KTH, Matematik
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**Final exam in SF1659, Mathematics, Basic course**
**6 October 2013 kl. 13:00-18:00**

For each question you can get at most 4 points. You need to give a full answer in order to get full points for a question. Solution must be well-presented and easy to follow. Explain your arguments, and write clearly and in an organized way.
Questions 1 and 2 correspond to the tests, respectively. If you passed test $j$, you get 4 points for question $j$.(that you don't need to solve)
Questions 3–6 correspond to basic knowledge and skills. Questions 7–9 are a little more advanced. If you want to recieve C or higher, you must get points from this part, called VG-question.
Preliminary grading is: A–31 points with at least 8 VG points B–26 points with at least 5 VG points, C–21 points with at least 2 VG points, D–17, E–15, Fx–13.
There is a chance to complete an Fx within 4 weeks. In such a case, contact Maria Saprykina (masha@kth.se).

**Questions that corresponds to the tests**

1 Solve the equation: $|x-1|+2|x|=4$.

2 Solve the equation: $\cos^2 x - \frac{1}{2}\sin x = 1$.

**G–questions**

3 Solve the equation $\ln(1+e^x) = \ln(1-e^x) + \ln(1+4e^x)$.

4. Given $\sin x = 3/5$ and $0 \leq x \leq \pi/2$. Calculate the value of $(\tan x + \sin x)\tan x$.

5. Solve the inequality $\dfrac{2x^2-3}{x+2} \leq x$.

6. Let $P(x)$ be the polynomial $(x^3-3x)^{10} - (x^2-2)^{15}$. What is the degree of $P(x)$? Compute the coefficient in front of the highest degree.

Turn over the page!

## VG–questions

7. Find all $x$ for which the following inequality holds:

$$\left(\ln\frac{1}{x}\right)\ln x^3 + 12\ln\sqrt{x} < 9.$$

8. Find the inverse of the function $f(x) = \frac{x}{\sqrt{x^2+1}}$. For the obtained inverse function, find the domain of definition and the image.

9. Find the constant $a$ such that the equation $x^3 - 3x + a = 0$ has a double root (that is two roots that are equal).