DT2118
Speech and Speaker Recognition
Language Modelling

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Outline

Introduction

Formal Language Theory

Stochastic Language Models (SLM)
  N-gram Language Models
  N-gram Smoothing
  Class N-grams
  Adaptive Language Models

Language Model Evaluation
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Language Model Evaluation
Components of ASR System

Speech Signal → Spectral Analysis → Feature Extraction

Constraints - Knowledge

Acoustic Models → Lexical Models → Language Models

Search and Match

Decoder

Recognised Words
Why do we need language models?

Bayes’ rule:

\[ P(\text{words}|\text{sounds}) = \frac{P(\text{sounds}|\text{words})P(\text{words})}{P(\text{sounds})} \]

where

\( P(\text{words}) \): *a priori* probability of the words

(Language Model)

We could use non informative priors

\( P(\text{words}) = 1/N \), but...
Branching Factor

- if we have $N$ words in the dictionary
- at every word boundary we have to consider $N$ equally likely alternatives
- $N$ can be in the order of millions
Ambiguity

“ice cream” vs “I scream”

/əɪ s k ʌ ɪː m/
Language Models in ASR

We want to:

1. limit the branching factor in the recognition network
2. augment and complete the acoustic probabilities
   - we are only interested to know if the sequence of words is plausible grammatically or not
   - this kind of grammar is integrated in the recognition network prior to decoding
Language Models in Dialogue Systems

- we want to assign a class to each word (noun, verb, attribute... parts of speech)
- parsing is usually performed on the output of a speech recogniser

The grammar is used twice in a Dialogue System!!
Language Models in ASR

- small vocabulary: often **formal** grammar specified by hand
- example: loop of digits as in the HTK exercise
- large vocabulary: often **stochastic** grammar estimated from data
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Formal Language Theory

**grammar:** formal specification of permissible structures for the language

**parser:** algorithm that can analyse a sentence and determine if its structure is compliant with the grammar
Chomsky’s formal grammar

Noam Chomsky: linguist, philosopher, . . .
Chomsky’s formal grammar

Noam Chomsky: linguist, philosopher, . . .

\[ G = (V, T, P, S) \]

where

- \( V \): set of non-terminal constituents
- \( T \): set of terminals (lexical items)
- \( P \): set of production rules
- \( S \): start symbol
Example

\[ S = \text{sentence} \]
\[ V = \{ \text{NP (noun phrase), NP1, VP (verb phrase), NAME, ADJ, V (verb), N (noun)} \} \]
\[ T = \{ \text{Mary, person, loves, that, \ldots} \} \]
\[ P = \{ S \rightarrow \text{NP VP} \]
\[ \text{NP} \rightarrow \text{NAME} \]
\[ \text{NP} \rightarrow \text{ADJ NP1} \]
\[ \text{NP1} \rightarrow \text{N} \]
\[ \text{VP} \rightarrow \text{VERB NP} \]
\[ \text{NAME} \rightarrow \text{Mary} \]
\[ \text{V} \rightarrow \text{loves} \]
\[ \text{N} \rightarrow \text{person} \]
\[ \text{ADJ} \rightarrow \text{that} \} \]
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\[ \text{NP1} \rightarrow \text{N} \]
\[ \text{VP} \rightarrow \text{VERB NP} \]
\[ \text{NAME} \rightarrow \text{Mary} \]
\[ V \rightarrow \text{loves} \]
\[ N \rightarrow \text{person} \]
\[ \text{ADJ} \rightarrow \text{that} \} \]
## Chomsky’s hierarchy

Greek letters: sequence of terminals or non-terminals  
Upper-case Latin letters: single non-terminal  
Lower-case Latin letters: single terminal

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<th>Automata</th>
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<td>Turing machine</td>
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<td>Linear bounded</td>
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Context-free and regular grammars are used in practice
Are languages context-free?

Mostly true, with exceptions

Swiss German:
“...das mer d’chind em Hans es huus lönd häfte aastriiche”

Word-by-word:
“...that we the children Hans the house let help paint”

Translation:
“...that we let the children help Hans paint the house”
Parsers

- assign each word in a sentence to a part of speech
- originally developed for programming languages (no ambiguities)
- only available for context-free and regular grammars
- top-down: start with $S$ and generate rules until you reach the words (terminal symbols)
- bottom-up: start with the words and work your way up until you reach $S$
Example: Top-down parser

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<td>NAME VP</td>
<td>NP → NAME</td>
</tr>
<tr>
<td>Mary VP</td>
<td></td>
</tr>
<tr>
<td>Mary loves NP</td>
<td></td>
</tr>
<tr>
<td>Mary loves ADJ NP</td>
<td></td>
</tr>
<tr>
<td>Mary loves that N</td>
<td></td>
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<tr>
<td>Mary loves that person N</td>
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<td>Mary loves NP</td>
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Top-down vs bottom-up parsers

- **Top-down characteristics:**
  + very predictive
  + only consider grammatical combinations
  - predict constituents that do not have a match in the text

- **Bottom-up characteristics:**
  + check input text only once
  + suitable for robust language processing
  - may build trees that do not lead to full parse

- All in all, similar performance
Chart parsing (dynamic programming)

Mary

loves

that

person

Name[1,1] → Mary
Chart parsing (dynamic programming)

S \rightarrow NP \circ VP

V[2,2] \rightarrow loves

Name \rightarrow Mary
NP \rightarrow Name
S \rightarrow NP \circ VP

Mary
loves
that
person
Chart parsing (dynamic programming)

Mary
loves
that
person

Name → Mary
NP → Name
S → NP °VP

V → loves
VP → V °NP

ADJ → that
Chart parsing (dynamic programming)
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Language Model Evaluation
Stochastic Language Models (SLM)

1. formal grammars lack coverage (for general domains)
2. spoken language does not follow strictly the grammar

Model sequences of words statistically:

\[ P(W) = P(w_1 w_2 \ldots w_n) \]
Probabilistic Context-free grammars (PCFGs)

Assign probabilities to generative rules:

\[ P(A \rightarrow \alpha | G) \]

Then calculate probability of generating a word sequence \( w_1w_2\ldots w_n \) as probability of the rules necessary to go from \( S \) to \( w_1w_2\ldots w_n \):

\[ P(S \Rightarrow w_1w_2\ldots w_n | G) \]
Training PCFGs

If annotated corpus, Maximum Likelihood estimate:

\[
P(A \rightarrow \alpha_j) = \frac{C(A \rightarrow \alpha_j)}{\sum_{i=1}^{m} C(A \rightarrow \alpha_i)}
\]

If non-annotated corpus: **inside-outside algorithm** (similar to HMM training, forward-backward)
Independence assumption

Mary loves that person
Inside-outside probabilities

Chomsky’s normal forms: \( A_i \rightarrow A_mA_n \) or \( A_i \rightarrow w_i \)

\[
\begin{align*}
\text{inside}(s, A_i, t) &= P(A_i \Rightarrow w_sw_{s+1} \ldots w_t) \\
\text{outside}(s, A_i, t) &= P(S \Rightarrow w_1 \ldots w_{s-1} A_i w_{t+1} \ldots w_T)
\end{align*}
\]
Probabilistic Context-free grammars: limitations

- probabilities help sorting alternative explanations, but
- still problem with coverage: the production rules are hand made

\[ P(A \rightarrow \alpha | G) \]
N-gram Language Models

Flat model: no hierarchical structure

\[
P(W) = P(w_1, w_2, \ldots, w_n)
\]
\[
= P(w_1)P(w_2|w_1)P(w_3|w_1, w_2) \cdots P(w_n|w_1, w_2 \ldots, w_{n-1})
\]
\[
= \prod_{i=1}^{n} P(w_i|w_1, w_2, \ldots, w_{i-1})
\]
N-gram Language Models

Flat model: no hierarchical structure

\[
P(\mathbf{w}) = P(w_1, w_2, \ldots, w_n) \\
= P(w_1)P(w_2|w_1)P(w_3|w_1, w_2) \cdots P(w_n|w_1, w_2 \ldots, w_{n-1}) \\
= \prod_{i=1}^{n} P(w_i|w_1, w_2, \ldots, w_{i-1})
\]

Approximations:

- \[P(w_i|w_1, w_2, \ldots, w_{i-1}) = P(w_i)\] (Unigram)
- \[P(w_i|w_1, w_2, \ldots, w_{i-1}) = P(w_i|w_{i-1})\] (Bigram)
- \[P(w_i|w_1, w_2, \ldots, w_{i-1}) = P(w_i|w_{i-2}, w_{i-1})\] (Trigram)
- \[P(w_i|w_1, w_2, \ldots, w_{i-1}) = P(w_i|w_{i-N+1}, \ldots, w_{i-1})\] (N-gram)
Example (Bigram)

\[
P(Mary, loves, that, person) = \\
\ P(Mary|<s>) P(loves|Mary) P(that|loves) \\
\ P(person|that) P(</s>|person)
\]
N-gram estimation (Maximum Likelihood)

\[
P(w_i | w_{i-N+1}, \ldots, w_{i-1}) = \frac{C(w_{i-N+1}, \ldots, w_{i-1}, w_i)}{C(w_{i-N+1}, \ldots, w_{i-1})} \\
= \frac{C(w_{i-N+1}, \ldots, w_{i-1}, w_i)}{\sum_{w_i} C(w_{i-N+1}, \ldots, w_{i-1}, w_i)}
\]
N-gram estimation (Maximum Likelihood)

\[ P(w_i | w_{i-N+1}, \ldots, w_{i-1}) = \frac{C(w_{i-N+1}, \ldots, w_{i-1}, w_i)}{\sum_{w_i} C(w_{i-N+1}, \ldots, w_{i-1}, w_i)} \]

Problem: data sparseness
N-gram estimation example

Corpus:

1: John read her book
2: I read a different book
3: John read a book by Mulan

\[
P(\text{John} | \langle s \rangle) = \frac{c(\langle s \rangle, \text{John})}{c(\langle s \rangle)} = \frac{2}{3}
\]

\[
P(\text{read} | \text{John}) = \frac{c(\text{John}, \text{read})}{c(\text{John})} = \frac{2}{2}
\]

\[
P(\text{a} | \text{read}) = \frac{c(\text{read}, \text{a})}{c(\text{read})} = \frac{2}{3}
\]

\[
P(\text{book} | \text{a}) = \frac{c(\text{a}, \text{book})}{c(\text{a})} = \frac{1}{2}
\]

\[
P(\langle /s > | \text{book}) = \frac{c(\text{book}, \langle /s >)}{c(\text{book})} = \frac{2}{3}
\]
N-gram estimation example

1: John read her book
Corpus: 2: I read a different book
  3: John read a book by Mulan

\[
P(\text{John} \mid <s>) = \frac{c(<s>, \text{John})}{c(<s>)} = \frac{2}{3}
\]
\[
P(\text{read} \mid \text{John}) = \frac{c(\text{John}, \text{read})}{c(\text{John})} = \frac{2}{2}
\]
\[
P(\text{a} \mid \text{read}) = \frac{c(\text{read}, \text{a})}{c(\text{read})} = \frac{2}{3}
\]
\[
P(\text{book} \mid \text{a}) = \frac{c(\text{a}, \text{book})}{c(\text{a})} = \frac{1}{2}
\]
\[
P(< /s > \mid \text{book}) = \frac{c(\text{book}, < /s >)}{c(\text{book})} = \frac{2}{3}
\]

\[
P(\text{John}, \text{read}, \text{a}, \text{book}) = P(\text{John} \mid <s>)P(\text{read} \mid \text{John})P(\text{a} \mid \text{read}) \cdots P(\text{book} \mid \text{a})P(< /s > \mid \text{book}) = 0.148
\]
N-gram estimation example

1: John read her book
Corpus: 2: I read a different book
3: John read a book by Mulan

\[
P(John \mid <s>) = \frac{c(<s>, John)}{c(<s>)} = \frac{2}{3}
\]

\[
P(read \mid John) = \frac{c(John, read)}{c(John)} = \frac{2}{2}
\]

\[
P(a \mid read) = \frac{c(read, a)}{c(read)} = \frac{2}{3}
\]

\[
P(book \mid a) = \frac{c(a, book)}{c(a)} = \frac{1}{2}
\]

\[
P(< /s > \mid book) = \frac{c(book, < /s >)}{c(book)} = \frac{2}{3}
\]

\[
P(John, read, a, book) = P(John \mid <s>)P(read \mid John)P(a \mid read) \cdots P(book \mid a)P(< /s > \mid book) = 0.148
\]

\[
P(Mulan, read, a, book) = P(Mulan \mid <s>) \cdots = 0
\]
N-gram Smoothing

Problem:

- Many very possible word sequences may have been observed in zero or very low numbers in the training data
- Leads to extremely low probabilities, effectively disabling this word sequence, no matter how strong the acoustic evidence is

Solution: smoothing

- produce more robust probabilities for unseen data at the cost of modelling the training data slightly worse
Simplest Smoothing technique

Instead of ML estimate

\[ P(w_i|w_{i-N+1}, \ldots, w_{i-1}) = \frac{C(w_{i-N+1}, \ldots, w_{i-1}, w_i)}{\sum_{w_i} C(w_{i-N+1}, \ldots, w_{i-1}, w_i)} \]

Use

\[ P(w_i|w_{i-N+1}, \ldots, w_{i-1}) = \frac{1 + C(w_{i-N+1}, \ldots, w_{i-1}, w_i)}{\sum_{w_i} (1 + C(w_{i-N+1}, \ldots, w_{i-1}, w_i))} \]

- prevents zero probabilities
- but still very low probabilities
N-gram simple smoothing example

1: John read her book

Corpus: 2: I read a different book
3: John read a book by Mulan

\[
P(\text{John}|<s>) = \frac{1+c(<s>,\text{John})}{11+c(<s>)} = \frac{3}{14}
\]

\[
P(\text{read}|\text{John}) = \frac{1+c(\text{John},\text{read})}{11+c(\text{John})} = \frac{3}{13}
\]

\[
\vdots
\]

\[
P(\text{Mulan}|<s>) = \frac{1+c(<s>,\text{Mulan})}{11+c(<s>)} = \frac{1}{14}
\]
N-gram simple smoothing example

Corpus:
1: John read her book
2: I read a different book
3: John read a book by Mulan

\[ P(\text{John} \mid < s >) = \frac{1 + c(<s>, \text{John})}{11 + c(<s>)} = \frac{3}{14} \]
\[ P(\text{read} \mid \text{John}) = \frac{1 + c(\text{John}, \text{read})}{11 + c(\text{John})} = \frac{3}{13} \]
\[ \ldots \]
\[ P(\text{Mulan} \mid < s >) = \frac{1 + c(<s>, \text{Mulan})}{11 + c(<s>)} = \frac{1}{14} \]

\[ P(\text{John, read, a, book}) = \ P(\text{John} \mid < s >)P(\text{read} \mid \text{John})P(\text{a} \mid \text{read}) \cdots \]
\[ P(\text{book} \mid \text{a})P(< /s > \mid \text{book}) = 0.00035(0.148) \]
N-gram simple smoothing example

Corpus:
1: John read her book
2: I read a different book
3: John read a book by Mulan

\[
P(John \mid <s>) = \frac{1 + c(<s>, John)}{11 + c(<s>)} = \frac{3}{14}
\]

\[
P(read \mid John) = \frac{1 + c(John, read)}{11 + c(John)} = \frac{3}{13}
\]

\[
P(Mulan \mid <s>) = \frac{1 + c(<s>, Mulan)}{11 + c(<s>)} = \frac{1}{14}
\]

\[
P(John, read, a, book) = P(John \mid <s>)P(read \mid John)P(a \mid read) \cdots
\]
\[
P(book \mid a)P(<s> \mid book) = 0.00035(0.148)
\]

\[
P(Mulan, read, a, book) = P(Mulan \mid <s>)P(read \mid Mulan)P(a \mid read) \cdots
\]
\[
P(book \mid a)P(<s> \mid book) = 0.000084(0)
\]
Interpolation vs Backoff smoothing

Interpolation models:

▶ Linear combination with lower order n-grams
▶ Modifies the probabilities of both nonzero and zero count n-grams

Backoff models:

▶ Use lower order n-grams when the requested n-gram has zero or very low count in the training data
▶ Nonzero count n-grams are unchanged
▶ Discounting: Reduce the probability of seen n-grams and distribute among unseen ones
Interpolation vs Backoff smoothing

**Interpolation models:**

\[
P_{\text{smooth}}(w_i|w_{i-N+1}, \ldots, w_{i-1}) = \lambda\underbrace{P_{\text{ML}}(w_i|w_{i-N+1}, \ldots, w_{i-1})}_{N-1} + (1 - \lambda)\underbrace{P_{\text{smooth}}(w_i|w_{i-N+2}, \ldots, w_{i-1})}_{N}
\]

**Backoff models:**

\[
P_{\text{smooth}}(w_i|w_{i-N+1}, \ldots, w_{i-1}) = \begin{cases} 
\alpha\underbrace{P(w_i|w_{i-N+1}, \ldots, w_{i-1})}_{N-1} & \text{if } C(w_i|w_{i-N+1}, \ldots, w_{i-1}) > 0 \\
\gamma P_{\text{smooth}}(w_i|w_{i-N+2}, \ldots, w_{i-1}) & \text{if } C(w_i|w_{i-N+1}, \ldots, w_{i-1}) = 0
\end{cases}
\]
Deleted interpolation smoothing

Recursively interpolate with n-grams of lower order:
if \( \text{history}_n = w_{i-n+1}, \ldots, w_{i-1} \)

\[
P_I(w_i|\text{history}_n) = \lambda_{\text{history}_n} P(w_i|\text{history}_n) + (1 - \lambda_{\text{history}_n}) P_I(w_i|\text{history}_{n-1})
\]

- hard to estimate \( \lambda_{\text{history}_n} \) for every history
- cluster into moderate number of weights
Backoff smoothing

Use $P(w_i|\text{history}_{n-1})$ only if you lack data for $P(w_i|\text{history}_n)$
Good-Turing estimate

- Partition n-grams into groups depending on their frequency in the training data
- Change the number of occurrences of an n-gram according to

$$r^* = (r + 1) \frac{n_{r+1}}{n_r}$$

where $r$ is the occurrence number, $n_r$ is the number of n-grams that occur $r$ times
Katz smoothing

based on Good-Turing: combine higher and lower order n-grams

For every N-gram:

1. if count $r$ is large ($> 5$ or $8$), do not change it
2. if count $r$ is small but non-zero, discount with $\approx r^*$
3. if count $r = 0$, reassign discounted counts with lower order N-gram

$$C^*(w_{i-1}, w_i) = \alpha(w_{i-1})P(w_i)$$
Kneser-Ney smoothing: motivation

Background

- Lower order n-grams are often used as backoff model if the count of a higher-order n-gram is too low (e.g. unigram instead of bigram)

Problem

- Some words with relatively high unigram probability only occur in a few bigrams. E.g. Francisco, which is mainly found in San Francisco. However, infrequent word pairs, such as New Francisco, will be given too high probability if the unigram probabilities of New and Francisco are used. Maybe instead, the Francisco unigram should have a lower value to prevent it from occurring in other contexts.

I can’t see without my reading...
Kneser-Ney intuition

If a word has been seen in many contexts it is more likely to be seen in new contexts as well.

- instead of backing off to lower order n-gram, use continuation probability

Example: instead of unigram $P(w_i)$, use

$$P_{CONTINUATION}(w_i) = \frac{|\{w_{i-1} : C(w_{i-1}w_i) > 0\}|}{\sum_{w_i} |\{w_{i-1} : C(w_{i-1}w_i) > 0\}|}$$
Kneser-Ney intuition

If a word has been seen in many contexts it is more likely to be seen in new contexts as well.

▶ instead of backing off to lower order n-gram, use continuation probability

Example: instead of unigram $P(w_i)$, use

$$P_{CONTINUATION}(w_i) = \frac{|\{w_{i-1} : C(w_{i-1}w_i) > 0\}|}{\sum_{w_i} |\{w_{i-1} : C(w_{i-1}w_i) > 0\}|}$$

I can’t see without my reading... glasses
Class N-grams

1. Group words into semantic or grammatical classes

2. build n-grams for class sequences:

\[ P(w_i | c_{i-N+1} \ldots c_{i-1}) = P(w_i | c_i)P(c_i | c_{i-N+1} \ldots c_{i-1}) \]

- rapid adaptation, small training sets, small models
- works on limited domains
- classes can be rule-based or data-driven
Combining PCFGs and N-grams

Only N-grams:

Meeting at three with Zhou Li
Meeting at four PM with Derek

\[ P(\text{Zhou}|\text{three, with}) \text{ and } P(\text{Derek}|\text{PM, with}) ]

N-grams + CFGs:

Meeting \{at three: TIME\} with \{Zhou Li: NAME\}
Meeting \{at four PM: TIME\} with \{Derek: NAME\}

\[ P(\text{NAME}|\text{TIME, with}) \]
Adaptive Language Models

- conversational topic is not stationary
- topic stationary over some period of time
- build more specialised models that can adapt in time

Techniques

- Cache Language Models
- Topic-Adaptive Models
- Maximum Entropy Models
Cache Language Models

1. build a full static n-gram model
2. during conversation accumulate low order n-grams
3. interpolate between 1 and 2
Topic-Adaptive Models

1. cluster documents into topics (manually or data-driven)
2. use information retrieval techniques with current recognition output to select the right cluster
3. if off-line run recognition in several passes
Maximum Entropy Models

Instead of linear combination:

1. reformulate information sources into constraints
2. choose maximum entropy distribution that satisfies the constraints
Maximum Entropy Models

Instead of linear combination:

1. reformulate information sources into constraints
2. choose maximum entropy distribution that satisfies the constraints

Constraints general form:

$$\sum_{X} P(X) f_i(X) = E_i$$

Example: unigram

$$f_{w_i} = \begin{cases} 
1 & \text{if } w = w_i \\
0 & \text{otherwise} 
\end{cases}$$
Outline

Introduction

Formal Language Theory

Stochastic Language Models (SLM)
  N-gram Language Models
  N-gram Smoothing
  Class N-grams
  Adaptive Language Models

Language Model Evaluation
Language Model Evaluation

- Evaluation in combination with Speech Recogniser
  - hard to separate contribution of the two
- Evaluation based on probabilities assigned to text in the training and test set
Information, Entropy, Perplexity

Information:

\[ I(x_i) = \log \frac{1}{P(x_i)} \]

Entropy:

\[ H(X) = E[I(X)] = -\sum_i P(x_i) \log P(x_i) \]

Perplexity:

\[ PP(X) = 2^{H(X)} \]
Perplexity of a model

We do not know the “true” distribution $p(w_1, \ldots, w_n)$. But we have a model $m(w_1, \ldots, w_n)$. The cross-entropy is:

$$H(p, m) = - \sum_{w_1, \ldots, w_n} p(w_1, \ldots, w_n) \log m(w_1, \ldots, w_n)$$

Cross-entropy is upper bound to entropy:

$$H \leq H(p, m)$$

The better the model, the lower the cross-entropy and the lower the perplexity (on the same data)
Test-set Perplexity

Estimate the distribution $p(w_1, \ldots, w_n)$ on the training data
Evaluate it on the test data

$$H = - \sum_{w_1, \ldots, w_n \in \text{test set}} p(w_1, \ldots, w_n) \log p(w_1, \ldots, w_n)$$

$$PP = 2^H$$
Perplexity and branching factor

Perplexity is roughly the geometric mean of the branching factor

\[ \text{word} \rightarrow \text{word1} \]
\[ \text{word} \rightarrow \text{word2} \]
\[ \text{...} \]
\[ \text{word}/N \]

Shannon: 2.39 for English letters and 130 for English words
Digit strings: 10
n-gram English: 50–1000
Wall Street Journal test set: 180 (bigram) 91 (trigram)
Performance of N-grams

<table>
<thead>
<tr>
<th>Models</th>
<th>Perplexity</th>
<th>Word Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unigram Katz</td>
<td>1196.45</td>
<td>14.85%</td>
</tr>
<tr>
<td>Unigram Kneser-Ney</td>
<td>1199.59</td>
<td>14.86%</td>
</tr>
<tr>
<td>Bigram Katz</td>
<td>176.31</td>
<td>11.38%</td>
</tr>
<tr>
<td>Bigram Kneser-Ney</td>
<td>176.11</td>
<td>11.34%</td>
</tr>
<tr>
<td>Trigram Katz</td>
<td>95.19</td>
<td>9.69%</td>
</tr>
<tr>
<td>Trigram Kneser-Ney</td>
<td>91.47</td>
<td>9.60%</td>
</tr>
</tbody>
</table>

Wall Street Journal database Dictionary: 60 000 words
Training set: 260 000 000 words