Mini-groupwork 1, solutions, 2016

a)

$$h = \frac{17}{2} \cdot 6378\,\text{km} = 54000 \text{ km}$$

The thermal energy is divided into motion in the three dimensions, two of which only give rise to a gyro motion around the magnetic field lines, with the motion along the magnetic field corresponding to an energy

$$E = \frac{k_B T}{2} = \frac{1.38 \cdot 10^{-23} \cdot 2 \cdot 10^{6}}{2} = 1.4 \cdot 10^{-17}\ \text{J}$$

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 1.4 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 5500\ \text{km/s}$$

Approximating the loop with a quarter-circle, the electron has to travel a length
$s = \pi h/2 = 85\ 000$ km

Then we get $t = 15$ s.

b)

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\frac{qB}{m} \quad \Rightarrow$$

$$B = \frac{2\pi\ f_c m}{q} = \frac{2\pi \cdot 8 \cdot 10^{9} \cdot 0.91 \cdot 10^{-30}}{1.6 \cdot 10^{-19}} = 0.29\ \text{T}$$

The perpendicular energy is given by

$$E = \frac{2k_B T}{2} = \frac{2 \cdot 1.38 \cdot 10^{-23} \cdot 2 \cdot 10^{6}}{2} = 2.8 \cdot 10^{-17}\ \text{J}$$

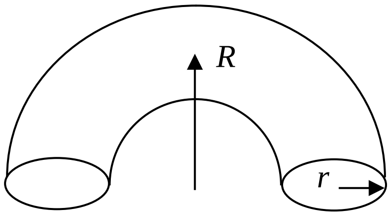
$$\Rightarrow$$

$$v_\perp = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 2.8 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 7.8 \cdot 10^{6}\,\text{m/s}$$

$$\rho = \frac{m_e v_\perp}{qB} = \frac{0.91 \cdot 10^{-30} \cdot 7.8 \cdot 10^{6}}{1.6 \cdot 10^{-19} \cdot 0.29} = 1.5 \cdot 10^{-4}\ \text{m}$$

c)

Model the flare by a half torus with minor axis *r*, and major axis *R*. From the figure I estimate *R* = 8 R_E, and r = 2 R_E.



Let this half-torus be filled with a magnetic field of strength B ~ 0.29 T (using the value in b)). If the volume of the half-torus is V and the magnetic energy density is p_B, the total energy is

$$W = Vp_B = \pi R \pi r^2 \frac{B^2}{2\mu_0} = \pi^2 \cdot 8 \cdot 2^2 {R_E}^3 \frac{B^2}{2\mu_0} = \pi^2 \cdot 32 \cdot (6378 \cdot 10^3)^3 \frac{(0.29)^2}{2\mu_0}$$
$$= 5.5 \cdot 10^{27} \text{ J}$$

which is enough to produce a solar flare, even if only a fraction of the available magnetic field energy is converted into kinetic energy of the plasma.