Final exam in SF1659, Mathematics, Basic course
6 October 2013 kl. 13:00-18:00

For each question you can get at most 4 points. You need to give a full answer in order to get full points for a question. Solution must be well-presented and easy to follow. Explain your arguments, and write clearly and in an organized way.
Questions 1 and 2 correspond to the tests, respectively. If you passed test \( j \), you get 4 points for question \( j \). (that you don’t need to solve)
Questions 3–6 correspond to basic knowledge and skills. Questions 7–9 are a little more advanced. If you want to receive C or higher, you must get points from this part, called VG-question.
Preliminary grading is: A–31 points with at least 8 VG points B–26 points with at least 5 VG points, C–21 points with at least 2 VG points, D–17, E–15, Fx–13.
There is a chance to complete an Fx within 4 weeks. In such a case, contact Maria Saprykina (masha@kth.se).

Questions that corresponds to the tests

1 Solve the equation: \(|x - 1| + 2|x| = 4\).
2 Solve the equation: \(\cos^2 x - \frac{1}{2} \sin x = 1\).

G–questions

3 Solve the equation \(\ln(1 + e^x) = \ln(1 - e^x) + \ln(1 + 4e^x)\).
4. Given \(\sin x = \frac{3}{5}\) and \(0 \leq x \leq \pi/2\). Calculate the value of \((\tan x + \sin x) \tan x\).
5. Solve the inequality \(\frac{2x^2 - 3}{x + 2} \leq x\).
6. Let \(P(x)\) be the polynomial \((x^3 - 3x)^{10} - (x^2 - 2)^{15}\). What is the degree of \(P(x)\)? Compute the coefficient in front of the highest degree.

Turn over the page!
VG–questions

7. Find all $x$ for which the following inequality holds:

$$
\left( \ln \frac{1}{x} \right) \ln x^3 + 12 \ln \sqrt{x} < 9.
$$

8. Find the inverse of the function $f(x) = \frac{x}{\sqrt{x^2+1}}$. For the obtained inverse function, find the domain of definition and the image.

9. Find the constant $a$ such that the equation $x^3 - 3x + a = 0$ has a double root (that is two roots that are equal).