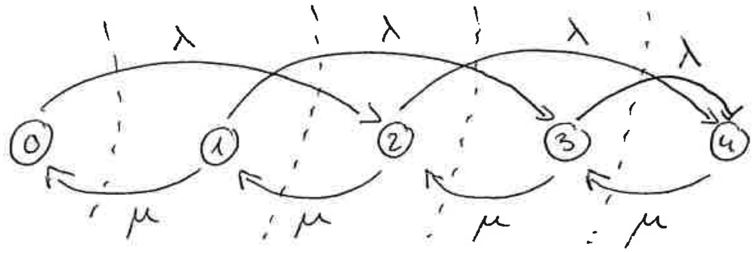
2016 june 11.

(1)

a)



$\lambda = \frac{1}{2}$ pairs/sec.

$\mu = 1$ customer/sec.

$$p_0\lambda = p_1 \cdot \mu \quad \Rightarrow \quad p_1 = \frac{1}{2}p_0$$

$$p_0 \cdot \lambda + p_1\lambda = p_2\mu \quad \Rightarrow \quad p_2 = \frac{1}{2}p_0 + \frac{1}{4}p_0 = \frac{3}{4}p_0$$

$$p_1\lambda + p_2\lambda = p_3\mu \quad \Rightarrow \quad p_3 = \frac{1}{4}p_0 + \frac{3}{8}p_0 = \frac{5}{8}p_0$$

$$p_2\lambda + p_3\lambda = p_4\mu \quad \Rightarrow \quad p_4 = \frac{3}{8}p_0 + \frac{5}{16}p_0 = \frac{11}{16}p_0$$

$$\sum_{i=0}^{4} p_i = 1$$

$$p_0 = \frac{16}{57} \quad p_1 = \frac{8}{57} \quad p_2 = \frac{12}{57} \quad p_3 = \frac{10}{57} \quad p_4 = \frac{11}{57}$$

b) $P(\text{block}) = \frac{1}{2}p_3 + p_4 = \frac{16}{57}$

c) $N_q = 0 \cdot p_0 + 0 \cdot p_1 + 1 \cdot p_2 + 2 \cdot p_3 + 3 \cdot p_4 = \frac{12 + 20 + 33}{57} = \frac{65}{57}$

$\lambda_{eff} = 2 \cdot \lambda \cdot (1 - P(\text{block})) = \frac{41}{57}$

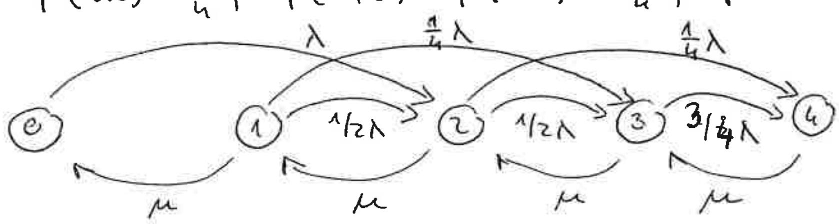
$W = \frac{N_q}{\lambda_{eff}} = \frac{65}{41} = 1.58$

d) P(at least 3 customer blocked in a raw) =

P(new arrival before service) P(~~first~~ arrival in 3 or 4) =

$\frac{\lambda}{\mu+\lambda}(P_3 + P_4) = \frac{1}{3} \cdot \frac{21}{57} = \frac{7}{57}$

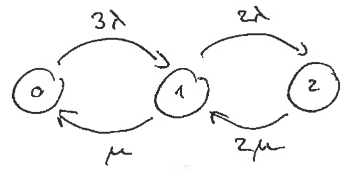
e) $p(hh) = \frac{1}{4}$, $p(h,l) = p(lh) = \frac{1}{4}$, $p(ll) = \frac{1}{4}$



Problem 2

$\lambda = 2$ calls/hour
$\mu = 4$ calls/hour
population : 3
servers : 2

a) states: # lines busy. $M'/M/2/2/3$



$\underline{p} = \left\{ \frac{4}{13}, \frac{6}{13}, \frac{3}{13} \right\}$

b) $P(\text{both lines busy}) = \{\text{time blocking probability}\} = p_2 = \frac{3}{13}$

$P(\text{arriving call blocked}) = \{\text{call blocking prob.}\} = \frac{\lambda p_2}{3\lambda p_0 + 2\lambda p_1 + \lambda p_2} = \frac{3}{27} = \frac{1}{9}$

= c)

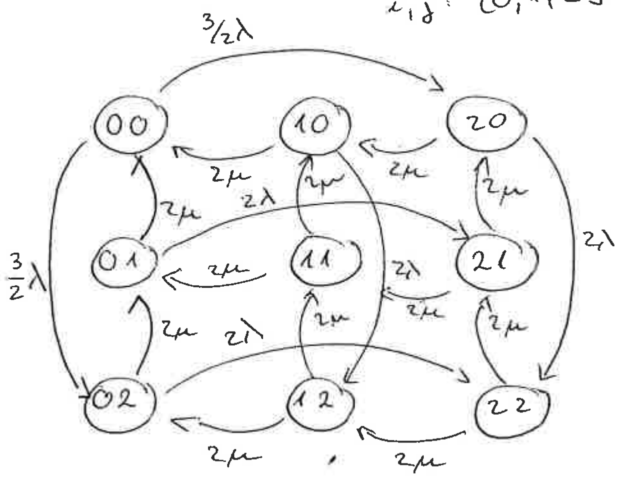
d) Call length: Erlang-2 . (Erlang-or: $c = \frac{\sigma}{m} = \frac{1}{\sqrt{2}}$)

$c_\lambda^2 = \frac{\sigma^2}{m^2} = \frac{1}{n}$

$M/E_2/2/2/3$

e) system state $(i,j) = $ (stages left on line 1 , stages left on line 2)

$i,j : \{0,1,2\}$



c) Average number of successful calls per hour =

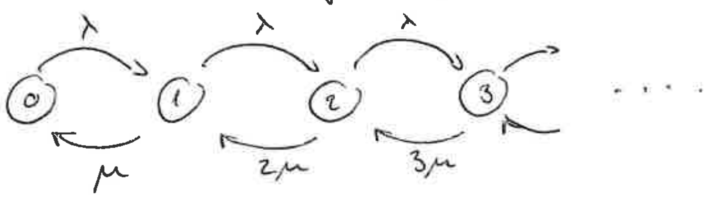
$\lambda_{eff} = 3\lambda p_0 + 2\lambda p_1 = 6 \cdot \frac{4}{13} + 4 \cdot \frac{6}{13} = \frac{48}{13}$

3. M / M / ∞ system

$\lambda = \frac{1}{6}$ arrival/sec

$\bar{x} = 5\,\text{min} = 300\,\text{sec}$, $\mu = \frac{1}{300}$

a) M / M / ∞ (everyone joins and get served immediately)



$\bar{N} = \bar{N}_S = \lambda \bar{x} = 50$ $(= \rho)$

b) State probabilities:

$p_k = p_0 \cdot \frac{\rho^k}{k!}$ (as in M/M/m for $k < m$)

$p_0 \underbrace{\sum_{i=0}^{\infty} \frac{\rho^k}{k!}}_{e^\rho} = 1 \Rightarrow p_k = \frac{\rho^k}{k!} e^{-\rho}$ (Poisson distribution!)

p(no one in the overlay at arrival) $= p_0 = e^{-\rho} = e^{-50}$

c) connection time = min (remaining time for uploader, remaining time for downloader)

$= \min(\text{Exp}(\mu), \text{Exp}(\mu)) = \text{Exp}(2\mu)$

E[connection time] $= \frac{1}{2\mu} = 2.5$ min.

d) Neither of the policies increases the expected connection time, due to the memoryless property of the ~~ex~~ Exponential distribution.

4

a) $E[x_1] = \frac{1}{\mu_1} = \frac{1}{5}$ msec $\qquad E[x_2] = \frac{1}{10}$ msec $\qquad \lambda_1 = 1$ /msec $\quad \lambda_2 = 2$ /msec

$E[x_1^2] = \frac{2}{\mu_1^2} = \frac{2}{25}$ msec^2 $\qquad E[x_2^2] = \frac{2}{100}$ msec $\qquad p_1 = \frac{1}{3}\,,\quad p_2 = \frac{2}{3}$

$$E[x] = p_1 E[x_1] + p_2 E[x_2] = \frac{2}{15} \text{ msec}$$

$$E[x^2] = p_1 E[x_1^2] + p_2 E[x_2^2] = \frac{1}{25} \text{ msec}^2$$

$$\rho = (\lambda_1 + \lambda_2) E[x] = \frac{2}{5}$$

$$W = \frac{\lambda E[x^2]}{2(1-\rho)} = 0.1 \text{ msec.}$$

b) Priority system, without preemption. High prio: stream 1

$R_S = \frac{1}{2} \sum_i \lambda_k \overline{x}_k^2 = \frac{3}{50}\,,\qquad \rho_1 = \lambda_1 E[x_1] = \frac{1}{5}\,,\quad \rho_2 = \lambda_2 E[x_2] = \frac{1}{5}$

$$\left.\begin{aligned} W_1 &= \frac{R_S}{1-\rho_1} = \frac{3}{40} \text{ ms} \\ W_2 &= \frac{R_S}{(1-\rho_1)(1-\rho_1-\rho_2)} = \frac{1}{8} \text{ ms} \end{aligned}\right\} \quad W = p_1 W_1 + p_2 W_2 \approx \frac{13}{120} \text{ ms} \approx 0.108$$

Prio 1 waiting is decreased, prio 2 is increased. Average is slightly increased, because long packets were prioritized.

c) $M/H_2/1$ with vacation. $\qquad V = 0.5,\quad E[V] = v = 0.5$ ms
$E[V^2] = 0.25$ ms^2

$$N_m = \frac{(1-\rho)T}{v} = 4.32 \cdot 10^6$$

d) $$W_1 = W_2 = W_{M/G/1} + \frac{E[V^2]}{2E[V]} = \frac{1}{10} + \frac{1}{4} = 0.35 \text{ ms}$$

⑤ Queuing network

a)

$\mu_1 = \frac{1}{3}$ min

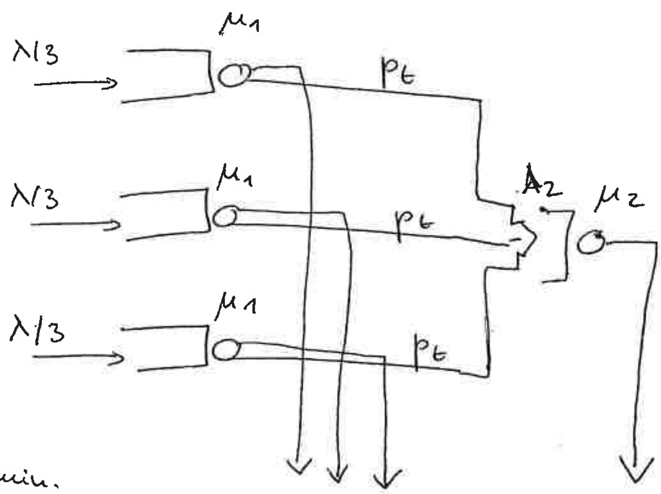
$\mu_2 = \frac{1}{8}$ min

$\lambda = \frac{1}{2}$ call/min

$p_t = 0.1$

$\lambda_1 = \frac{\lambda}{3} = \frac{1}{6}$ call/min

$\lambda_2 = \frac{1}{10} \cdot 3 \cdot \lambda_1 = \frac{1}{20}$ call/min.



b) $T = (1-p_t)T_1 + p_t(T_1+T_2) = T_1 + p_t \cdot T_2$

$$T_1 \overset{M/M/1}{=} \frac{1}{\mu_1-\lambda_1} = 6 \text{ min}$$

$$T_2 = \frac{1}{\mu_2-\lambda_2} = \frac{40}{3} \text{ min}$$

$\left.\right\}$ $T = 6\text{ min} + \frac{4}{3}\text{ min} \approx 7.3$ min.

c) P(service without waiting) = $\overset{(1-p_t)}{}$ P(first queue empty) +

p_t P(first queue empty) · P(second queue empty)

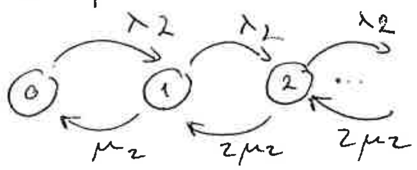
$$= \frac{9}{10} \cdot (1-\rho_1) + \frac{1}{10}(1-\rho_1)(1-\rho_2)$$

$$= \frac{48}{100}$$

/ $\rho_1 = \lambda_1/\mu_1 = \frac{1}{2}$

$\rho_2 = \lambda_2/\mu_2 = \frac{2}{5}$

d) Last queue becomes an M/M/2 system, with λ_2, μ_2.



$p_0\lambda_2 = p_1\mu \Rightarrow p_1 = \rho_2 p_0$

$p_1\lambda_2 = p_2 \cdot 2\mu \qquad p_2 = \frac{1}{2}\rho_2^2 p_0$

$p_2 \cdot \lambda_2 = p_3 2\mu \qquad p_3 = \frac{1}{4}\rho^4 p_0$

⋮

$$\sum_{i=0}^{\infty} p_i = 1 = \dots = p_0 + \rho \cdot \sum_{i=0}^{\infty}\left(\frac{\rho}{2}\right)^i p_0 = 1 \quad \Rightarrow p_0 = \frac{2}{3}$$

$$P(\text{wait}) = 1 - (p_0+p_1) = \frac{1}{15}$$

$$T_2 = \frac{1}{2\mu_2-\lambda_2} P_{\text{wait}} + \frac{1}{\mu_2} = \frac{25}{3}$$

$$T = T_1 + 0.1 \cdot T_2 \approx 6.8 \text{ min.}$$

Little decrease compared to the investment. Also, most of the time the new technician will be idle.