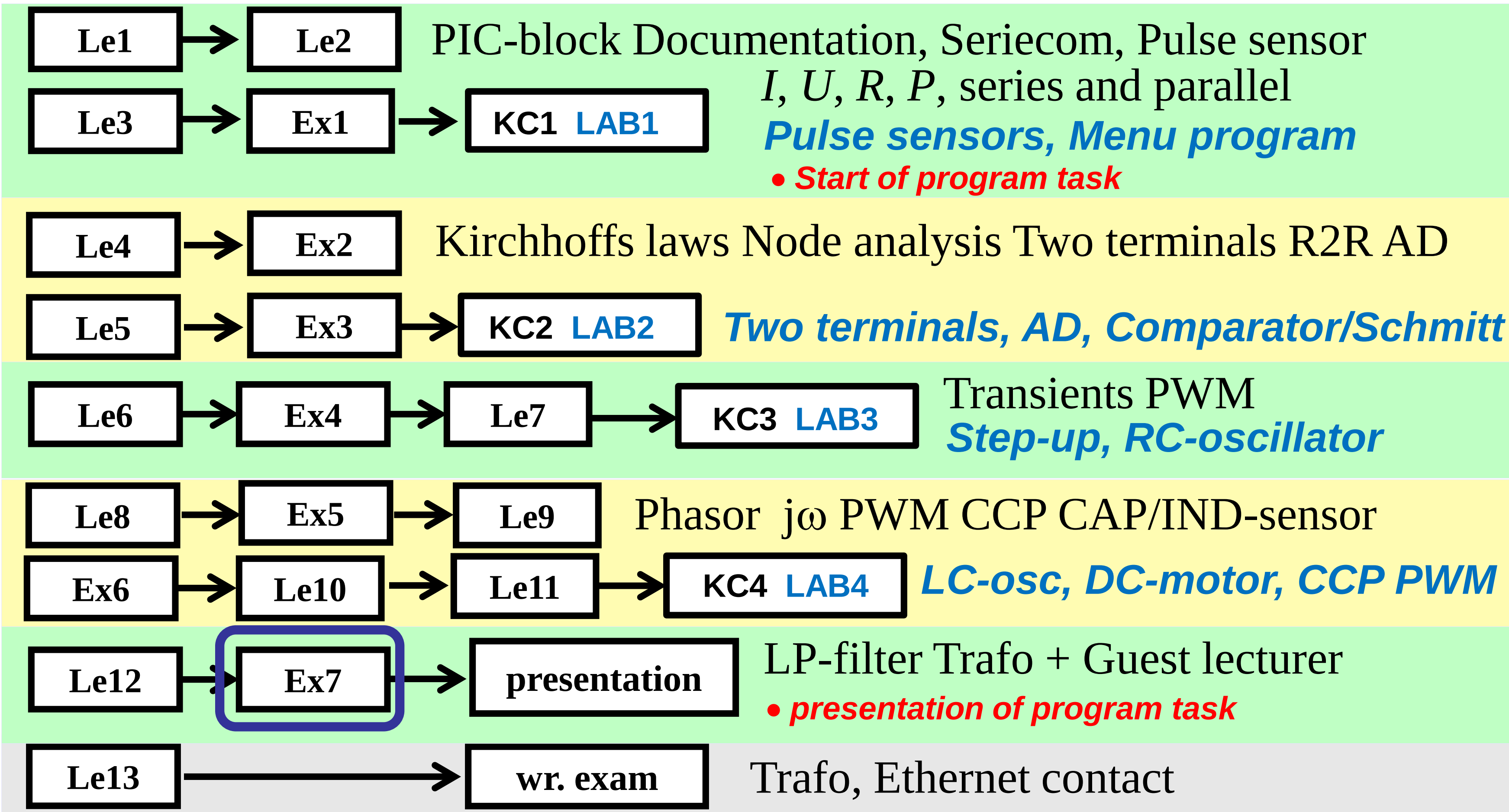
IE1206 Embedded Electronics



William Sandqvist william@kth.se

Complex phasors, jω-method

- Complex OHM's law for R L and C.

$$\underline{U}_{\mathrm{R}} = \underline{I}_{\mathrm{R}} \cdot \boxed{R}$$

$$\underline{U}_{\mathrm{L}} = \underline{I}_{\mathrm{L}} \cdot \mathrm{j} X_{\mathrm{L}} = \underline{I}_{\mathrm{L}} \cdot \boxed{\mathrm{j}\omega L} \qquad \omega = 2\pi \cdot f$$

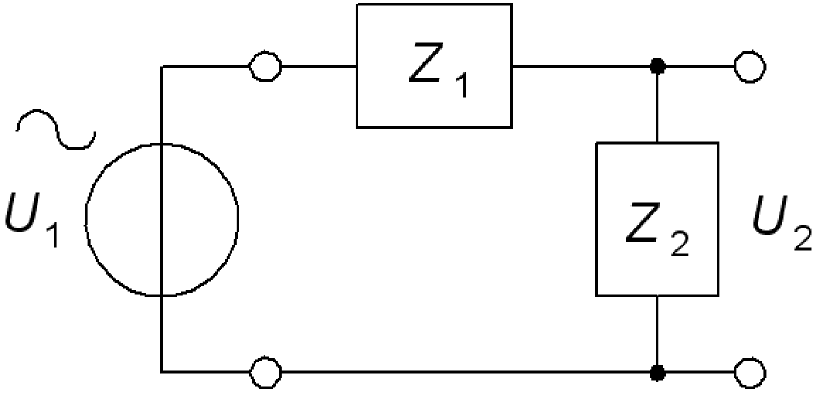
$$\underline{U}_{\mathrm{C}} = \underline{I}_{\mathrm{C}} \cdot \mathrm{j} X_{\mathrm{C}} = \underline{I}_{\mathrm{C}} \cdot \boxed{\frac{1}{\mathrm{j}\omega C}}$$

- Complex OHM's law for Z.

$$\boxed{\underline{U} = \underline{I} \cdot \underline{Z}} \qquad Z = \frac{U}{I} \qquad \varphi = \arg(\underline{Z}) = \arctan\left(\frac{\mathrm{Im}[\underline{Z}]}{\mathrm{Re}[\underline{Z}]}\right)$$

Voltage divider, Transfer function

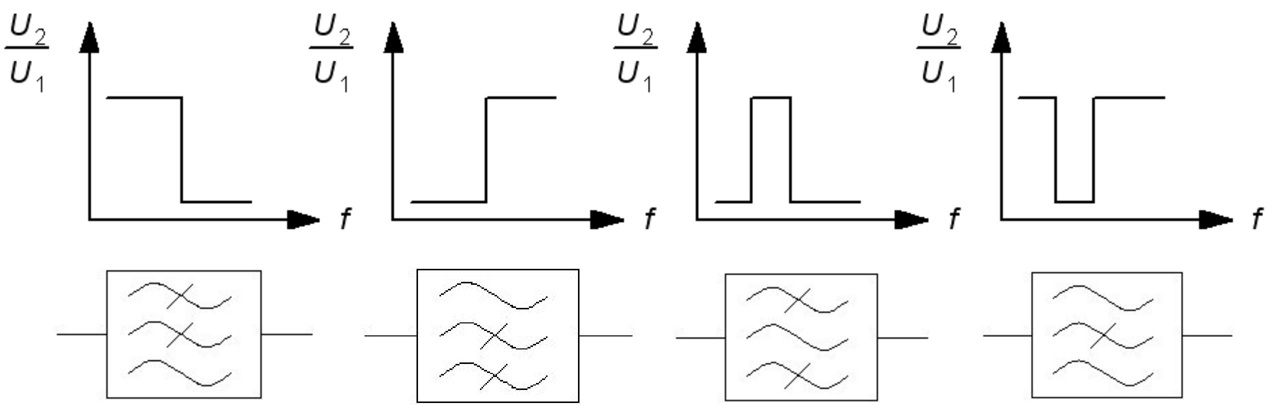
Simple filters are often designed as a voltage dividers.A filter **transfer function**, $H(\omega)$ or $H(f)$, is the ratio between output voltage and input voltage. This ratio we get directly from the voltage divider formula!



$$\underline{U}_2 = \underline{U}_1 \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \quad \Rightarrow \quad \boxed{\underline{H}(\omega) = \frac{\underline{U}_2}{\underline{U}_1} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}}$$

LP HP BP BS

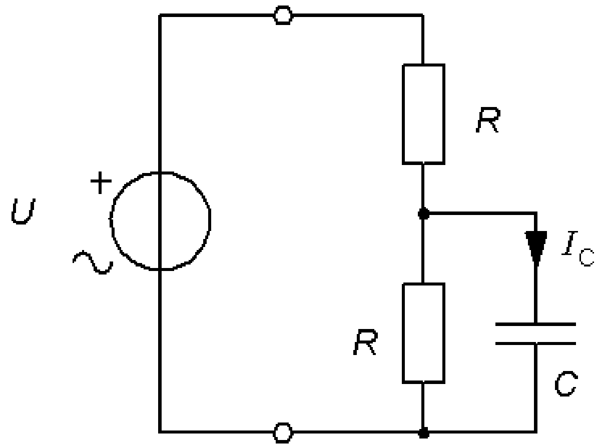
LP lowpass HP highpass BP bandpass BS bandstop



BP and BS filters can be seen as different combination of LP and HP filters.

William Sandqvist william@kth.se

Transfer function (14.2)



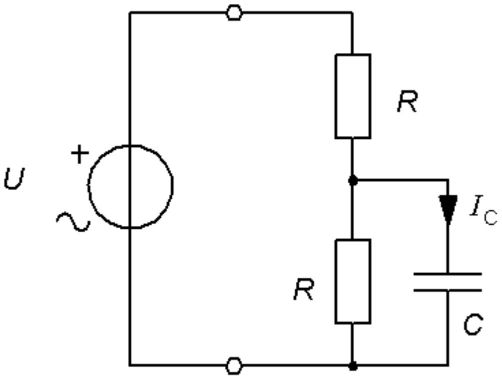
a) Set up an expression of $I_C = f(U, \omega, R, C)$.

b) Set up the transfer function I_C/U the **amount function** and the **phase function**.

c) What filter type is the transfer function, LP HP BP BS ?

d) What break frequency has the transfer function?

Transfer function (14.2)



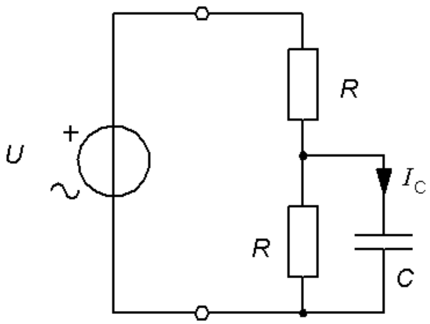
Answer a)

$$R \,\|\, C = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{R}{1 + j\omega RC}$$

$$\underline{I}_C = \frac{\underline{U}_C}{\frac{1}{j\omega C}} = \underline{U}_C \cdot j\omega C$$

William Sandqvist william@kth.se

Transfer function (14.2)

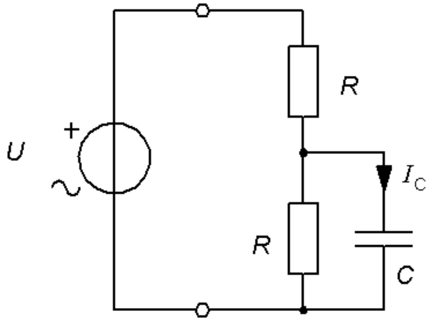


$$\underline{U}_{\mathrm{C}} = \underline{U}\,\frac{\dfrac{R}{1+\mathrm{j}\omega RC}}{R+\dfrac{R}{1+\mathrm{j}\omega RC}}\cdot\frac{\dfrac{1+\mathrm{j}\omega RC}{R}}{\dfrac{1+\mathrm{j}\omega RC}{R}} = \underline{U}\,\frac{1}{1+\mathrm{j}\omega RC+1} \quad \Rightarrow$$

$$\underline{I}_C = \underline{U}\,\frac{\mathrm{j}\omega C}{2+\mathrm{j}\omega RC}$$

Transfer function (14.2)

Answer b) I_C/U



$$\frac{\underline{I}_C}{\underline{U}} = \frac{j\omega C}{2 + j\omega RC}$$

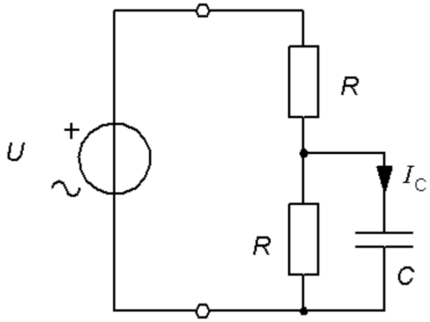
$$\frac{I_C}{U} = \frac{\omega C}{\sqrt{4 + (\omega RC)^2}}$$

$$\arg\left(\frac{\underline{I}_C}{\underline{U}}\right) = 90° - \arctan\left(\frac{\omega RC}{2}\right)$$

$$\arg\left(\frac{\underline{I}_C}{\underline{U}}\right) = \arctan\left(\frac{2}{\omega RC}\right)$$

Transfer function (14.2)

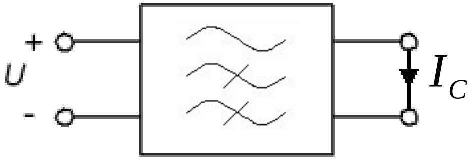
Answer c) LP HP BP BS?



$$\frac{\underline{I}_C}{\underline{U}} = \frac{\mathrm{j}\omega C}{2 + \mathrm{j}\omega RC}$$

$$\frac{\underline{I}_C}{\underline{U}}\left\{\omega = 0\right\} = \frac{0 \cdot j}{2 + 0 \cdot j} = 0 \quad \frac{\underline{I}_C}{\underline{U}}\left\{\omega = \infty\right\} = \frac{1}{R}$$

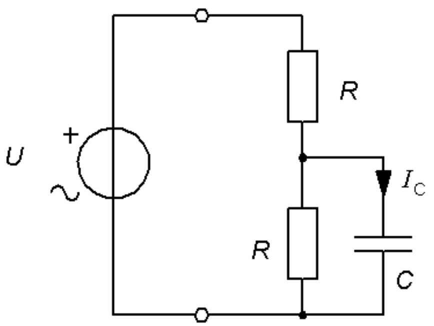
$\Rightarrow$ HP



Transfer function (14.2)

Answer d) Break frequency?

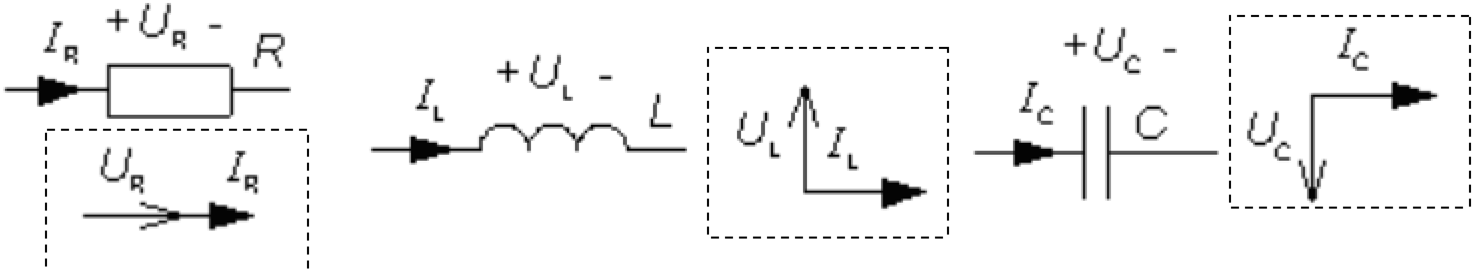
At the break frequency the numerator real part and imaginary part are equal.



$$\frac{\underline{I}_C}{\underline{U}} = \frac{\mathrm{j}\omega C}{2 + \mathrm{j}\omega RC} \qquad \omega RC = 2 \quad \Rightarrow \quad f_G = \frac{1}{2\pi} \cdot \frac{2}{RC}$$

$$\frac{\underline{I}_C}{\underline{U}} = \frac{\mathrm{j}\omega C}{2 + \mathrm{j}\omega RC} = \frac{\mathrm{j}\frac{2}{\mathrm{R}}}{2 + \mathrm{j}2} \quad \Rightarrow \quad \frac{I_C}{U} = \frac{\frac{2}{R}}{\sqrt{2^2 + 2^2}} = \frac{1}{R \cdot \sqrt{2}}$$

Phasor - vector



$$\omega = 2\pi\ f$$

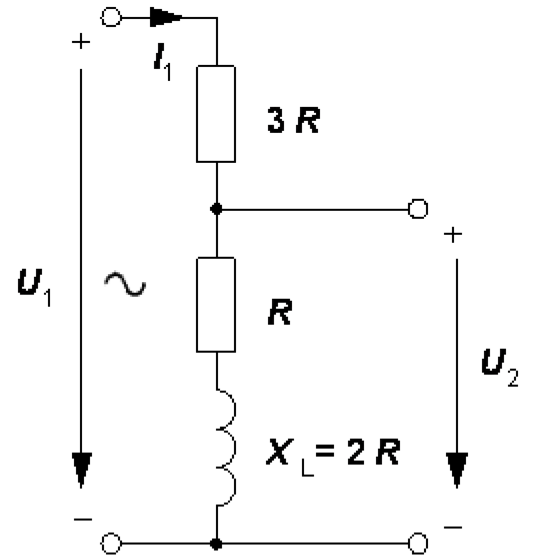
$$\left|X_L\right| = \omega \cdot L$$

$$\left|X_C\right| = \frac{1}{\omega \cdot C}$$

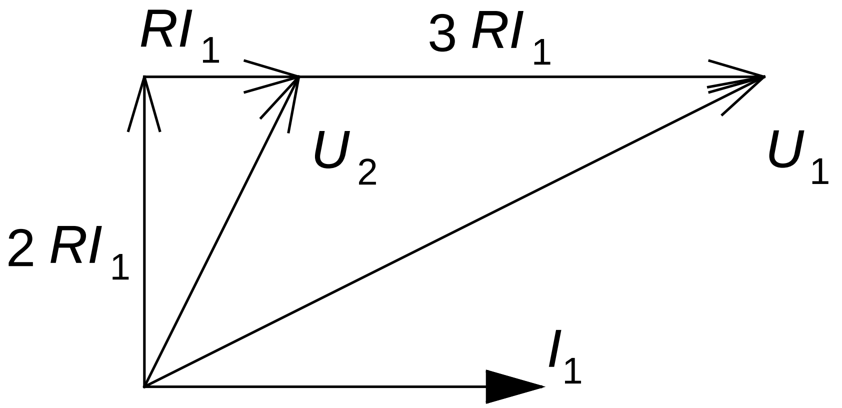
$$\boxed{Z = \frac{U}{I}}$$

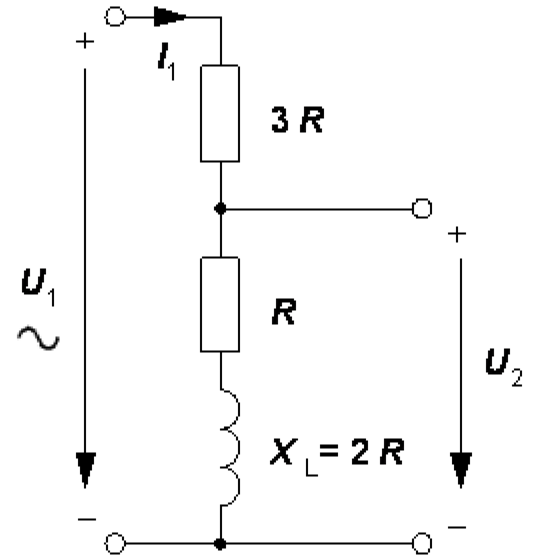
Phasor chart for voltage divider (11.8)

The figure shows a voltage divider. It is connected to an AC voltage source U_1 and it's output voltage is U_2. At a some frequency the reactance of the inductor is X_L = 2R.
Draw the phasor chart of this circuit with I_1, U_1 and U_2 at this frequency.
Use I_1 as reference phase (= horizontal).



Phasor chart for voltage divider (11.8)



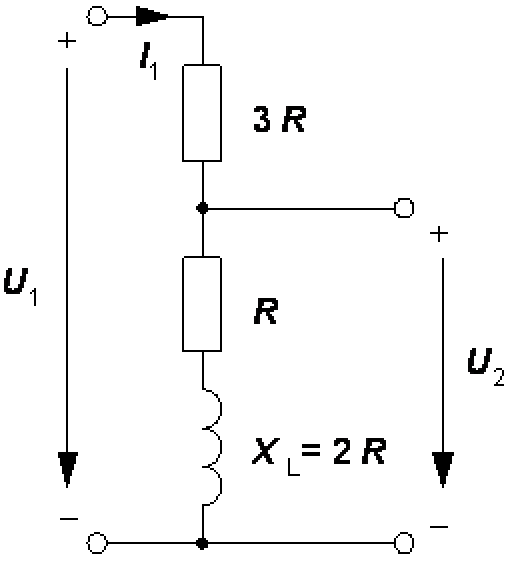


jω-calculation of the divided voltage

$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{R + \mathrm{j}\omega L}{4R + \mathrm{j}\omega L} \qquad \frac{U_2}{U_1} = \frac{\sqrt{R^2 + (\omega L)^2}}{\sqrt{16R^2 + (\omega L)^2}}$$

$$X_L = \omega L = 2R \quad \Rightarrow$$

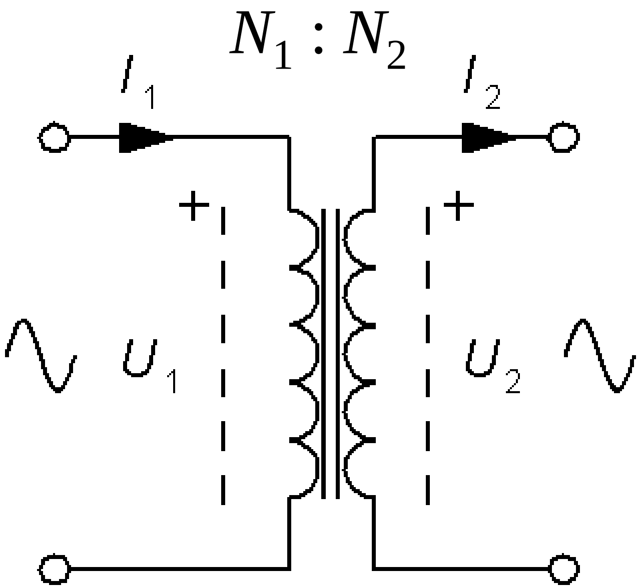
$$\boxed{\frac{U_2}{U_1}} = \frac{\sqrt{R^2 + (2R)^2}}{\sqrt{16R^2 + (2R)^2}} = \frac{\sqrt{5}}{\sqrt{20}} = \boxed{\frac{1}{2}}$$



Voltage ratio

$$U_1 = N_1 \frac{d\Phi}{dt} \quad U_2 = N_2 \frac{d\Phi}{dt}$$

$$\frac{U_1}{U_2} = \frac{N_1}{N_2}$$

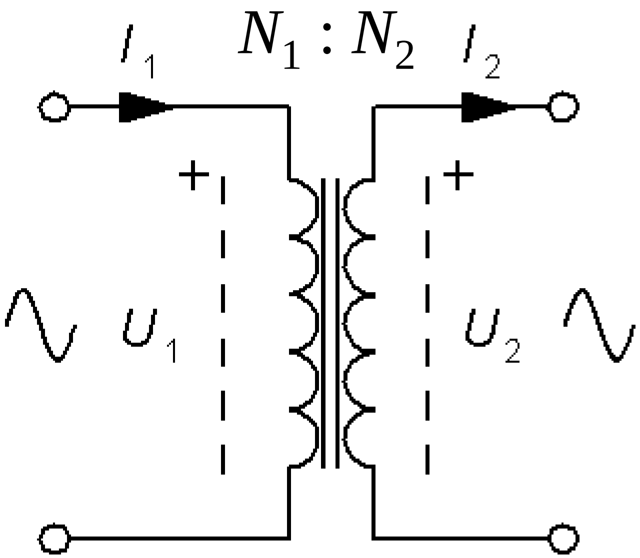


Current ratio

$$P_1 = P_2 \quad (P_0, I_0 = 0)$$

$$U_1 \cdot I_1 = U_2 \cdot I_2 \quad \Rightarrow$$

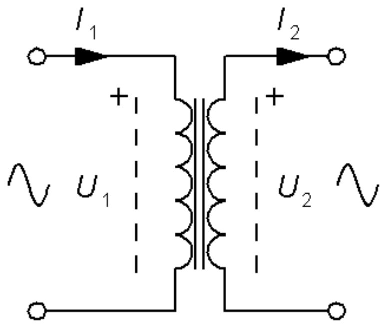
$$\frac{I_2}{I_1} \approx \frac{U_1}{U_2} = \frac{N_1}{N_2}$$



Two values are missing? (15.1)

For a transformer the following data was given:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
600	225 V	**?**	200	**?**	9 A

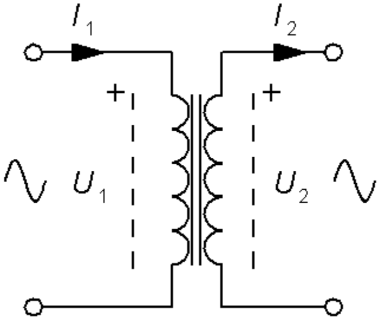


Calculate the two values that are missing. I_1 and U_2.

Two values are missing! (15.**1**)

For a transformer the following data was given:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
600	225 V	3A	200	75V	9 A



Calculate the two values that are missing. I_1 and U_2.

$$n = N_1/N_2 = 600/200 = 3$$

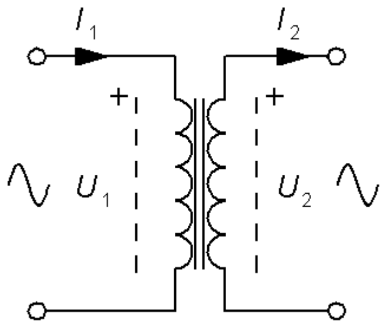
$$I_1 = \frac{1}{n} I_2 = \frac{9}{3} = 3 \qquad U_2 = \frac{1}{n} U_1 = \frac{225}{3} = 75$$

William Sandqvist william@kth.se

Two values are missing? (15.**2**)

For a transformer the following data was given:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
?	230 V	2A	150	**?**	12 A



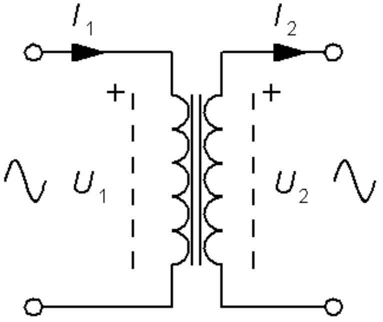
Calculate the two values that are missing. N_1 and U_2.

William Sandqvist william@kth.se

Two values are missing! (15.**2**)

For a transformer the following data was given:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
900	230 V	2A	150	38V	12 A



Calculate the two values that are missing. N_1 and U_2.

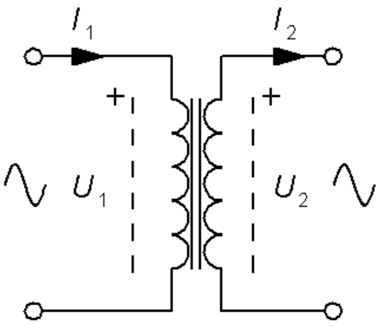
$$n = I_2/I_1 = 12/2 = 6$$

$$N_1 = N_2 \cdot n = 150 \cdot 6 = 900 \qquad U_2 = U_1/n = 230/6 = 38{,}3 \text{ V}$$

Two values are missing? (15.**3**)

For a transformer the following data was given:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
600	225 V	**?**	**?**	127 V	9 A



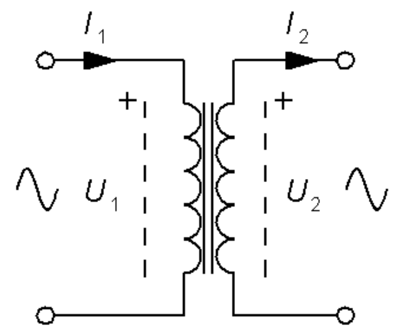
Calculate the two values that are missing. I_1 and N_2.

William Sandqvist william@kth.se

Two values are missing! (15.**3**)

For a transformer the following data was given:

Primary			Secondary		
N_1	U_1	I_1	N_2	U_2	I_2
600	225 V	5A	339	127 V	9 A



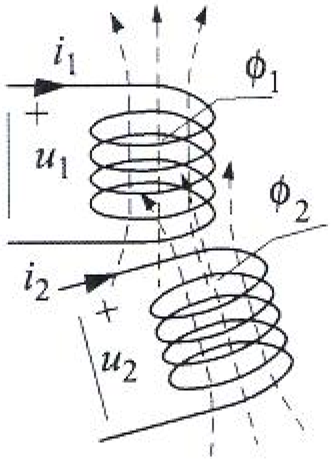
Calculate the two values that are missing. I_1 and N_2.

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} = \frac{225}{127} = 1{,}77 \quad \Rightarrow \quad N_2 = \frac{U_2}{U_1} N_1 = \frac{600 \cdot 127}{225} = 339$$

$$I_1 = \frac{N_2}{N_1} I_2 = \frac{339}{600} 9 = 5{,}08 \text{ A}$$

William Sandqvist william@kth.se

Inductive coupling



The coupling factor indicates how much of its flow a coil has in common with another coil? An ideal transformer has the coupling factor k = 1 (100%)

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$\pm M$ is called mutal inductance

- Series connected coils

$$L_{TOT} = L_1 + L_2 + 2M$$

- **Anti** series connected coils

$$L_{TOT} = L_1 + L_2 - 2M$$

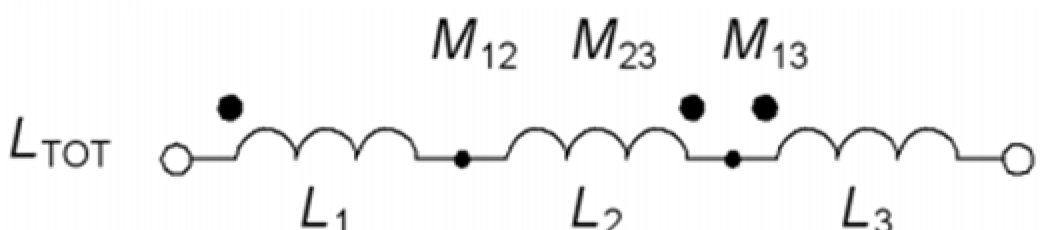
- Parallel connected coils

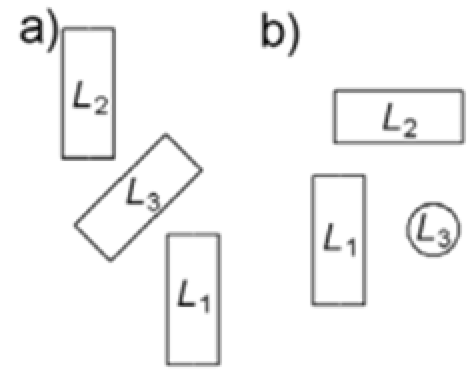
$$L_{TOT} = \frac{L_1 \cdot L_2 - M^2}{L_1 + L_2 - 2M}$$

- **Anti** parallel connected coils

$$L_{TOT} = \frac{L_1 \cdot L_2 - M^2}{L_1 + L_2 + 2M}$$

Mutal inductance (15.8)





Three inductors L_1 = 12, L_2 = 6, L_3 =5 [H] are series connected. When inductors are close to each other the placement on the circuit board can be important. In the figure to the left a) will inductors to have a portion of the magnetic lines in common. They then have the mutual inductances M_{12} = 3, M_{23} = 1, M_{13} = 1 [H].

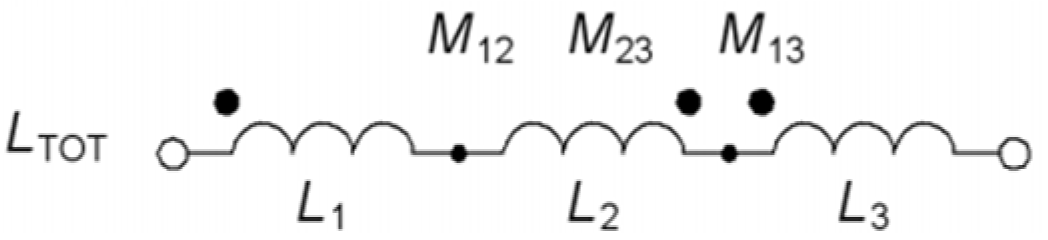
In the figure to the right b) the inductors are mounted three dimensional so that there are no shared power magnetic lines.

a) Calculate the total inductance for the arrangement in figure a). L_{TOT} = ?

b) Calculate the total inductance for the arrangement in figure b). L_{TOT} = ?

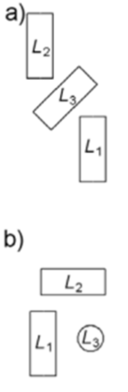
William Sandqvist william@kth.se

Mutal inductance (15.8)



a) $$L_{TOT} = L_1 - M_{12} + M_{13} + L_2 - M_{12} - M_{23} + L_3 - M_{23} + M_{13} =$$
$$= 12 - 3 + 1 + 6 - 3 - 1 + 5 - 1 + 1 = 17\,[\mathrm{H}]$$

b) $$L_{TOT} = L_1 + L_2 + L_3 = 12 + 6 + 5 = 23\,[\mathrm{H}]$$



William Sandqvist william@kth.se

Here is one more qualifying part example if time permits!

Qualifying part at exam   example

An AC voltage E with frequency f = 2 kHz feeds a circuit with a parallell capacitor C = 0,64 μF and an inductor L = 8,8 mH in series with a resistor R = 100 Ω.

We measure the voltage U_R = 5 V.

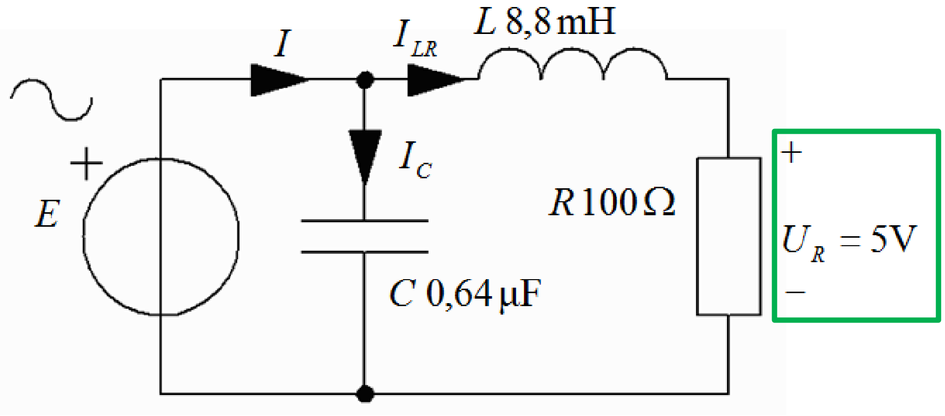
a) Calculate I_{LR} [mA]

b) Calculate E [V]

c) Calculate I_C [mA]

d) Calculate I [mA]

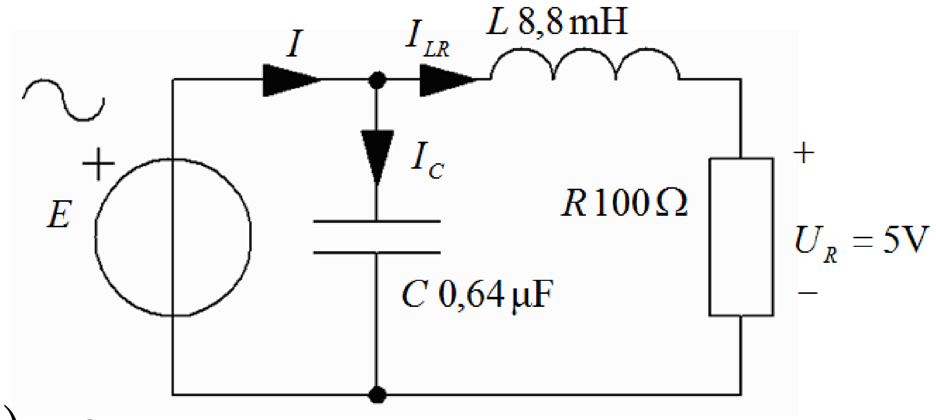
e) Draw principal phasor chart.
(I I_{LR} I_C E U_R)



William Sandqvist william@kth.se

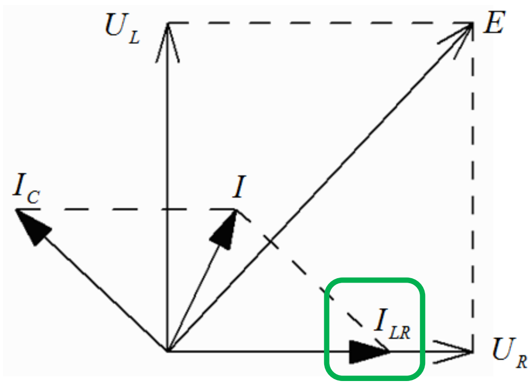
Qualifying part at exam   example

a) Calculate I_{LR} [mA]



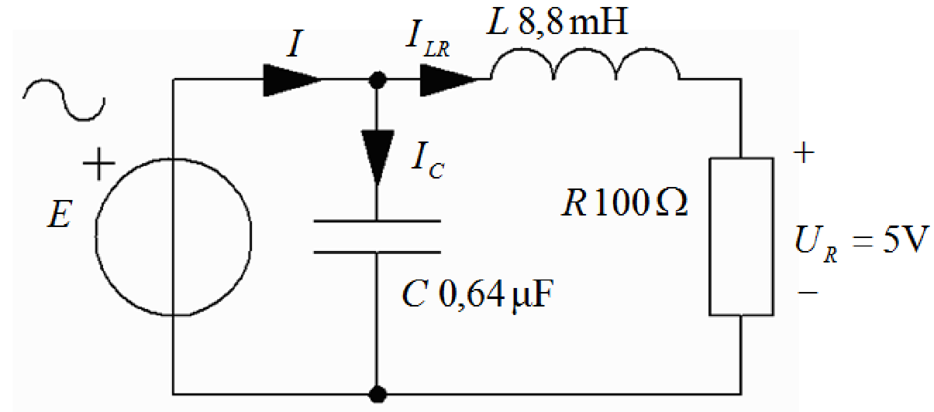
a) $\underline{U}_R$ *is reference* $\arg(\underline{U}_R) = 0$

$$U_R = 5 \quad I_{LR} = \frac{5}{100} = 50 \text{ mA}$$



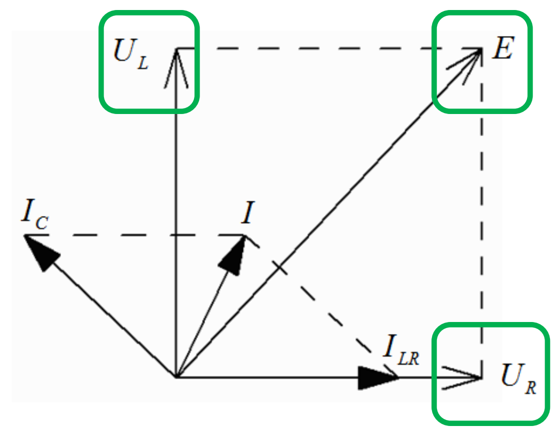
Qualifying part at exam   example

b) Calculate E [V]



$$\text{b)} \quad \underline{E} = \underline{U}_L + U_R = I_{LR} \cdot j \cdot 2\pi \cdot 2000 \cdot 8{,}8 \cdot 10^{-3} + 5 =$$

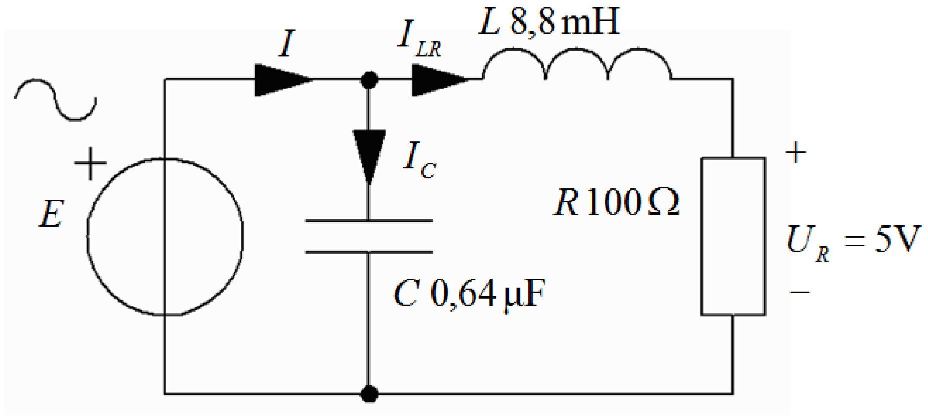
$$= 5{,}53j + 5 \quad E = \sqrt{5{,}53^2 + 5^2} = 7{,}45 \text{ V}$$



William Sandqvist william@kth.se

Qualifying part at exam   example

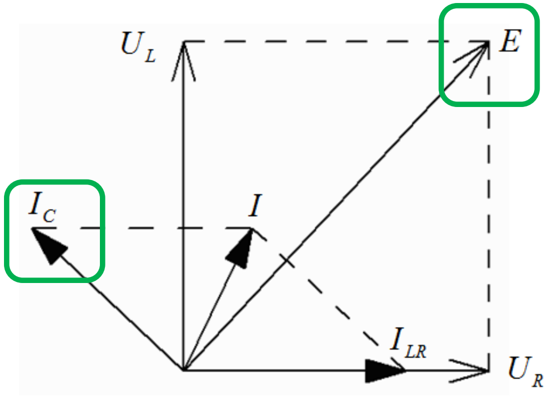
c) Calculate I_C [mA]



c) $\underline{I}_C = \dfrac{\underline{E}}{1/j\omega C} = \underline{E} \cdot j\omega C =$

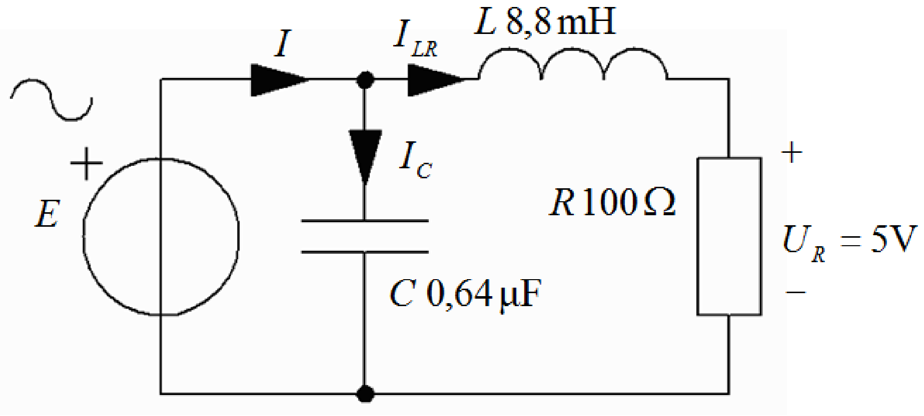
$= (5{,}53j + 5) \cdot j \cdot 2\pi \cdot 2000 \cdot 0{,}64 \cdot 10^{-6} =$

$= (-45 + 40j) \cdot 10^{-3} \quad I_C = \sqrt{40^2 + 45^2} \cdot 10^{-3} = 60 \text{ mA}$



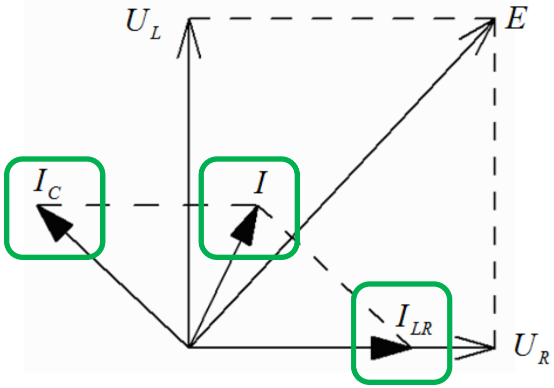
Qualifying part at exam   example

d) Calculate I [mA]

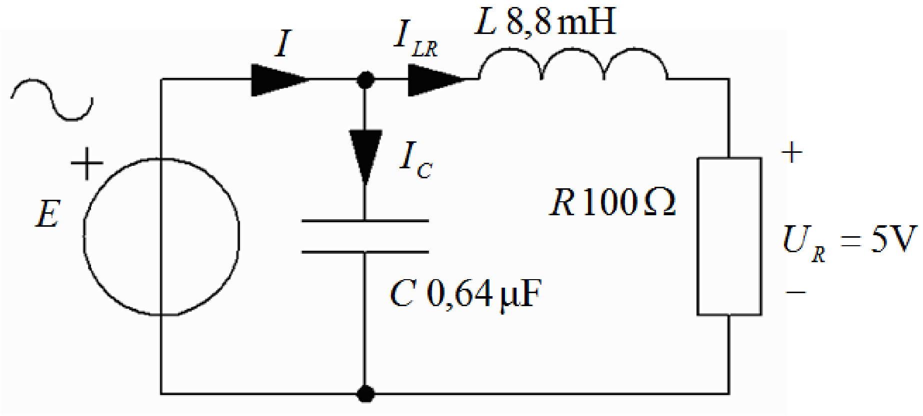


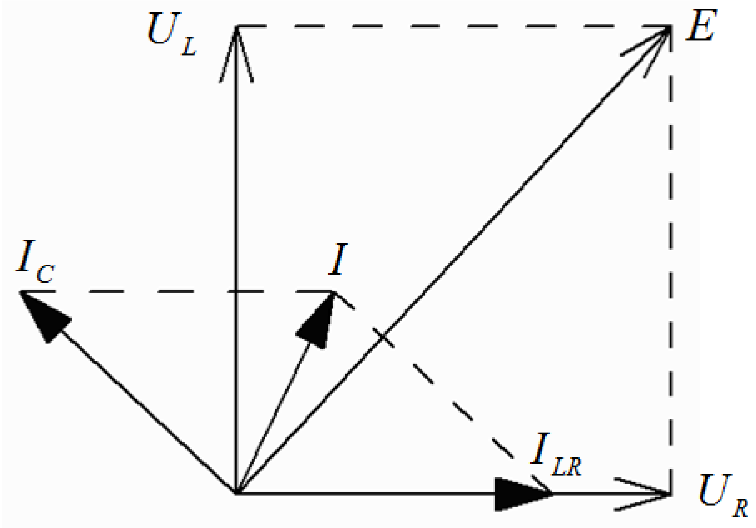
d) $\underline{I} = I_{LR} + \underline{I}_C = (50 - 45 + 40j) \cdot 10^{-3} =$

$= (5 + 40j) \cdot 10^{-3} \quad I = 10^{-3} \cdot \sqrt{5^2 + 40^2} = 40{,}3\ \text{mA}$



Qualifying part at exam   example

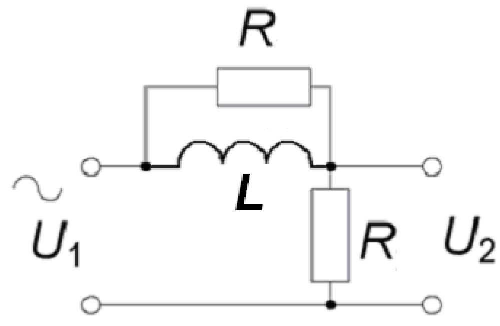




William Sandqvist william@kth.se

Here are some more ”filters” if time permits!

Filter RLR (14.7)



The figure shows a simple filter with two *R* and one *L*.

a) Derive the filter complex transfer function $\underline{U}_2/\underline{U}_1$.

b) At what angle frequency ω_X will the amount function be $|\underline{U}_2|/|\underline{U}_1| = 1/\sqrt{2}$

Give an expresson for this frequency ω_X with *R L*.

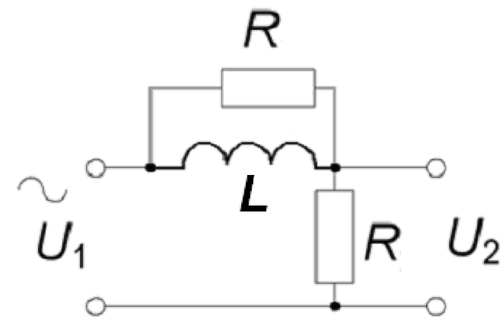
c) What value has the amount of the transfer function at very low frequencys, $\omega \approx 0$? What value has the phase function at very low frequencys?

d) What value has the amount of the transfer function at very high frequencys, $\omega \approx \infty$? What value has the phase function at very high frequencys?

$$a)\ \frac{\underline{U}_2}{\underline{U}_1} = ? \quad b)\ \omega_X \Rightarrow \quad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = \frac{1}{\sqrt{2}} \quad \omega_X(R,L) = ? \quad c)\ \omega \approx 0 \Rightarrow \quad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = ? \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = ?$$

$$d)\ \omega \approx \infty \Rightarrow \quad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = ? \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = ?$$

Filter RLR (14.7)



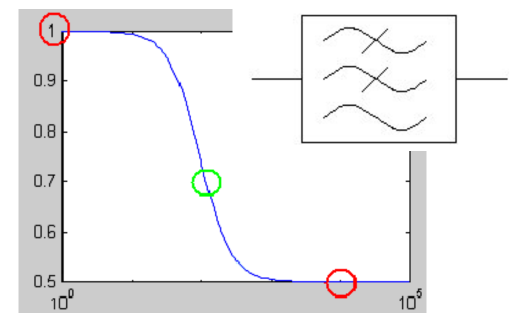
a) $$R \| L = \frac{R\cdot j\omega L}{R+j\omega L} \qquad \frac{\underline{U}_2}{\underline{U}_1} = \frac{R}{R+\frac{R\cdot j\omega L}{R+j\omega L}} = \frac{1}{1+\frac{1\cdot j\omega L}{R+j\omega L}} = \frac{\frac{R+j\omega L}{R+j\omega L}}{\frac{R+j\omega L+j\omega L}{R+j\omega L}} = \frac{R+j\omega L}{R+j2\omega L}$$

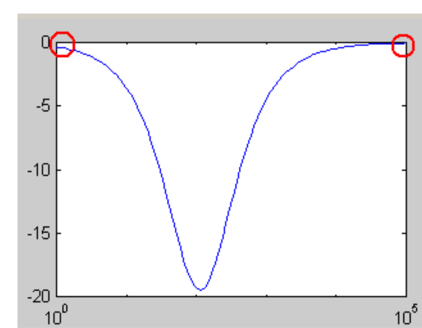
b) $$\left|\frac{\underline{U}_2}{\underline{U}_1}\right| = \left|\frac{R+j\omega L}{R+j2\omega L}\right| = \frac{1}{\sqrt{2}} \qquad \frac{\sqrt{R^2+(\omega L)^2}}{\sqrt{R^2+(2\omega L)^2}} = \frac{1}{\sqrt{2}} \qquad 2R^2+2(\omega L)^2) = R^2+4(\omega L)^2$$

$$R^2 = 2(\omega L)^2 \quad \Rightarrow \quad \omega_X = \frac{R}{L\sqrt{2}}$$

c) $$\frac{R+j\omega L}{R+j2\omega L} \quad \omega \to 0 \quad \frac{R+0}{R+0} = 1 \quad \Rightarrow \quad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = 1 \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = 0°$$

d) $$\frac{R+j\omega L}{R+j2\omega L} \quad \Rightarrow \quad \frac{\frac{R}{\omega}+jL}{\frac{R}{\omega}+j2L} \quad \omega \to \infty \quad \frac{0+jL}{0+j2L} = \frac{1}{2} \quad \Rightarrow \quad \left|\frac{\underline{U}_2}{\underline{U}_1}\right| = 0{,}5 \quad \arg\left(\frac{\underline{U}_2}{\underline{U}_1}\right) = 0°$$



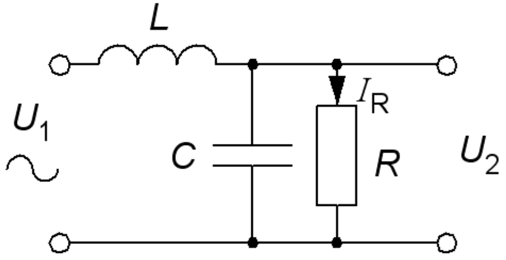


Filter LCR if time … (14.8)

The figure shows a simple filter with L C and R.

a) Derive the filter transfer function $\underline{U}_2/\underline{U}_1$.

b) At what angular frequency ω_x will the denominator be purely imaginary? Give an expression of this frequency ω_x with R L and C.



c) What value has the amount function at this angular frequency, ω_x?

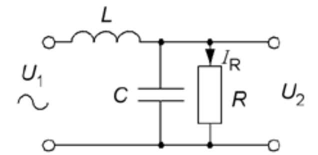
d) What value has the phase function at this angular frequency, ω_x ?

e) Give an expression of the transfer function between $\underline{I}_R/\underline{U}_1$

(Note! You already have the transferfunction $\underline{U}_2/\underline{U}_1$ from a)

$$a)\ \frac{\underline{U}_2(\omega)}{\underline{U}_1(\omega)}=? \quad b)\ \omega_X(R,L,C)=? \quad c)\ \left|\frac{\underline{U}_2(\omega_X)}{\underline{U}_1(\omega_X)}\right|=? \quad d)\ \arg\left(\frac{\underline{U}_2(\omega_X)}{\underline{U}_1(\omega_X)}\right)=? \quad e)\ \frac{\underline{I}_R(\omega)}{\underline{U}_1(\omega)}=?$$

Filter LCR if time … (14.8)



$$a)\ b) \qquad R \parallel C = \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{R}{1 + j\omega RC}$$

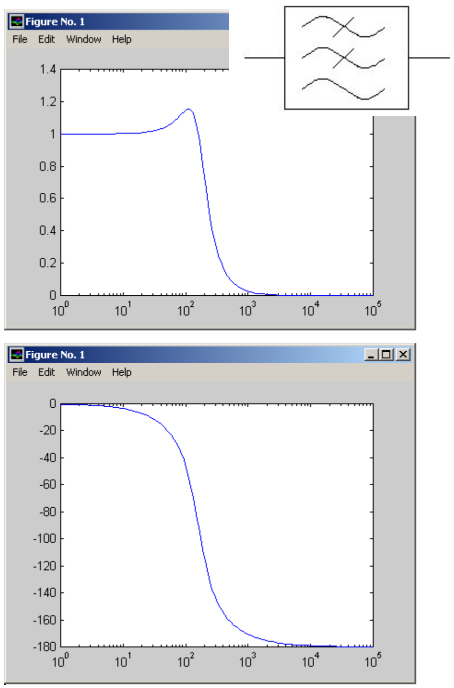
$$\frac{\underline{U}_2}{\underline{U}_1} = \frac{\frac{R}{1 + j\omega RC}}{j\omega L + \frac{R}{1 + j\omega RC}} \cdot \frac{1 + j\omega RC}{1 + j\omega RC} = \frac{R}{j\omega L(1 + j\omega RC) + R} =$$

$$= \frac{R}{(R - \omega^2 RLC) + j\omega L} \qquad RE\left[\frac{\underline{U}_2}{\underline{U}_1}\right] = 0 \;\Rightarrow\; \omega^2 RLC = R \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$c) \quad \frac{\underline{U}_2}{\underline{U}_1} = \frac{R}{(R - \omega^2 RLC) + j\omega L} = \left\{ \omega = \frac{1}{\sqrt{LC}} \right\} = \frac{R}{0 + j\sqrt{\frac{L}{C}}} \qquad \frac{U_2}{U_1} = \frac{R}{\sqrt{\frac{L}{C}}} = R\sqrt{\frac{C}{L}}$$

$$d) \quad \arg\left[\frac{\underline{U}_2}{\underline{U}_1}\right] = \arg\left[\frac{R}{j\sqrt{\frac{L}{C}}}\right] = -90^\circ$$

$$e) \quad \frac{\underline{I}_R}{\underline{U}_1} = ? \qquad \underline{I}_R = \frac{\underline{U}_2}{R} \;\Rightarrow\; \frac{\underline{I}_R}{\underline{U}_1} = \frac{\underline{U}_2}{\underline{U}_1} \cdot \frac{1}{R} = \frac{1}{(R - \omega^2 RLC) + j\omega L}$$



William Sandqvist william@kth.se