EP2200 Queuing Theory and Teletraffic Systems

Saturday, June 11$^{\text{th}}$, 2016
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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas and Laplace transforms. Erlang tables are not attached as they are not needed for this exam.

1. Consider a single-server system with exponentially distributed service time with a mean of 1 second. There buffer has place for maximum three waiting customers. Customers arrive in pairs and are served one by one. The inter-arrival times of the pairs are exponentially distributed with a mean of 2 seconds. If there is one free position in the buffer when the pair of customers arrives, then one of the customers is accepted, the other is blocked.
a) Draw the state transition diagram and calculate the state probabilities. (2p)
b) Calculate the probability that a customer is blocked. (2p)
c) Calculate the expected waiting time of all accepted customers. (2p)
d) What is the probability that at least 3 customers are blocked right after each other? (2p)
e) Assume that low and high priority customers arrive with equal probability. Low priority customers are blocked if there are customers in the system. Draw the state transition diagram. (2p)

2. Consider three secretaries sharing two phone lines in an office. If both lines are busy when a secretary attempts to make a call, the call is blocked. The length of the phone calls is exponentially distributed with a mean of 15 minutes. Between phone-calls the secretaries do paperwork for an exponentially distributed time of 30 minutes.
a) Define the states of the system and draw the state transition diagram. (2p)
b) Calculate the probability that both lines are busy at an arbitrary point of time, and the probability that a secretary finds both lines busy when attempting a call. (3p)
c) Calculate the mean number of successful calls per hour. (2p)
Now assume that the length of the phone calls is Erlang distributed with the same mean of 15 minutes and a coefficient of variation $c_x^2 = 1/2$.
d) Give the Kendall notation of the system in this case. (1p)
e) Define the states of the system and draw the state transition diagram. (2p)

3. In a peer-to-peer file-sharing overlay (e.g., BitTorrent) peers offer their upload capacity to transmit files to other peers. Assume that peers join the overlay with exponential inter-arrival time, with an average of 6 second. Peers stay in the overlay for an exponentially distributed time of 5 minutes in average.
a) Give the Kendall notation and the state diagram of the queuing system that describes the how the number of peers changes in the overlay, and calculate the average number of peers. (3p)
b) Derive the state probabilities and give the probability that an arriving peer finds no other peers in the overlay. (3p)
c) Consider a peer that downloads a file from another peer. The connection is up for the time while both the uploading and the downloading peers stay in the overlay. Give the distribution of the connection time and its average. (2p)
d) A peer would like to maximize the connection time, and tries two policies to select peer to download from: Policy A: it selects the peer that arrived just before him. Policy B: it selects the peer that has been in the overlay for the longest time. Can the peer increase its connection time with these policies? Motivate your answer. (2p)

4. Packets arrive to a multiplexer from two sources. Arrivals from both of the sources can be considered as Poissonian. The packet arrival rate from source 1 is λ_1=1 packet/msec, the packet arrival rate from source 2 is λ_2=2 packets/msec. The transmission time of the packets of source 1 is exponentially distributed with a mean value of 0.2msec. The transmission time of the packets of source 2 is exponential as well, with a mean value of 0.1msec. The buffer capacity is infinite.
a) Give the mean and the second moment of the service time, calculate the server utilization, and the mean waiting time. (3p)
Consider now that packets of stream 1 are transmitted with high priority, but without interrupting the

ongoing service of stream 2 packets.
b) Calculate the mean waiting time of stream 1 and stream 2 packets, and the average waiting time of all packets. Discuss how the waiting times change compared to the case without priorities. (3p)
Consider again the system without priorities, but now the multiplexer transmits a large packet with management information every time the queue becomes empty. The transmission of this management packet takes exactly 0.5msec.
c) Give the system model, and calculate the average number of management packets transmitted within an hour. (2p)
d) Calculate the average waiting time of the stream 1 and stream 2 packets in this case. (2p)

5. A broadband network operator asks you to evaluate the efficiency of their call center. The call center works as follows. When calling, customers may select between options of *Question about TV service*, *Question about Internet connection* and *Question about fixed telephony service*, where there is one-one clerk answering the questions. Customers select one of the three options with equal probability, and wait for service in independent queues. The service time at each of these queues is exponentially distributed with a mean of 3 minutes. However, with probability 0.1, customers are then forwarded to the *Technical support*, with a shared queue and one technician answering the questions. The service time here is exponentially distributed, and the mean service time is 8 minutes. Assume that calls to the call center arrive according to a Poisson process with a rate of 0.5 call per minute, and the length of the queues is not limited.
a) Draw the queuing network model of the call center, and give all the arrival and service time parameters. (2p)
b) What is the average time a customer spends in the system, from the time of calling until completed service? (3p)
c) Calculate the probability that a customer is served without any waiting. (2p)
d) Assume now that the company decides to employ one more technician, so now customers waiting in the technical support queue are served by two technicians. How much does it decrease the average time customers spend in the system? Does it seem to be a good investment? (3p).