KTH Teknikvetenskap

SF1624 Algebra och geometri
Exam
Thursday, June 9, 2016

Time: 08:00am–1:00pm
No books/notes/calculators etc. allowed
Examiner: Tilman Bauer

This exam consists of nine problems, each worth 4 points.

Part A comprises the first three problems. The bonus points from the seminars will be automatically added to the total score of this part, which however cannot exceed 12 points.

The next three problems constitute part B, and the last three problems part C. The latter is mostly for achieving a high grade.

The thresholds for the respective grades are as follows:

Grade	A	B	C	D	E	Fx
Total sum	27	24	21	18	16	15
of which in part C	6	3	–	–	–	–

To get full score on a problem, your solution must be well-presented and easy to follow. In particular, you should define your notation; clearly explain the logical structure of your argument in words or symbols; and motivate and explain your argument. Solutions severely lacking in these respects will achieve at most 2 points.

p.t.o.

PART A

1. The plane H_1 is given by the equation $3x + 2y + 2z = 0$, and H_2 is given by the equation $x + 2y - 2z = 0$. The line L is the intersection of H_1 and H_2.
   (a) Compute a basis for the intersection line L. **(2 p)**
   (b) Determine whether the line L is contained in the subspace $V = \text{Span}(\vec{u}, \vec{v}, \vec{w})$, where **(2 p)**

$$\vec{u} = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \text{and} \quad \vec{w} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}.$$

2. Climate statistics show that the average winter temperature in Stockholm county changed according to the following table (temperatures are rounded to integers)

Period 0 (1961-1970)	$-5°C$
Period 1 (1971-1980)	$-2°C$
Period 2 (1981-1990)	$-3°C$
Period 3 (1991-2000)	$-1°C$
Period 4 (2001-2010)	$-1°C$

   Compute a function of the form $T(k) = Ak + B$ which best approximates these values using the least-squares method. Here k is the number of the period and $T(k)$ the average temperature in period k. **(4 p)**

3. Let
$$A = \begin{bmatrix} 3 & -4 & 8 \\ 2 & -3 & 8 \\ 0 & 0 & 1 \end{bmatrix}.$$
   (a) Compute all the eigenvalues and eigenvectors for the matrix A. **(2 p)**
   (b) Compute $A^{11}\vec{v}$, where $\vec{v} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

PART B

4. Let U be the subset of $\mathbb{R}^3$ which is the solution set of the equation $2x + y = 0$. Let $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 115 \end{bmatrix}$.
   (a) Compute an orthonormal basis β for U **(2 p)**
   (b) Extend the basis β to an orthonormal basis for $\mathbb{R}^3$. **(1 p)**
   (c) Compute the vector $\mathrm{proj}_U(\vec{v})$. **(1 p)**

5. Is there some value of a for which the three planes

$$ax + y - z = 1, \quad y + 2z = 7, \quad x + z = 2,$$

   share a common line? If so, determine this line in parametric form for all such a.
   (4 p)

6. The map $R\colon \mathbb{R}^3 \to \mathbb{R}^3$ is a rotation with the following properties: the axis of rotation l is the line $x_1 = x_2 = x_3$; the positive x_1-axis is mapped to the positive x_2-axis; the positive x_2-axis is mapped to the positive x_3-axis; the positive x_3-axis is mapped to the positive x_2-axis.
   (a) Compute a matrix representation of the map R in the standard basis. **(1 p)**
   (b) Compute all eigenvalues and eigenvectors of the map. **(1 p)**
   (c) In the plane orthogonal to the line l, the map R acts as a rotation. Determine the angle of rotation. **(2 p)**

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PART C

7. (a) Find a 2×2-matrix A whose null space and column space agree. **(2 p)**
   (b) Show that there is no 3×3-matrix with the above property. **(2 p)**

8. Which conditions on the numbers a, b, c are necessary for the matrix
$$\begin{bmatrix} a & 1 & 2 \\ 0 & b & -1 \\ 0 & 0 & c \end{bmatrix}$$
to be diagonalizable? **(4 p)**

9. Let V be an n-dimensional vector space and $L\colon V \to V$ a linear map which satisfies that $L(L(v)) = L(v)$ for all $v \in V$.
   (a) Show that the only vector which lies in both $\mathrm{Range}(L)$ and $\mathrm{Null}(L)$ is the zero vector. **(2 p)**
   (b) Show that there is a basis $\mathcal{B}$ of V such that the matrix representation of L with respect to the basis $\mathcal{B}$ is a diagonal matrix which has only zeroes and ones on the diagonal. **(2 p)**