SF1625 Calculus in one variable
Exam
Friday June 10th 2016

Time: 08.00-13.00
No calculators, formula sheets etcetera allowed
Examiner: Lars Filipsson

This exam consists of nine problems, each worth four points, hence the maximal score is 36. Part A consists of the three first problems. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically. You can check how many bonus points you have on your results page.

The following three problems constitute part B and the last three problems part C. You need a certain amount of points from part C to obtain the highest grades.

The grading will be performed according to this table:

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

To obtain a maximal 4 for a solution to a problem on the exam, your solution must be well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

Please turn page!

PART A

1. Differentiate these functions with respect to x and state in each case for what x the derivative exists. Only answers are necessary, no motivations needed.

   A. $f(x) = \arctan \dfrac{1}{x}$

   B. $g(x) = 2^x$

   C. $h(x) = xe^{-x^2}$

   D. $k(x) = \dfrac{\sqrt{x}}{\ln x}$

2. Compute the integrals and simplify your answers.

   A. $\displaystyle\int \tan x \, dx$ (you may want to use the substitution $u = \cos x$)

   B. $\displaystyle\int x^2 \cos x \, dx$ (you may want to use repeated integration by parts)

3. Decide whether the function $f(x) = |2x - 1| + \arcsin x$ assumes maximum and minimum values, and if so, find these. Simplify your answer.

PART B

4. Assume the function f to be three times differentiable on the real axis. Assume further that $f(1) = 2$, $f'(1) = -3$ and $|f''(x)| \leq 5$ for all x.
   A. Find an approximate value of $f(1.1)$ using linear approximation (Taylor polynomial of degree 1).
   B. Find as good a bound as possible for the error in your approximation.

5. Compute the integral

$$\int_0^{\sqrt{3}} \arctan x \, dx.$$

   (For a maximum score the integral should be computed exactly, but an approximate computation may be awarded partial score. Simplify your answer.)

6. We study the curve given by $2x^2 + 4xy + 3y^2 + 2y = 10$.
   A. Find an equation for the tangent to the curve at the point $(x_0, y_0) = (-1, 2)$.
   B. Using the tangent, find an approximate value of the y-coordinate of a point on the curve with x-coordinate -0.8.
   C. Can there be more than one point on the curve with x-coordinate -0.8?

PART C

7. We study the function f given by

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$$

   A. Is f odd? Is f even?
   B. At what points is f continuous?
   C. At what points is f differentiable?
   D. Is f integrable on the interval $-\pi \leq x \leq \pi$?

8. Find the smallest possible real number M such that $|f''(x)| \leq M$ for all $x \in \mathbb{R}$, if $f(x) = \arctan x$.

9. The curves $y = x^{2/3}$ and $y = x^{3/2}$ bounds a domain in the first quadrant. Compute the length of the boundary curve of that domain.