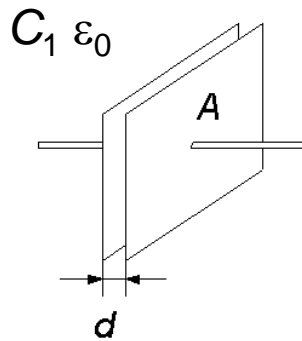
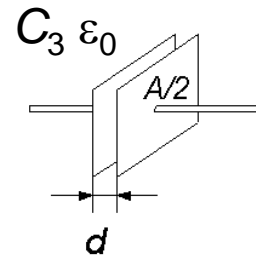
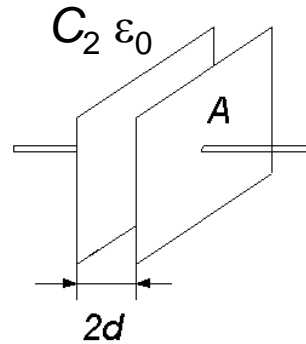
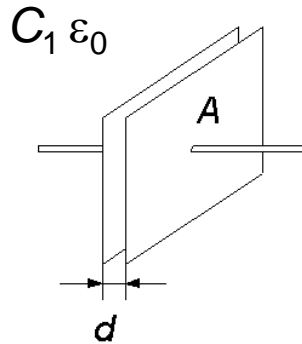
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Plate capacitor

$$C = \frac{Q}{U} \quad C = \varepsilon \frac{A}{d}$$

A capacitor capacitance C is proportional to the area A and inversely proportional to flat distance d .

If the insulation material between the plates is polarizable (ε) the capacitance is increased.



$$C = \frac{Q}{U}$$

$$C_1 = \varepsilon_0 \frac{A}{d} > C_2 = \varepsilon_0 \frac{A}{2d} = C_3 = \varepsilon_0 \frac{A/2}{d}$$

$$C_1 = \varepsilon_0 \frac{A}{d} < C_4 = \varepsilon \frac{A}{d} \quad \varepsilon = \varepsilon_r \cdot \varepsilon_0 \quad \varepsilon_0 = 8,85 \text{ pF/m}$$

Dielectric

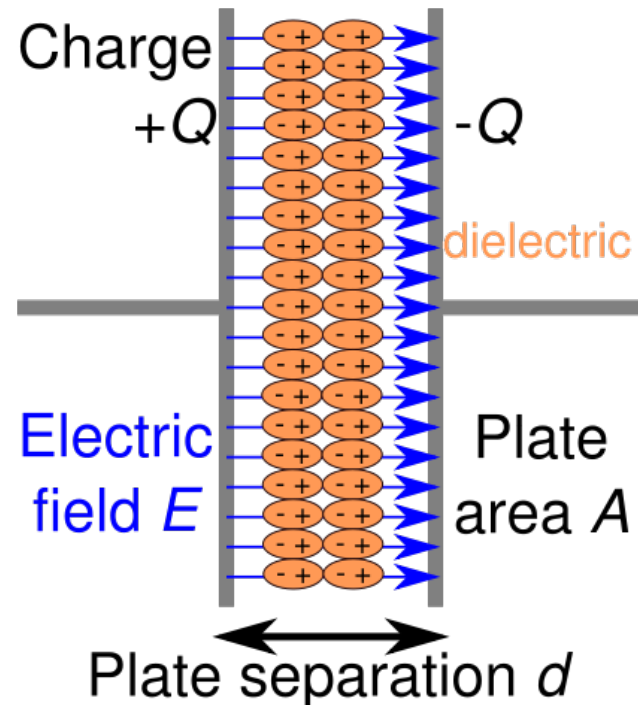
Most materials are polarizable, and will then increase the electric field, and the capacitance of the capacitor if placed between the plates.

Titanite used in ceramic capacitors, the increases the capacitance 7500 times in comparison to vacuum or air.

$$\epsilon_r = 7500$$

ϵ_r is playing the same role for the electric field as μ_r does for the magnetic field.

$$\epsilon = \epsilon_r \cdot \epsilon_0 \quad \epsilon_0 = 8,85 \text{ pF/m}$$

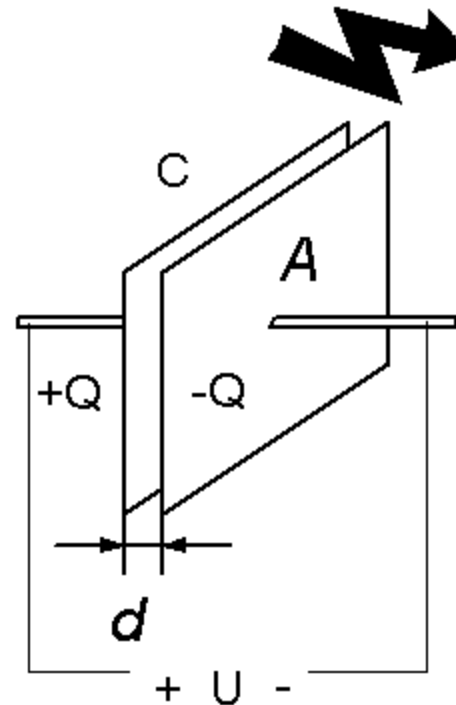


Short d , Voltage rating

High capacitance value could be obtained with a small flat distance d .

The drawback is that the risk increases for arcing between the plates. Each capacitor then has a maximum rated voltage which must not be exceeded.

A capacitor for higher rated voltage are necessarily larger than a lower rated voltage if the capacitance is the same.

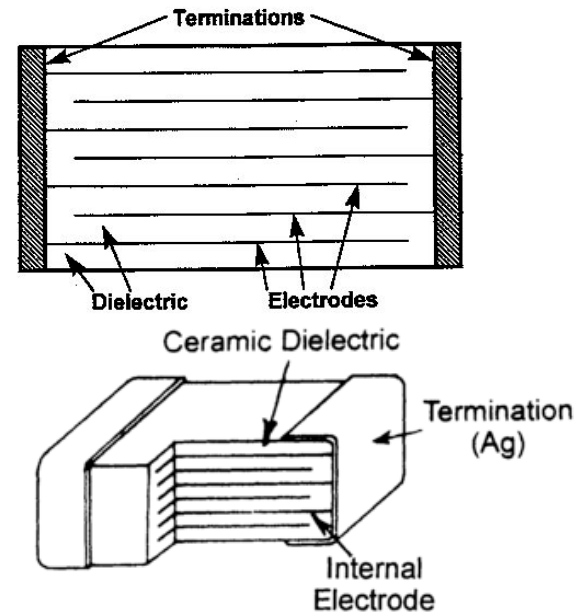
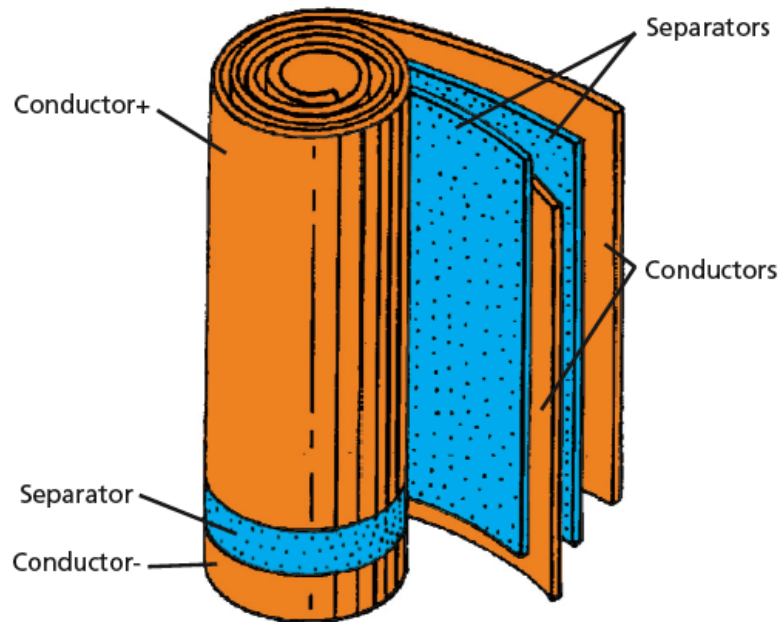


$$C = \frac{Q}{U} \quad E = \frac{U}{d}$$

The electric field E of the capacitor is $E=U/d$. The air can withstand 2.5 kV/mm before arcing!

Big area A

High capacitance one can get with large area A . The capacitor can then be rolled, or type by multilayer type, so that "the component surface" is minimized despite the large inner surface.



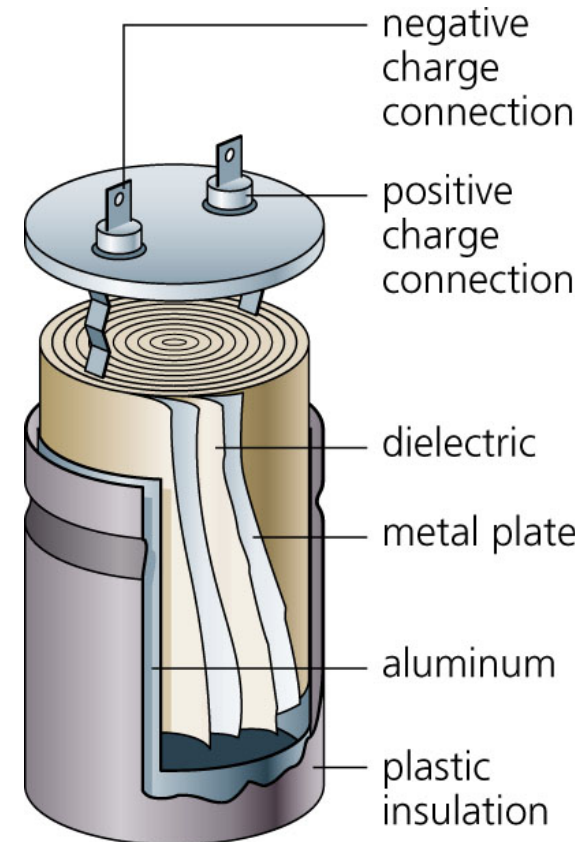
Multilayer Capacitor with ceramic dielectrics ($= \text{high } \epsilon_r$).

Very short distance d

The **electrolytic capacitor** is based on extremely small distance d between the electrodes. One electrode is an aluminum foil, and the dielectric is a thin insulating oxide layer on the foil. The other electrode is the electrolyte itself which of course is in close contact with the surface of the foil.

The capacitor must be polarized correctly, with the same polarity as when the oxide layer was formed. Otherwise the oxide layer is destroyed and the capacitor is shorted!

The capacitor is also destroyed if the rated voltage is exceeded.



Big area A and very short distance d

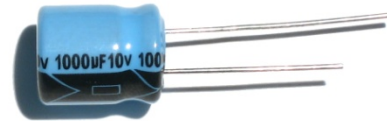
Tantal electrolytic capacitor have a "sponge formed" electrode.

The total inner surface A becomes extremely large. The insulation consists of an oxide layer so even d is small.

A 3.5 mm×2.5 mm× 5.5 mm, 4.7μF **tantal electrolytic capacitor** has the equivalent inner area of 40 cm² !



Capacitors



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Supercap (9.2)



$$C = \frac{Q}{U} \quad I = \frac{Q}{t}$$

The backup capacitors of the type "Supercap" can be used as a power backup for memories - if one for example needs to move the phone from one room to another without the phone forgetting its settings.

Make a rough estimate of how long the charge in the capacitor will last?

Assume that $C = 1 \text{ F}$ and U is initially 5V . The equipment draws $I = 10 \text{ mA}$ and operates down to 2.5V .

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$$\Delta Q = C \cdot \Delta U = 1 \cdot (5 - 2,5) = 2,5 \text{ As} \quad t = \frac{\Delta Q}{I} = \frac{2,5}{10 \cdot 10^{-3}} = 250 \text{ s} = 4 \text{ min}$$

School's "biggest" supercap?

3000 F × 16

Research is going on for energy storage for routers in places where batteries would have 'inappropriate' temperatures.

For example, in the desert or in the arctic.



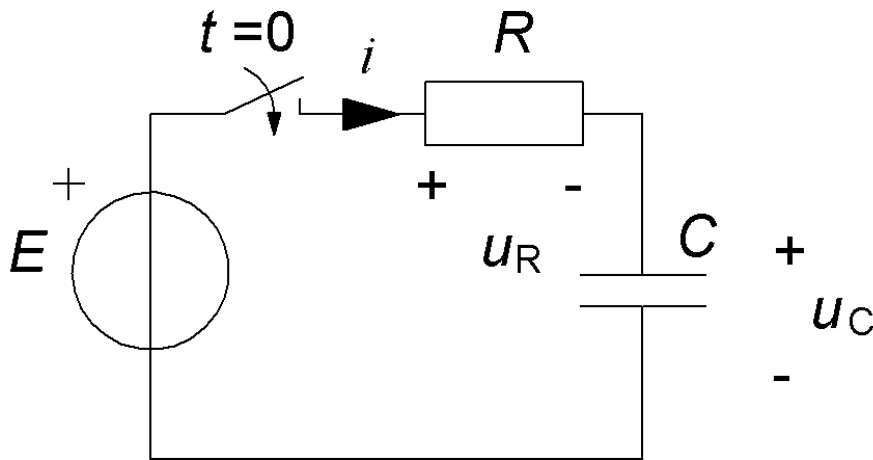
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Capacitor transients

$$\tau = R \cdot C$$



The voltage across the capacitor originates from the collected charge.

$$u_C(t) = \frac{q(t)}{C} = \frac{\int_0^t i(z) dz}{C}$$

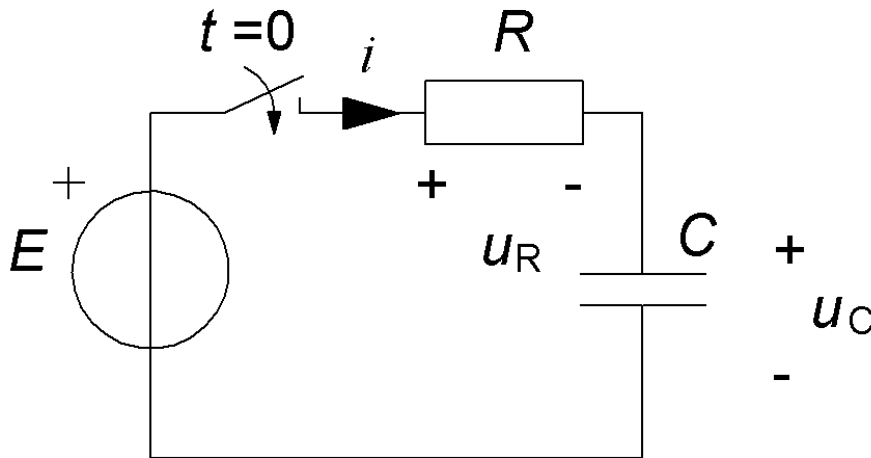
$$E = u_R + u_C \Leftrightarrow E = i(t) \cdot R + \frac{1}{C} \int_0^t i(z) dz$$

$$\frac{d}{dt} E = \frac{d}{dt} i(t) \cdot R + \frac{d}{dt} \frac{1}{C} \int_0^t i(z) dz \Rightarrow 0 = R \frac{di(t)}{dt} + \frac{1}{C} i(t) \Leftrightarrow 0 = R \cdot C \frac{di(t)}{dt} + i(t)$$

$$i(t) = \frac{E}{R} \cdot e^{-\frac{t}{\tau}} \quad \tau = R \cdot C$$

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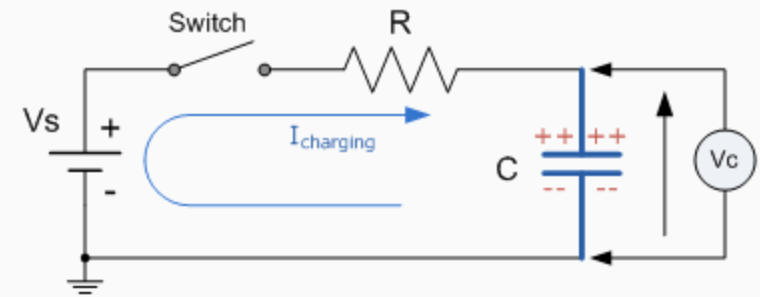
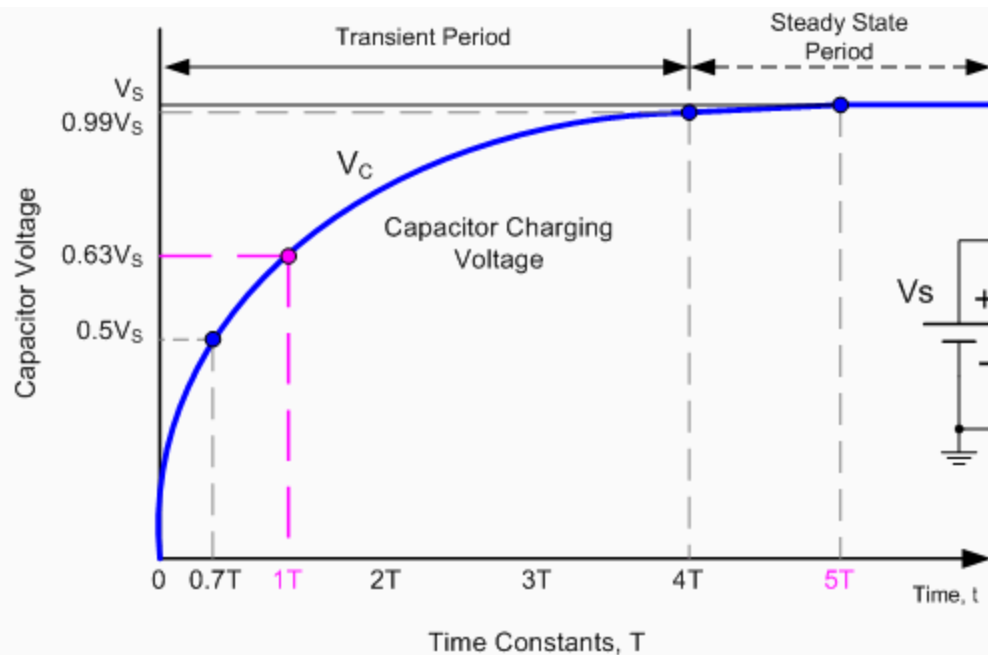
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The differential equation has the solution:

$$\boxed{i(t) = \frac{E}{R} \cdot e^{-\frac{t}{\tau}} \quad \tau = R \cdot C}$$

Charging a capacitor



Time constant
 $T = R \cdot C$

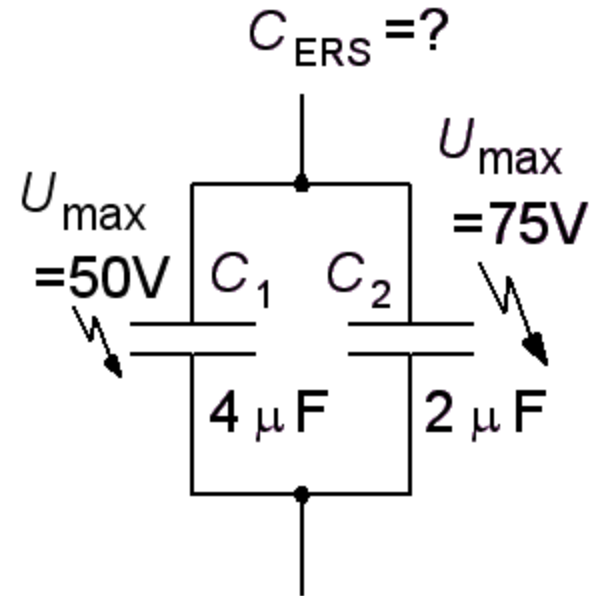
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Parallel connected capacitors

(Ex. 9.3) Two capacitors parallel-connected. What about the equivalent capacitance and its rated voltage?

$$C_1 = 4 \mu\text{F} \ 50\text{V}$$

$$C_2 = 2 \mu\text{F} \ 75\text{V}$$

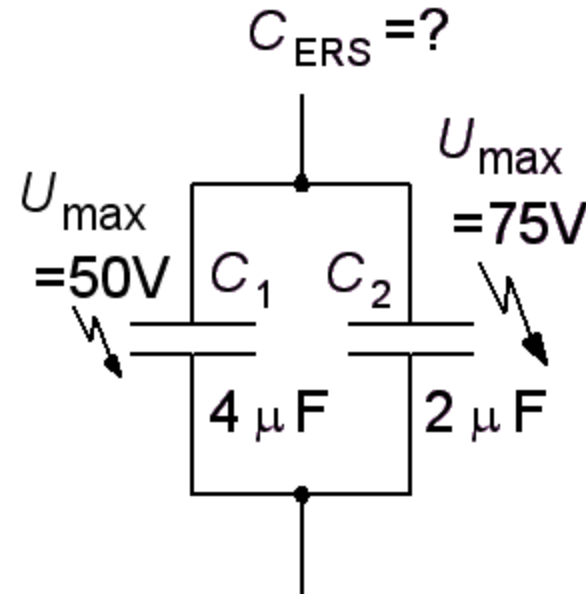


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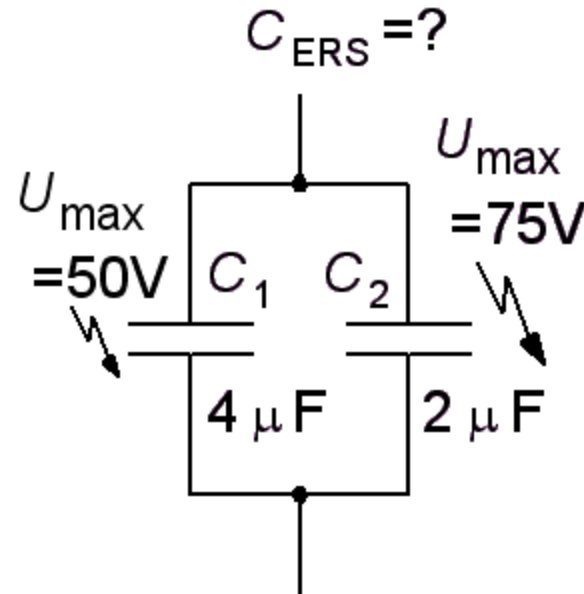
Capacitance is added, the parallel connection is the same as if plate surfaces were added. The capacitor with the worst withstanding voltage determines the equivalent capacitor rated voltage. It is in this capacitor the impact would occur.

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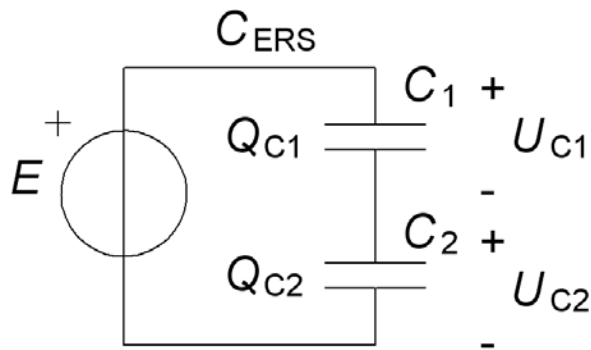
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$$C_{ERS} = C_1 + C_2 = 4 + 2 = 6 \mu\text{F} \ 50\text{V}$$

Series connected capacitors

$$E = U_{C1} + U_{C2} \quad U = \frac{Q}{C} \Rightarrow E = \frac{Q}{C_{ERS}} = \frac{Q_{C1}}{C_{C1}} + \frac{Q_{C2}}{C_{C2}} \quad Q = Q_{C1} = Q_{C2}$$

$$\Rightarrow \frac{1}{C_{ERS}} = \frac{1}{C_{C1}} + \frac{1}{C_{C2}}$$



$$C_{ERS} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Parallel coupling formula for resistors is comparable to series coupling capacitors formula!

In a capacitive voltage divider the voltages are divided inversely with the capacitor capacitances. The smallest capacitor will have the highest voltage – will it withstand it?

Example. Series connected capacitors

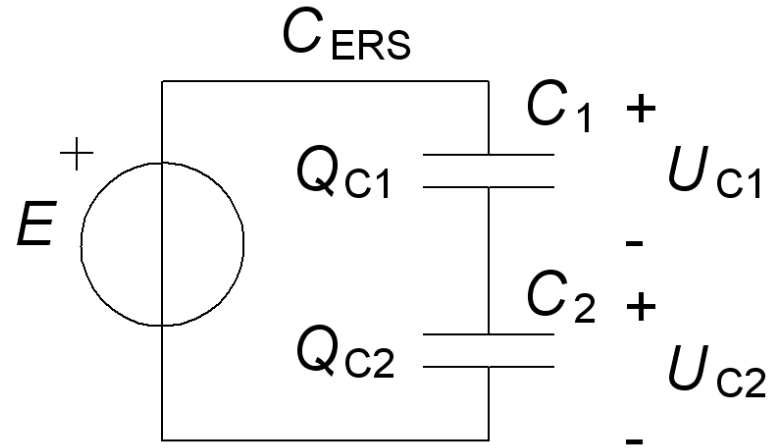
(Ex. 9.4) Two capacitors are connected in series. Calculate the equivalent capacitance and specify how the voltage is divided between the capacitors.

$$E = 10 \text{ V}$$

$$C_1 = 6 \mu\text{F}$$

$$C_2 = 12 \mu\text{F}$$

$$C_{\text{ERS}} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$



Example. Series connected capacitors

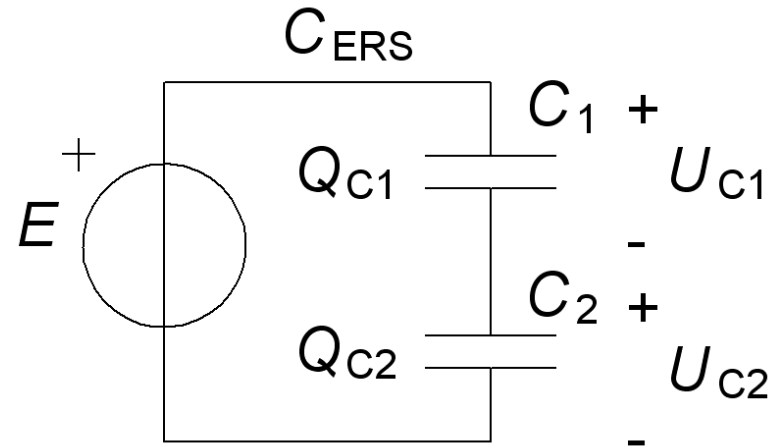
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No current/charge can pass through a capacitor. Two series-connected capacitors must therefore always have the same charge! $Q_{C1} = Q_{C2}$.

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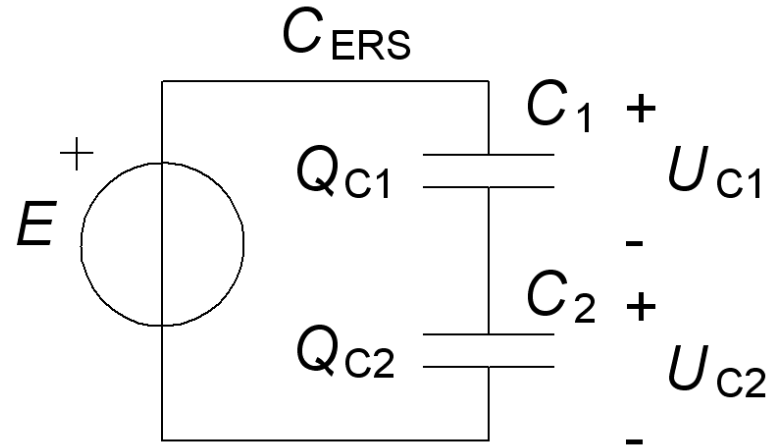
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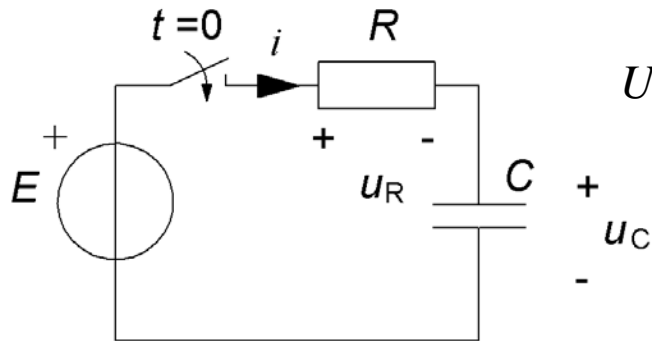
$$Q_{C1} = Q_{C2} = Q = C_{\text{ERS}} \cdot E = C_1 \cdot U_{C1} = C_2 \cdot U_{C2}$$

$$C_{\text{ERS}} = \frac{6 \cdot 12}{6 + 12} = 4 \text{ } \mu\text{F} \quad Q = 4 \cdot 10^{-6} \cdot 10 = 40 \text{ } \mu\text{C}$$

$$U_{C1} = \frac{Q}{C_1} = \frac{40 \cdot 10^{-6}}{6 \cdot 10^{-6}} = 6,66 \text{ V} \quad U_{C2} = E - U_{C1} = 10 - 6,66 = 3,33 \text{ V}$$

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Energy in capacitor



$$U = \frac{Q}{C} \Rightarrow \frac{du_C}{dt} = \frac{1}{C} \cdot \frac{dq}{dt} \Rightarrow \frac{dq}{dt} = i = C \frac{du_C}{dt}$$

Instantaneous power:

$$p = i \cdot u_C = C \frac{du_C}{dt} \cdot u_C \Rightarrow$$

Energy:

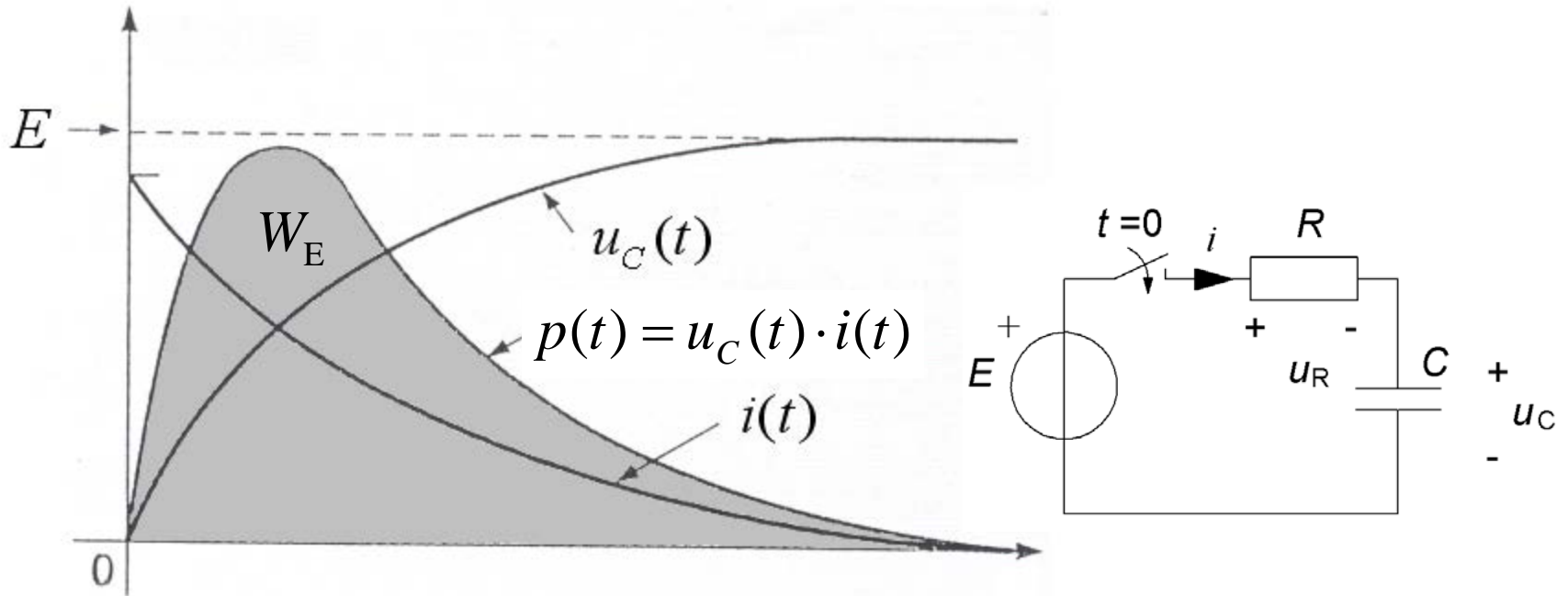
$$W = \int_{t=0}^{t=\infty} p dt = \int_{t=0}^{t=\infty} C \cdot u_C \cdot \frac{du_C}{dt} dt = \int_{u=0}^{u=E} C \cdot u_C du_C = \frac{1}{2} \cdot C \cdot E^2$$

Stored energy in
the electrical field:

$$W_E = \frac{1}{2} C \cdot U^2$$

*Remember the formula, but
its allowed to skip the
derivation ...*

Energy in capacitor

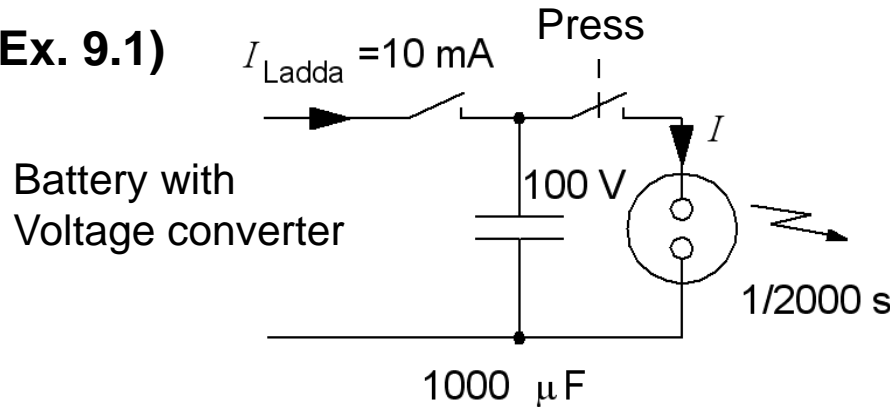


$$W_E = \frac{1}{2} C \cdot E^2$$

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Camera Flash

(Ex. 9.1)



$$W = \frac{1}{2} \cdot C \cdot U^2$$

$$Q = C \cdot U$$

$$I = \frac{Q}{t}$$

$$P = \frac{W}{t}$$

Electric energy in capacitor W ?

$$W = \frac{1}{2} \cdot C \cdot U^2 = \frac{1}{2} \cdot 1000 \cdot 10^{-6} \cdot 100^2 = 5 \text{ J, Ws}$$

Capacitor charge Q ?

$$Q = C \cdot U = 1000 \cdot 10^{-6} \cdot 100 = 0,1 \text{ C, As}$$

The lightning current (mean value) I ?

$$I = \frac{Q}{t} = \frac{0,1}{1/2000} = 200 \text{ A}$$

Power during flash discharge P ?

$$P = \frac{W}{t} = \frac{5}{1/2000} = 10 \text{ kW}$$

How long to wait for next flash t_{Ladda} ?

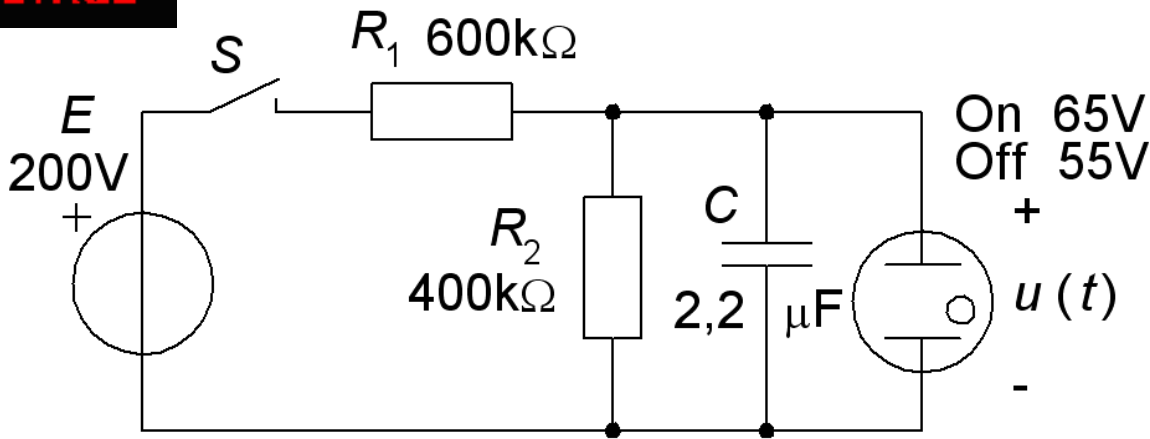
$$U = \frac{Q}{C} = \frac{I_{Ladda} \cdot t_{Ladda}}{C} \Rightarrow t_{Ladda} = \frac{C \cdot U}{I_{Ladda}} = \frac{1000 \cdot 10^{-6} \cdot 100}{10 \cdot 10^{-3}} = 10 \text{ s}$$

Nowdays
LED
Flash?



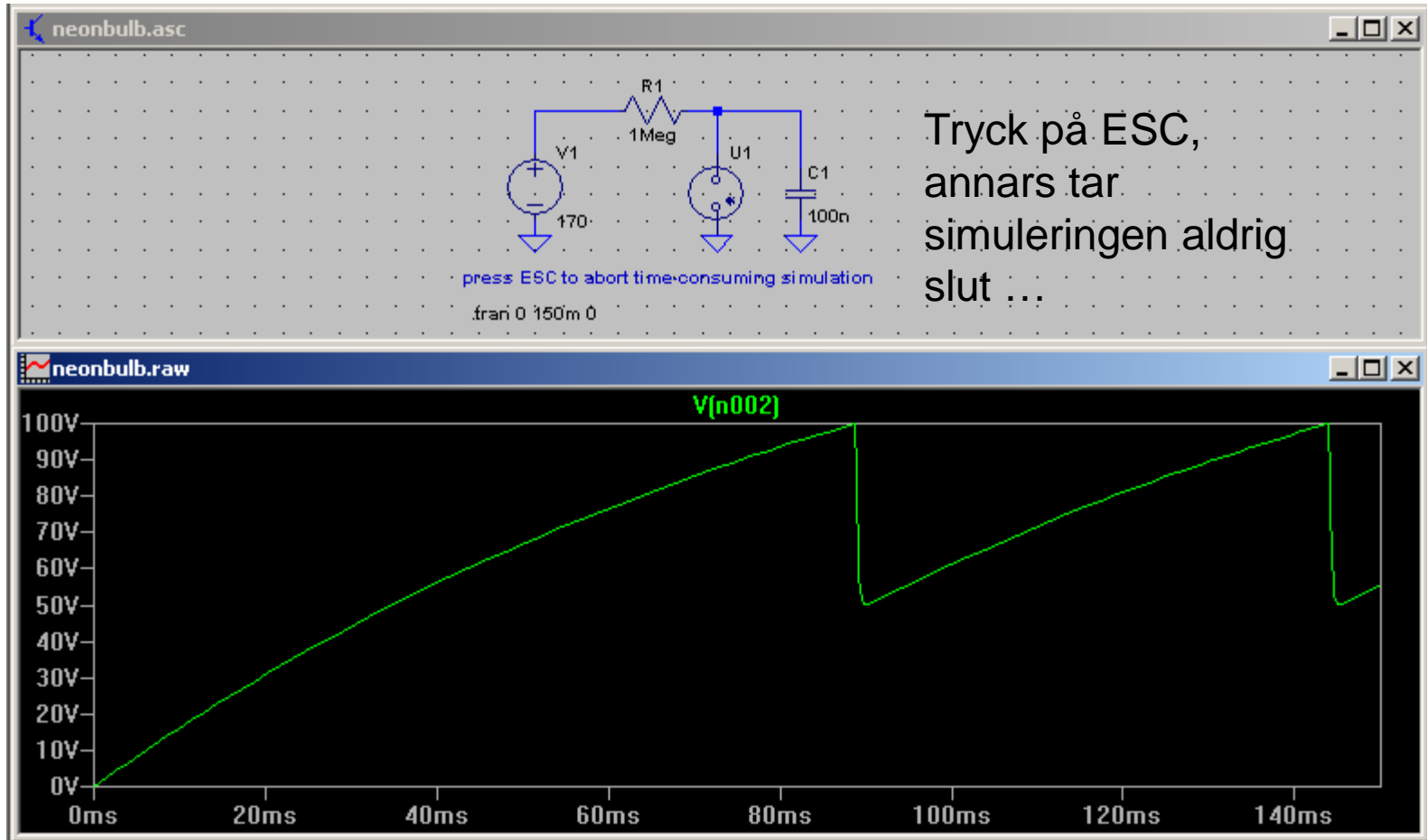
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(Ex. 10.9) Neon lamp



Blink-circuit with Neon-lamp
at exercise ...

Simulate Neon lamp



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