

# KONSTRUKTION AV ALGORITMER

## METOD 3 DEKOMPOSITION

(DIVIDE AND CONQUER, SÖNDRA OCH HÄRSKA)

- DELA UPP I MINDRE PROBLEM
- LÖS DELPROBLEMEN REKURSIVT
- KOMBINERA RESULTATEN

ANALYS: ANVÄND EN REKURSIONSRELATION

EXEMPEL: MATRISMULTIPLIKATION

$$C = AB \iff \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

MUL(A, B, n) =

IF  $n=1$  THEN RETURN  $A \cdot B$

$C_{11} = \text{ADD} \left( \text{MUL} \left( A_{11}, B_{11}, \frac{n}{2} \right), \text{MUL} \left( A_{12}, B_{21}, \frac{n}{2} \right), \frac{n}{2} \right)$

$C_{12} = \text{ADD} \left( \text{MUL} \left( A_{11}, B_{12}, \frac{n}{2} \right), \text{MUL} \left( A_{12}, B_{22}, \frac{n}{2} \right), \frac{n}{2} \right)$

$C_{21} = \text{ADD} \left( \text{MUL} \left( A_{21}, B_{11}, \frac{n}{2} \right), \text{MUL} \left( A_{22}, B_{21}, \frac{n}{2} \right), \frac{n}{2} \right)$

$C_{22} = \text{ADD} \left( \text{MUL} \left( A_{21}, B_{12}, \frac{n}{2} \right), \text{MUL} \left( A_{22}, B_{22}, \frac{n}{2} \right), \frac{n}{2} \right)$

RETURN C

FUNGERAR OM  $n=2^k$  FÖR NÅGOT  $k$ .

ANALYS:  $T(1)=1$   
 $T(n) = 8T\left(\frac{n}{2}\right) + 4 \cdot \left(\frac{n}{2}\right)^2 \} \Rightarrow T(n) = O(n^3)$

## SNABB MULTIPLIKATION AV 2x2-MATRISER (STRASSEN)

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

KAN BERÄKNAS GENOM

$$m_1 = (a_{12} - a_{22}) \cdot (b_{21} + b_{22})$$

$$m_2 = (a_{11} + a_{22}) \cdot (b_{11} + b_{22})$$

$$m_3 = (a_{11} - a_{21}) \cdot (b_{11} + b_{12})$$

$$m_4 = (a_{11} + a_{12}) \cdot b_{22}$$

$$m_5 = a_{11} \cdot (b_{12} - b_{22})$$

$$m_6 = a_{22} \cdot (b_{21} - b_{11})$$

$$m_7 = (a_{21} + a_{22}) \cdot b_{11}$$

$$c_{11} = m_1 + m_2 - m_4 + m_6$$

$$c_{12} = m_4 + m_5$$

$$c_{21} = m_6 + m_7$$

$$c_{22} = m_2 - m_3 + m_5 - m_7$$

SAMMANLAGT 7 MULTIPLIKATIONER

18 ADDITIONER OCH SUBTRAKTIONER

MULTIPLIKATION AV TVÅ  $n \times n$ -MATRISER TAR TID

$$T(n) = 7 T\left(\frac{n}{2}\right) + 18 \left(\frac{n}{2}\right)^2, \quad n \geq 2$$

$$T(1) = 1$$

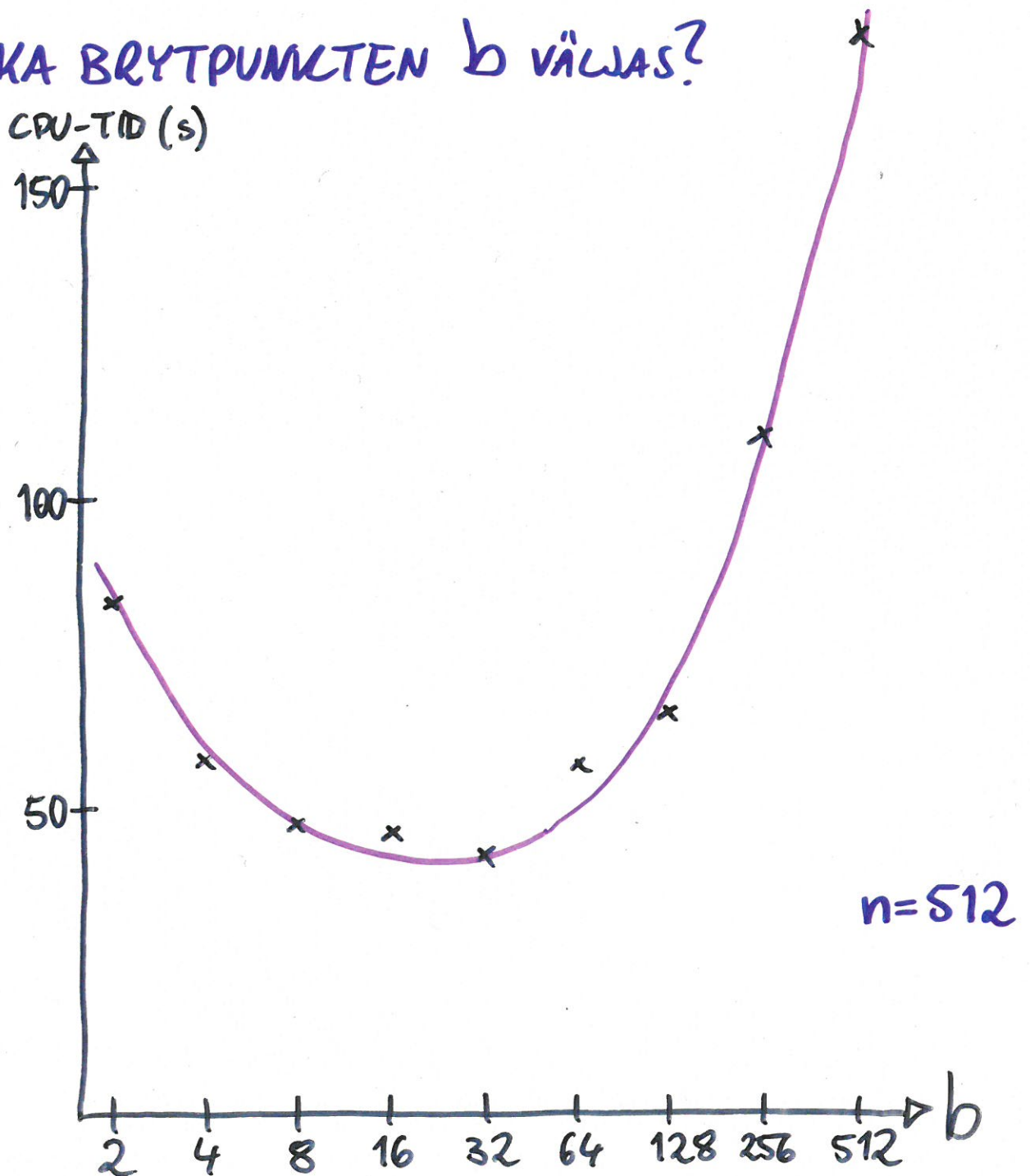
$$\Rightarrow T(n) \sim n^{2.81}$$

# STRASSEN I PRAKTIKEN

ANVÄND STRASSENS MULTIPLIKATIONSALGORITM FÖR STORA MATRISER ( $n > b$ ).

ANVÄND VANLIG MATRISMULTIPLIKATION FÖR MINDRE MATRISER ( $n \leq b$ ).

HUR SKA BRYTPUNKTEN  $b$  VÄLJAS?



(Karatsuba)

# MULTIPLIKATION AV BINÄRA TAL

$$x = \underbrace{x_{n-1} x_{n-2} \dots x_{\frac{n}{2}}}_{a} \underbrace{x_{\frac{n}{2}-1} \dots x_1 x_0}_{b} = a \cdot 2^{\frac{n}{2}} + b$$

$$y = \underbrace{y_{n-1} y_{n-2} \dots y_{\frac{n}{2}}}_{c} \underbrace{y_{\frac{n}{2}-1} \dots y_1 y_0}_{d} = c \cdot 2^{\frac{n}{2}} + d$$

$$x \cdot y = a \cdot c \cdot 2^n + (a \cdot d + b \cdot c) 2^{\frac{n}{2}} + b \cdot d$$

ANTA ATT  $n = 2^k$

mult(x, y, k) =

IF  $k=0$  THEN RETURN  $x * y$

ELSE

$[a, b] \leftarrow x$

$[c, d] \leftarrow y$

$p_1 \leftarrow \text{mult}(a, c, k-1)$

$p_2 \leftarrow \text{mult}(b, d, k-1)$

$p_3 \leftarrow \text{mult}(a, d, k-1)$

$p_4 \leftarrow \text{mult}(b, c, k-1)$

RETURN  $p_1 \cdot 2^n + (p_3 + p_4) \cdot 2^{\frac{n}{2}} + p_2$

$$T(1) = \Theta(1)$$

$$T(n) = 4 \cdot T\left(\frac{n}{2}\right) + \Theta(n) \left. \vphantom{T(n)} \right\} T(n) = \Theta(n^{\log_2 4}) = \Theta(n^2)$$

# SMARTARE MULTIPLIKATION

$$A \leftarrow a \cdot c$$

$$B \leftarrow b \cdot d$$

$$C \leftarrow (a+b) \cdot (c+d)$$

$$D \leftarrow A \cdot 2^n + (C - A - B) \cdot 2^{\frac{n}{2}} + B$$

$$D = a \cdot c \cdot 2^n + (a \cdot d + b \cdot c) \cdot 2^{\frac{n}{2}} + b \cdot d = x \cdot y$$

smartmult(x, y, k) =

IF  $k \leq 4$  THEN RETURN  $x * y$

ELSE

$$[a, b] \leftarrow x$$

$$[c, d] \leftarrow y$$

$$A \leftarrow \text{smartmult}(a, c, k-1)$$

$$B \leftarrow \text{smartmult}(b, d, k-1)$$

$$C \leftarrow \text{smartmult}(a+b, c+d, k-1)$$

$$\text{RETURN } A \cdot 2^n + (C - A - B) \cdot 2^{\frac{n}{2}} + B$$

$$T(4) = \Theta(1)$$

$$T(n) = 3 \cdot T\left(\frac{n}{2}\right) + \Theta(n) \left. \vphantom{T(n)} \right\} T(n) = \mathcal{O}(n^{\log_2 3}) =$$

$$= \mathcal{O}(n^{1.58})$$

(BÄSTA KÄNDA ALGORITMEN  $\mathcal{O}(n \log n \log \log n)$ .)