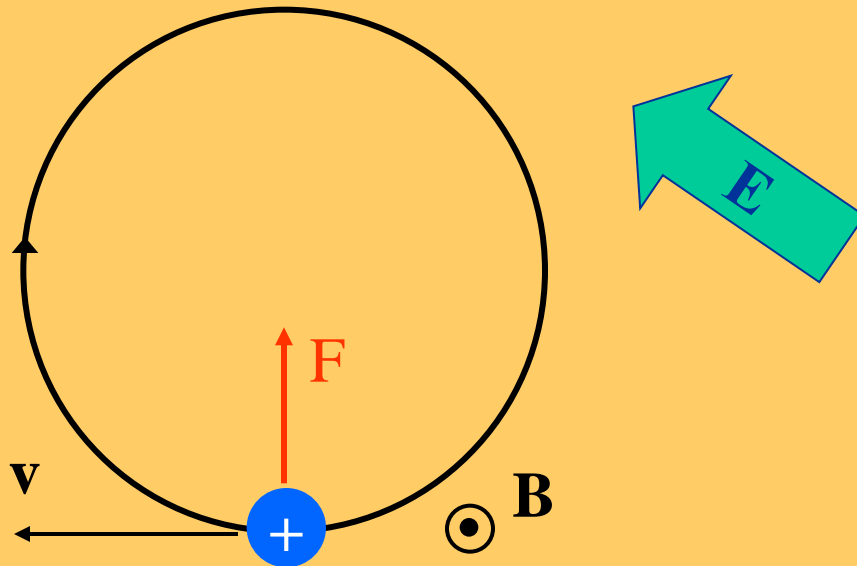


Think about this:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$



What happens if you add an electric field \mathbf{E} ?



Last lecture (3)

- Solar activity
- Solar wind – basic facts
- Solar wind – magnetic structure

Today's lecture (4)

- Ionosphere
 - layers
 - radio wave reflection



Schedule

10×2 h Lectures

6×2 h Tutorials

L = Lecture, T = Tutorial

Activity	Date	Time	Room	Subject	Litterature
L1	29/8	13-15	E52	Course description, Introduction, The Sun 1, Plasma physics 1	CGF Ch 1, 5, (p 110-113)
L2	1/9	15-17	L52	The Sun 2, Plasma physics 2	CGF Ch 5 (p 114-121), 6.3
L3	5/9	13-15	E51	Solar wind, The ionosphere and atmosphere 1, Plasma physics 3	CGF Ch 6.1, 2.1-2.6, 3.1-3.2, 3.5, LL Ch III, Extra material
T1	8/9	15-17	D41	Mini-group work 1	
L4	12/9	13-15	E35	The ionosphere 2, Plasma physics 4	CGF Ch 3.4, 3.7, 3.8
L5	14/9	10-12	V32	The Earth's magnetosphere 1, Plasma physics 5	CGF 4.1-4.3, LL Ch I, II, IV.A
T2	15/9	15-17	E51	Mini-group work 2	
L6	19/9	13-15	M33	The Earth's magnetosphere 2, Other magnetospheres	CGF Ch 4.6-4.9, LL Ch V.
T3	22/9	15-17	E51	Mini-group work 3	
L7	26/9	13-15	E31	Aurora, Measurement methods in space plasmas and data analysis 1	CGF Ch 4.5, 10, LL Ch VI, Extra material
L8	28/9	10-12	L52	Space weather and geomagnetic storms	CGF Ch 4.4, LL Ch IV.B-C, VII.A-C
T4	29/9	15-17	M31	Mini-group work 4	
L9	3/10	13-15	E52	Interstellar and intergalactic plasma, Cosmic radiation,	CGF Ch 7-9
T5	6/10	15-17	E31	Mini-group work 5	
L10	10/10	13-15	E52	Swedish and international space physics research.	
T6	13/10	15-17	E31	Round-up, old exams.	
Written examination	26/10	8-13	F2		



Mini groupwork 1

$$h = \frac{17}{2} \cdot 6378 \text{ km}$$

a)

The thermal energy is divided into motion in the three dimensions, two of which only give rise to a gyro motion around the magnetic field lines, with the motion along the magnetic field corresponding to an energy

$$E = \frac{k_B T}{2} = \frac{1.38 \cdot 10^{-23} \cdot 2 \cdot 10^6}{2} = 1.4 \cdot 10^{-17} \text{ J}$$

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 1.4 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 5500 \text{ km/s}$$

Approximating the loop with a quarter-circle, the electron has to travel a length

$$s = \pi h / 2 = 85 \text{ 000 km}$$

Then we get $t = 15 \text{ s}$.

Energy - temperature

Average energy of molecule/atom:

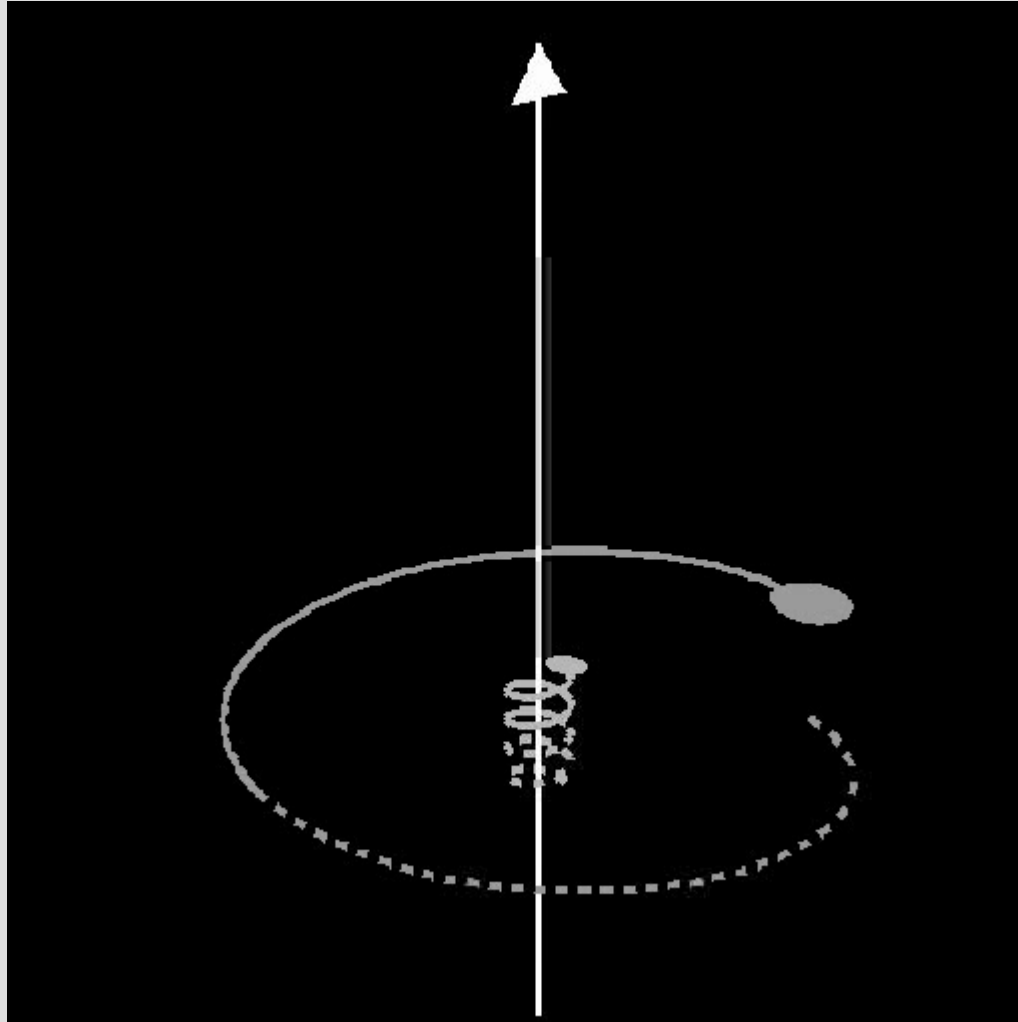
$$E = \frac{3}{2} k_B T \Rightarrow$$

$$T = \frac{2E}{3k_B}$$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J} \Rightarrow$$

$$T = \frac{2E}{3k_B} = \frac{2 \cdot 1.6 \cdot 10^{-19} \text{ J}}{3 \cdot 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}} = 7729 \text{ K}$$

Gyro motion



Equipartition principle

Statistically the kinetic energy is equally distributed along the three dimensions:

$$E_{\parallel} = \frac{1}{2} k_B T$$

$$E_{\perp} = \frac{2}{2} k_B T$$

Mini groupwork 1

$$\text{b) } f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi} \frac{qB}{m} \Rightarrow$$
$$B = \frac{2\pi f_c m}{q} = \frac{2\pi \cdot 8 \cdot 10^9 \cdot 0.91 \cdot 10^{-30}}{1.6 \cdot 10^{-19}} = 0.29 \text{ T}$$

The perpendicular energy is given by

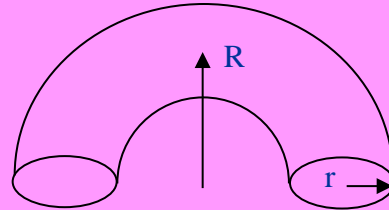
$$E = \frac{2k_B T}{2} = \frac{2 \cdot 1.38 \cdot 10^{-23} \cdot 2 \cdot 10^6}{2} = 2.8 \cdot 10^{-17} \text{ J}$$

$$v_{\perp} = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \cdot 2.8 \cdot 10^{-17}}{0.91 \cdot 10^{-30}}} = 7.8 \cdot 10^6 \text{ m/s}$$

$$\rho = \frac{m_e v_{\perp}}{qB} = \frac{0.91 \cdot 10^{-30} \cdot 7.8 \cdot 10^6}{1.6 \cdot 10^{-19} \cdot 0.29} = 1.5 \cdot 10^{-4} \text{ m}$$

Mini groupwork 1

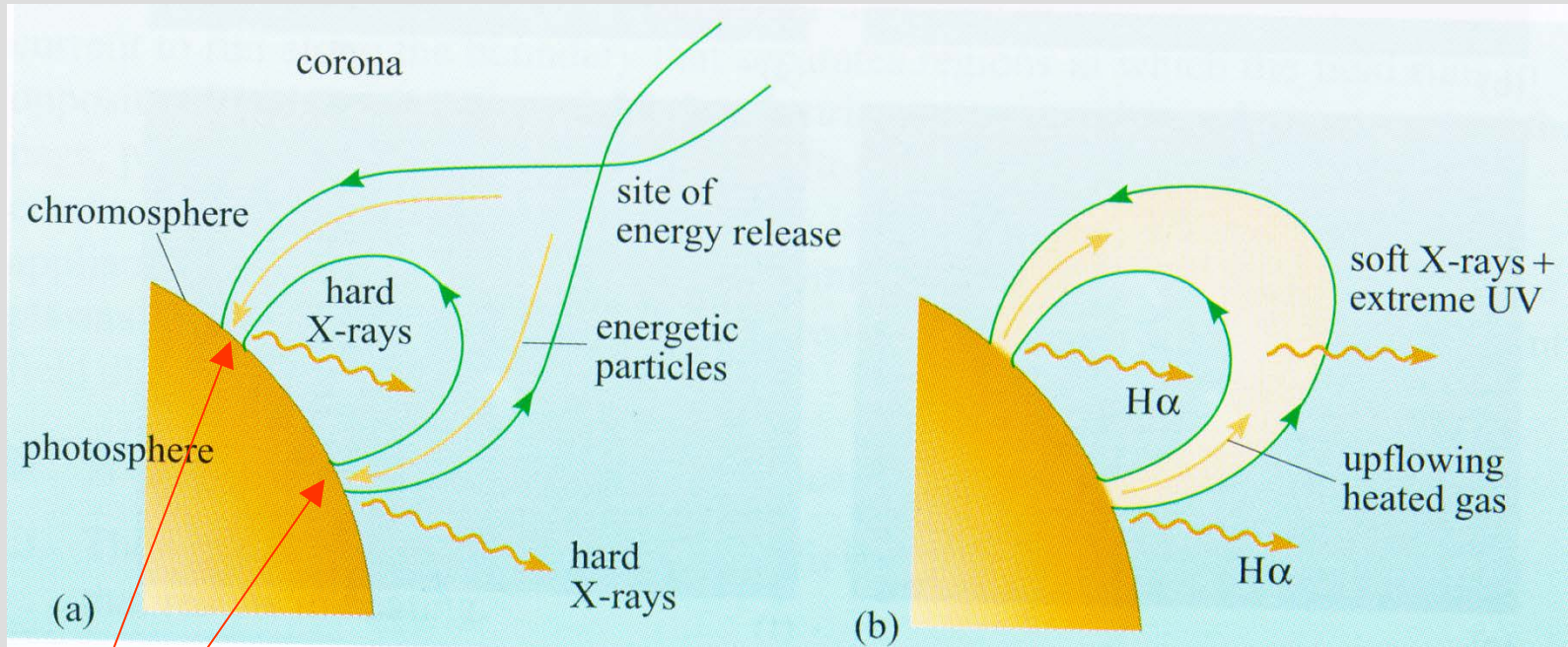
- c) Model the flare by a half torus with minor axis r , and major axis. From the figure, estimate $R = 8 R_E$, and $r = 2 R_E$.



Let this half-torus be filled with a magnetic field of strength $B \sim 0.36$ T (using the value in b)). If the volume of the half-torus is V and the magnetic energy density is p_B , the total energy is

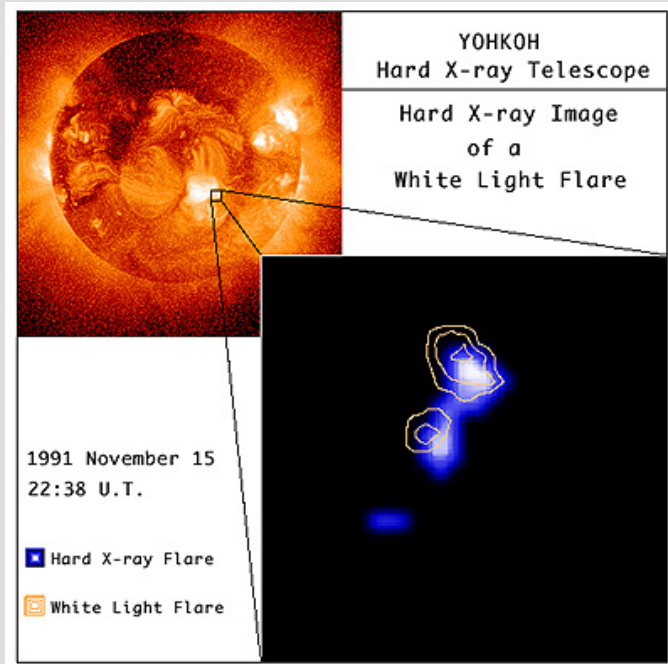
$$\begin{aligned}
 W &= V p_B = \pi R \pi r^2 \frac{B^2}{2\mu_0} = \pi^2 \cdot 8 \cdot 2^2 R_E^3 \frac{B^2}{2\mu_0} \\
 &= \pi^2 \cdot 32 \cdot (6378 \cdot 10^3)^3 \frac{(0.29)^2}{2\mu_0} \\
 &= 5.5 \cdot 10^{27} \text{ J}
 \end{aligned}$$

Solar flare mechanism

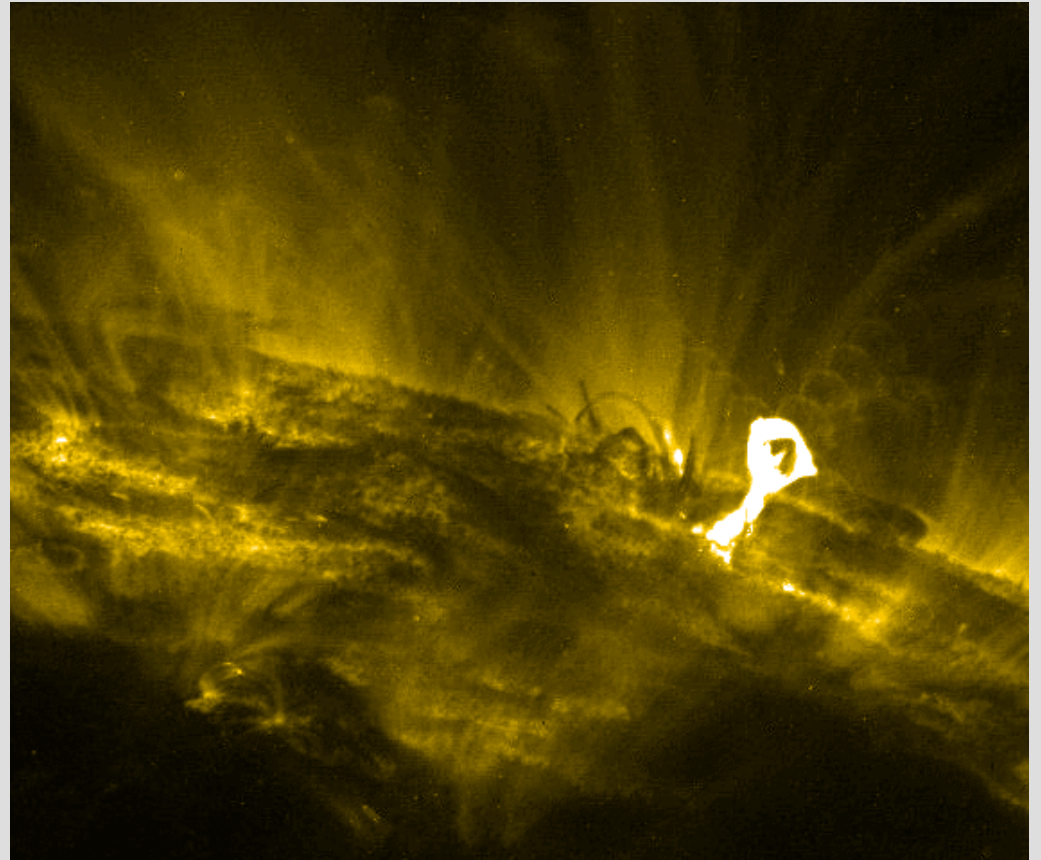


Electrons are accelerated, collide with solar surface (photosphere) and emit bremsstrahlung (X-rays).

Solar flare observations



(a) double signature of x-ray emissions at foot of flare



(b) coronal loop filled with hot gas

Frozen in magnetic flux *PROOF II*

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{v} \times \mathbf{B})}_A + \underbrace{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}}_B$$

Order of magnitude estimate:

$$\frac{A}{B} = \frac{\nabla \times (\mathbf{v} \times \mathbf{B})}{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}} \approx \frac{\frac{v \Delta B}{L}}{\frac{\Delta B}{\mu_0 \sigma L^2}} = v L \mu_0 \sigma \equiv R_m$$

Magnetic Reynolds number R_m :

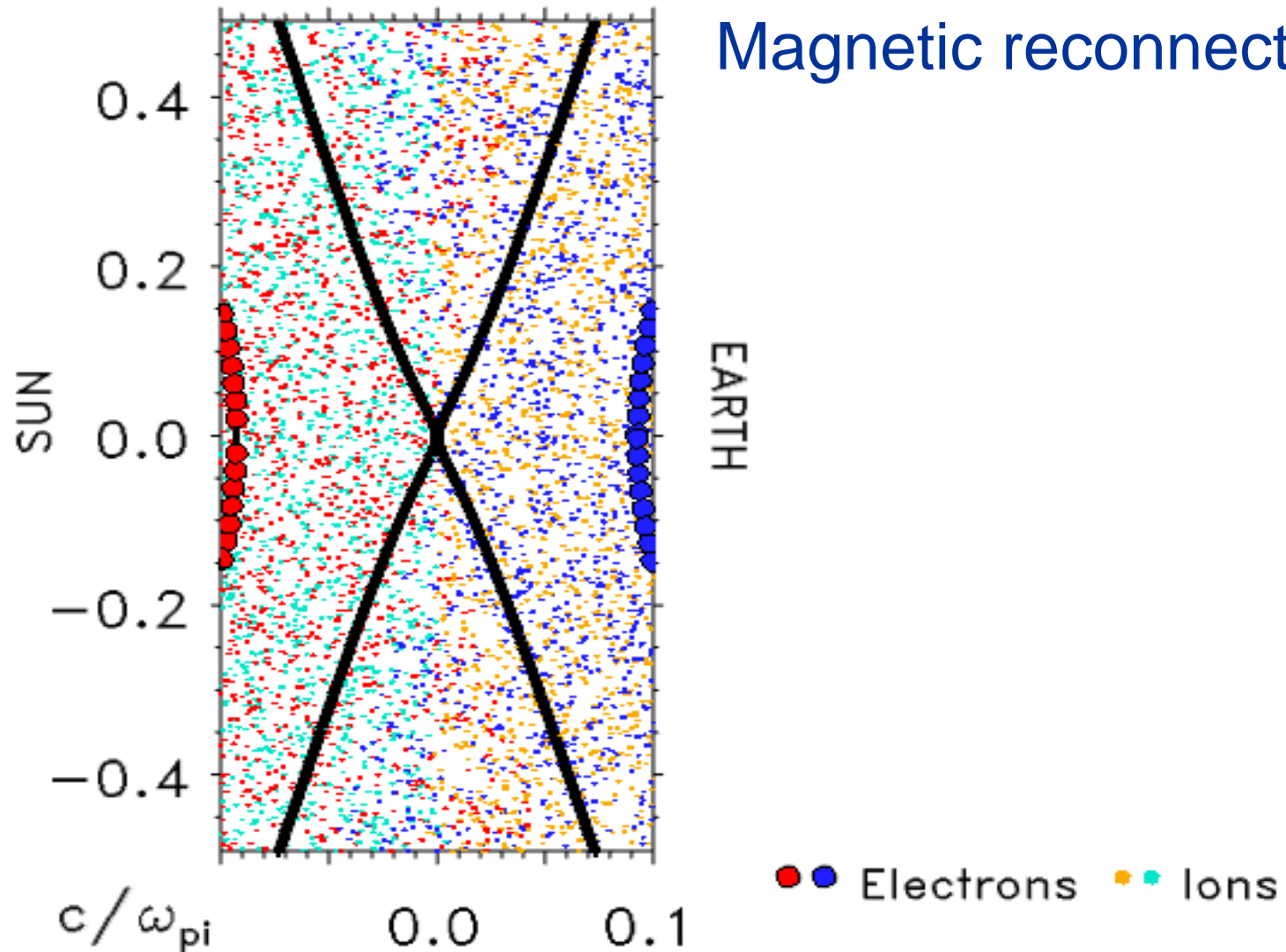
$$R_m \gg 1 \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Frozen-in fields!

$$R_m \ll 1 \Rightarrow \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Diffusion equation!

Magnetic reconnection



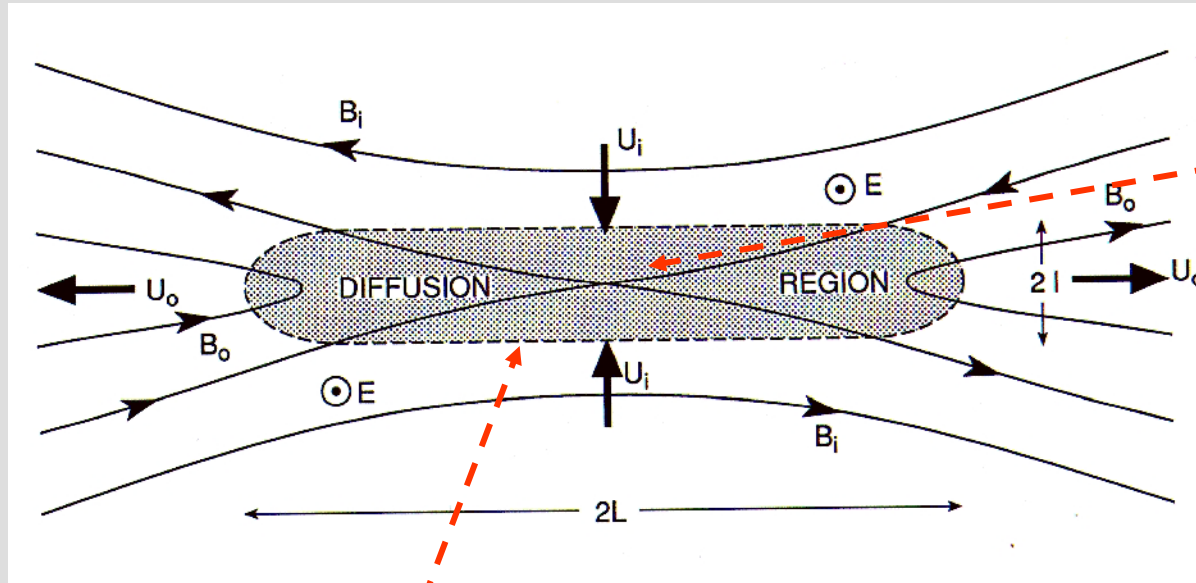
Reconnection

In 'diffusion region':

$$R_m = \mu_0 \sigma l v \sim 1$$

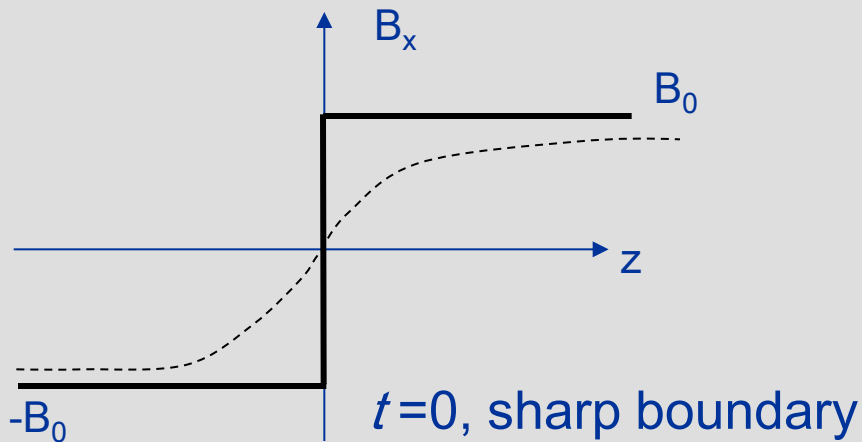
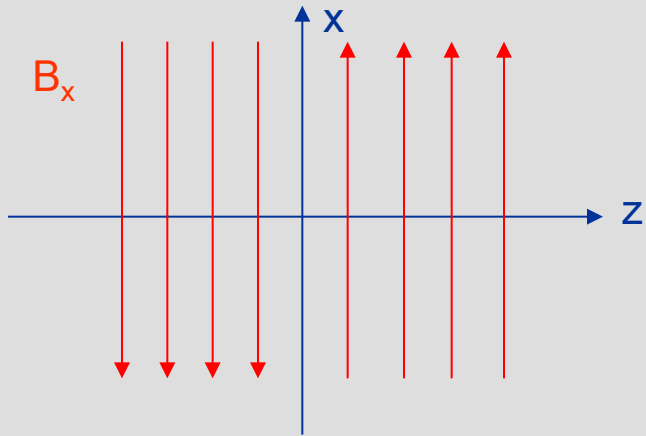
Thus: **condition** for frozen-in magnetic field breaks down.

A second **condition** is that there are two regions of magnetic field pointing in *opposite* direction:



- Field lines are “cut” and can be re-connected to other field lines
- **Magnetic energy is transformed into kinetic energy ($U_0 \gg U_i$)**
- **Plasma from different field lines can mix**

Reconnection in 1D



$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \quad \rightarrow \quad \frac{\partial B_x}{\partial t} = \frac{1}{\mu_0 \sigma} \frac{\partial^2 B_x}{\partial z^2}$$

Diffusion equation! Has solution

$$B_x(z, t) = B_0 \operatorname{erf} \left(\left[\frac{\mu_0 \sigma}{4t} \right]^{1/2} z \right)$$

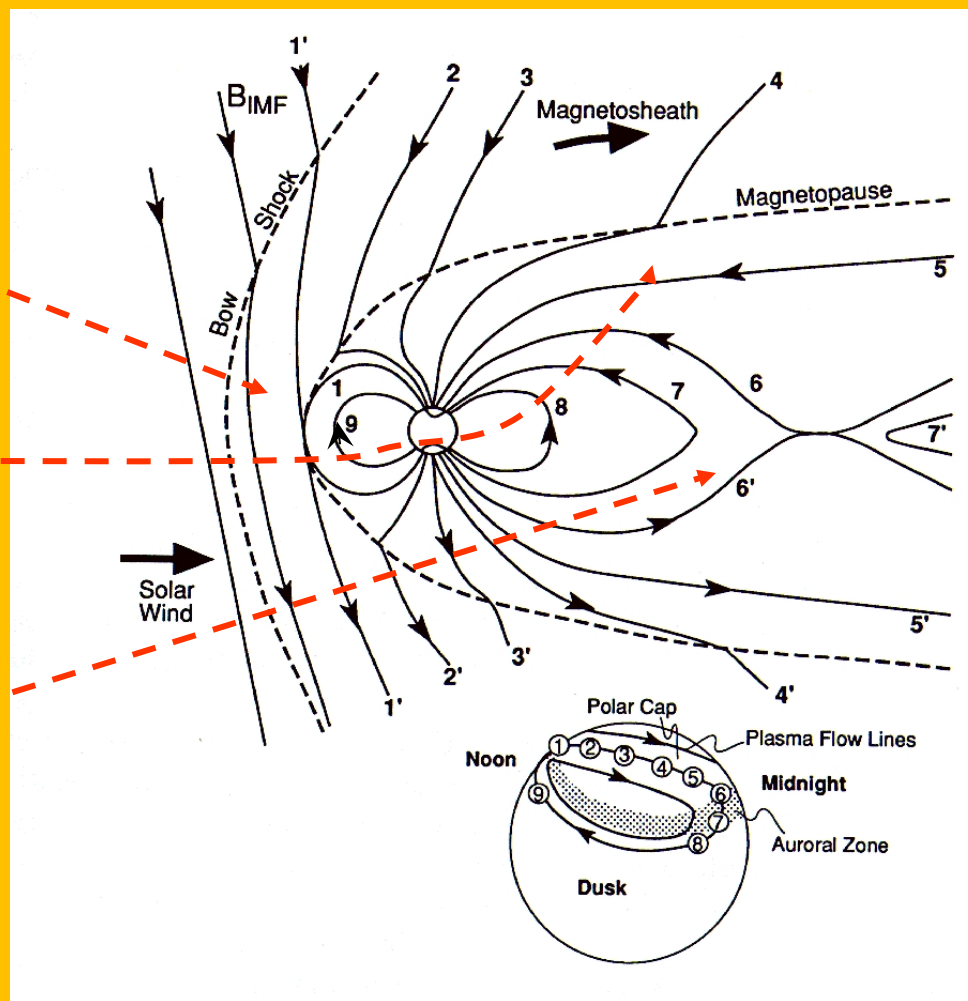
The total magnetic energy then decreases with time:

$$W_B = \int_{-\infty}^{\infty} \frac{B^2}{2\mu_0} dz$$

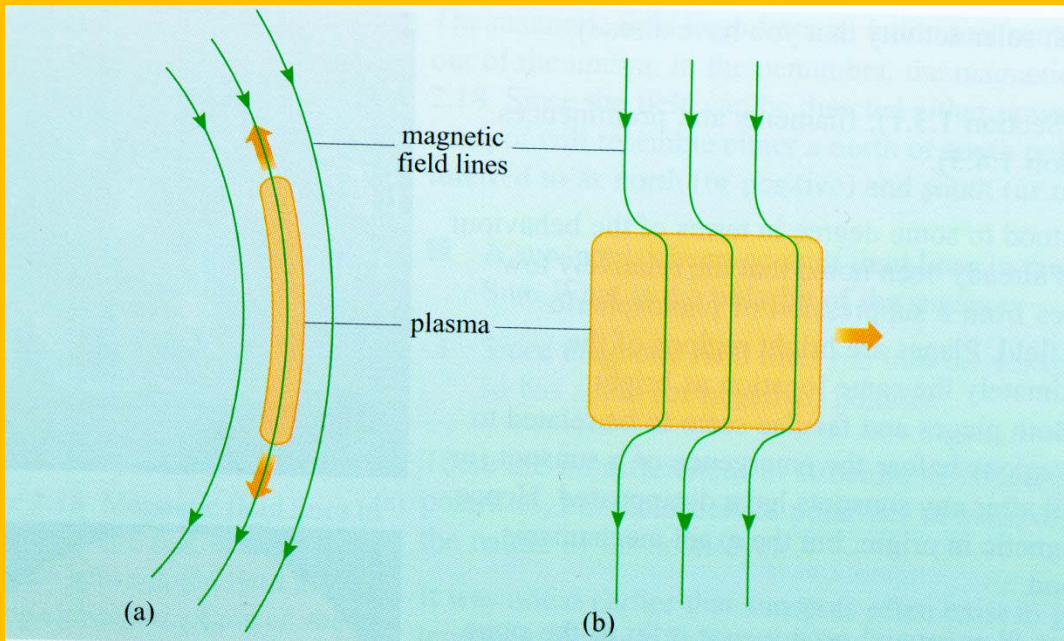
The magnetic energy is converted into heat and kinetic energy in 2D

Reconnection och plasma convection

- Reconnection on the dayside “re-connects” the solar wind magnetic field and the geomagnetic field
- In this way the plasma convection in the outer magnetosphere is driven
- In the night side a second reconnection region drives the convection in the inner magnetosphere. The reconnection also heats the plasmasheet plasma.



Does the plasma follow the magnetic field (a) or the other way around (b)?



$$\beta \ll 1$$

$$\beta \gg 1$$

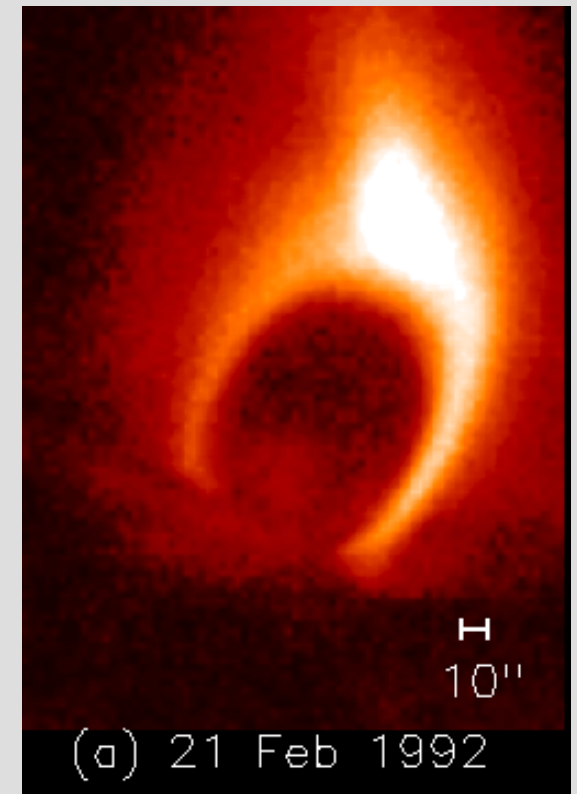
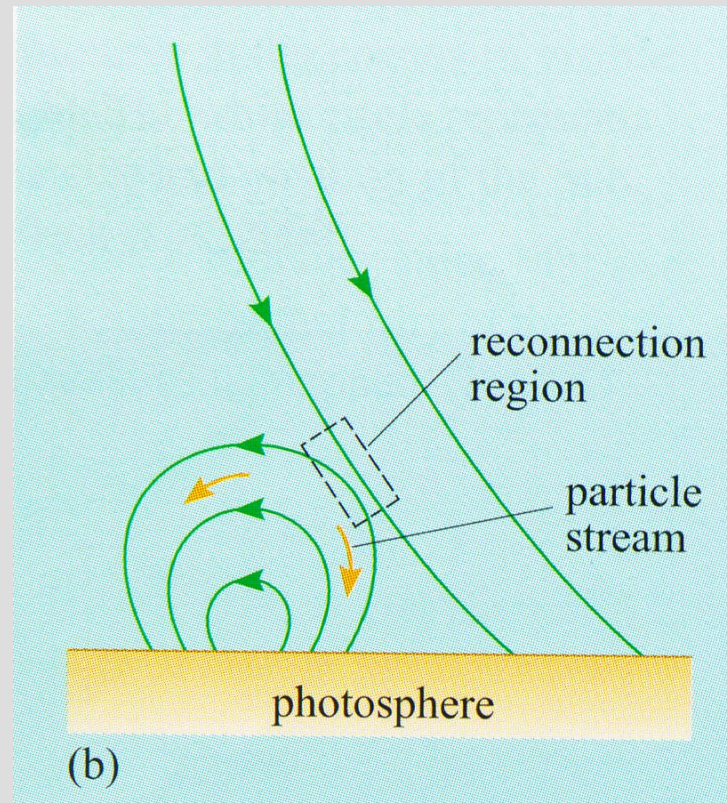
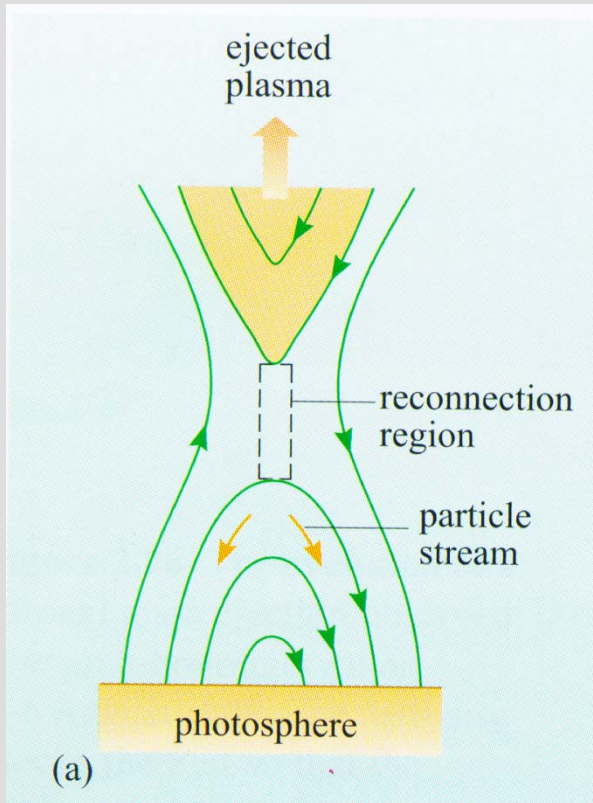
Depends on relative energy density (pressure)

$$p_{pl} = nk_B T$$

$$p_B = \frac{B^2}{2\mu_0}$$

$$\beta = \frac{p_{pl}}{p_B}$$

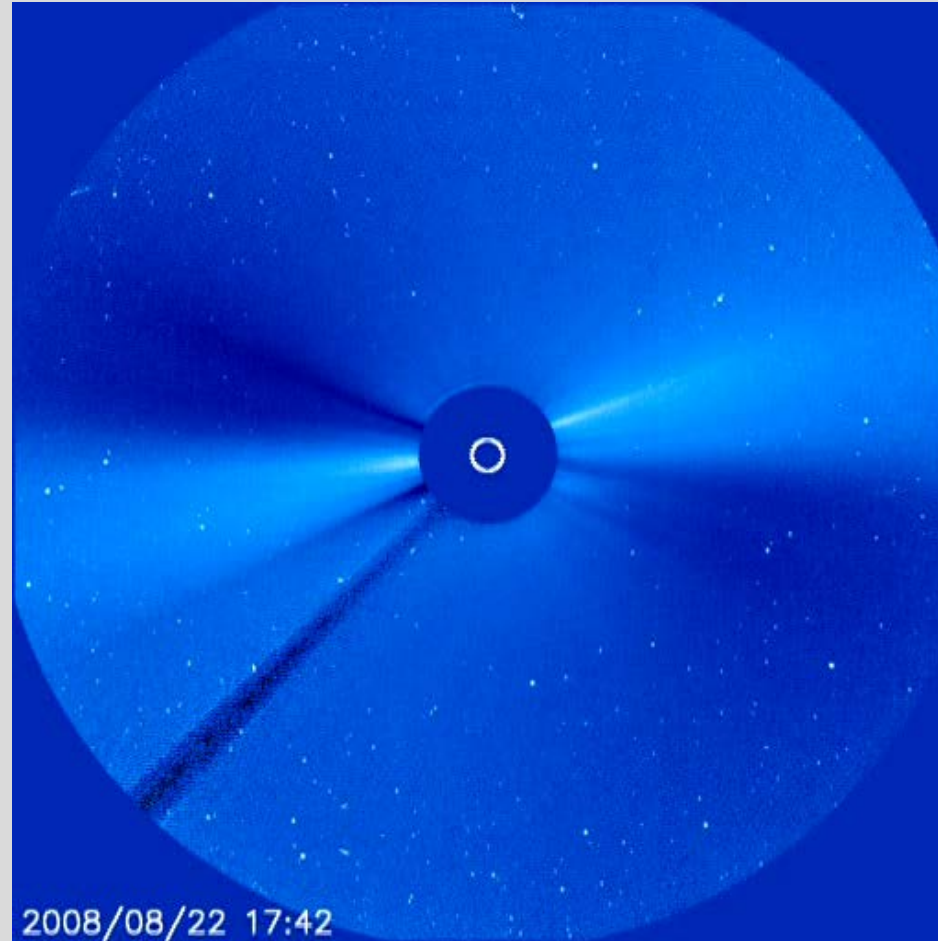
Solar flare *energization mechanism*



Two possible reconnection geometries

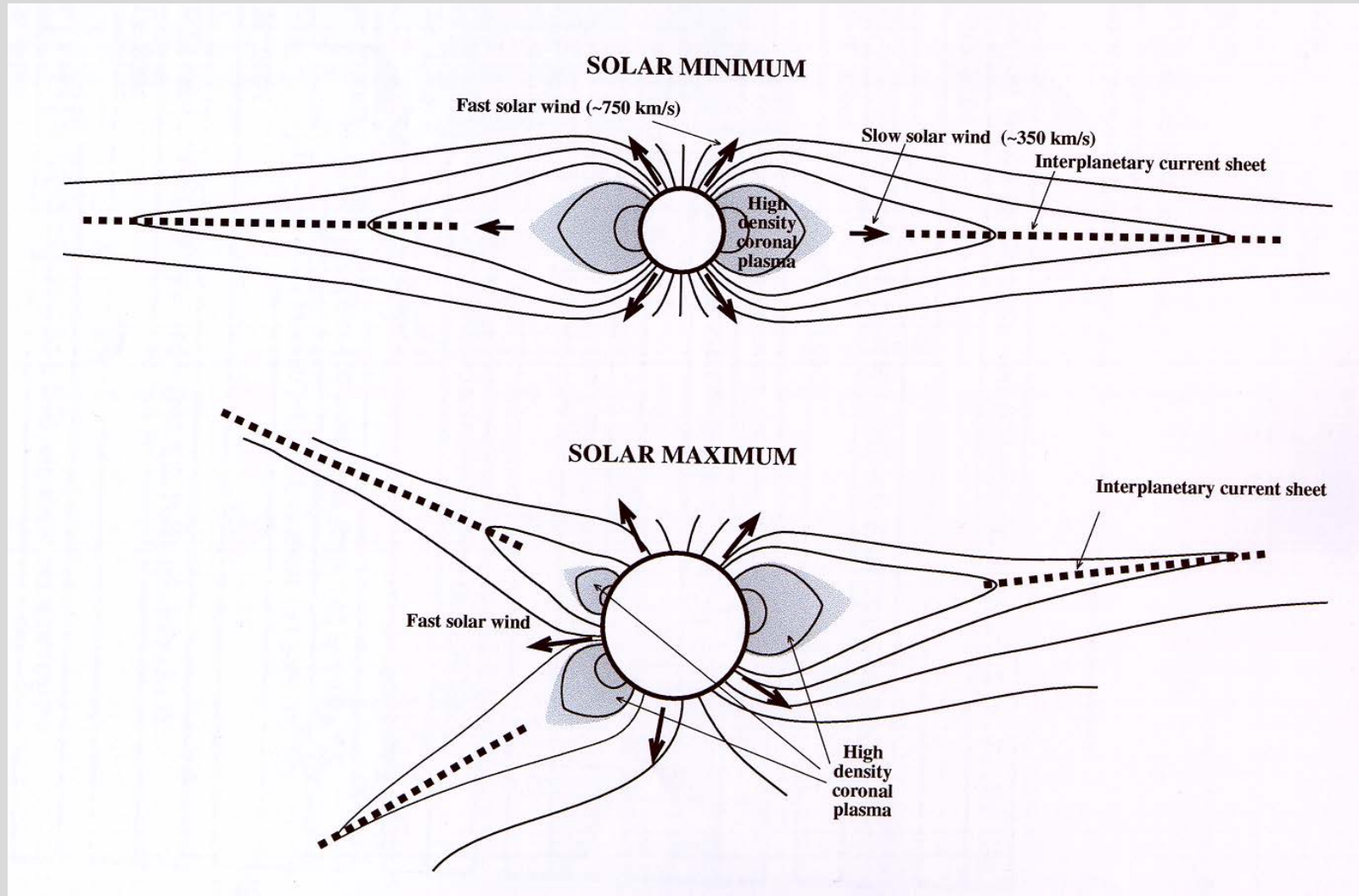
Solar wind

***Corona
continuously
merges into
solar wind***



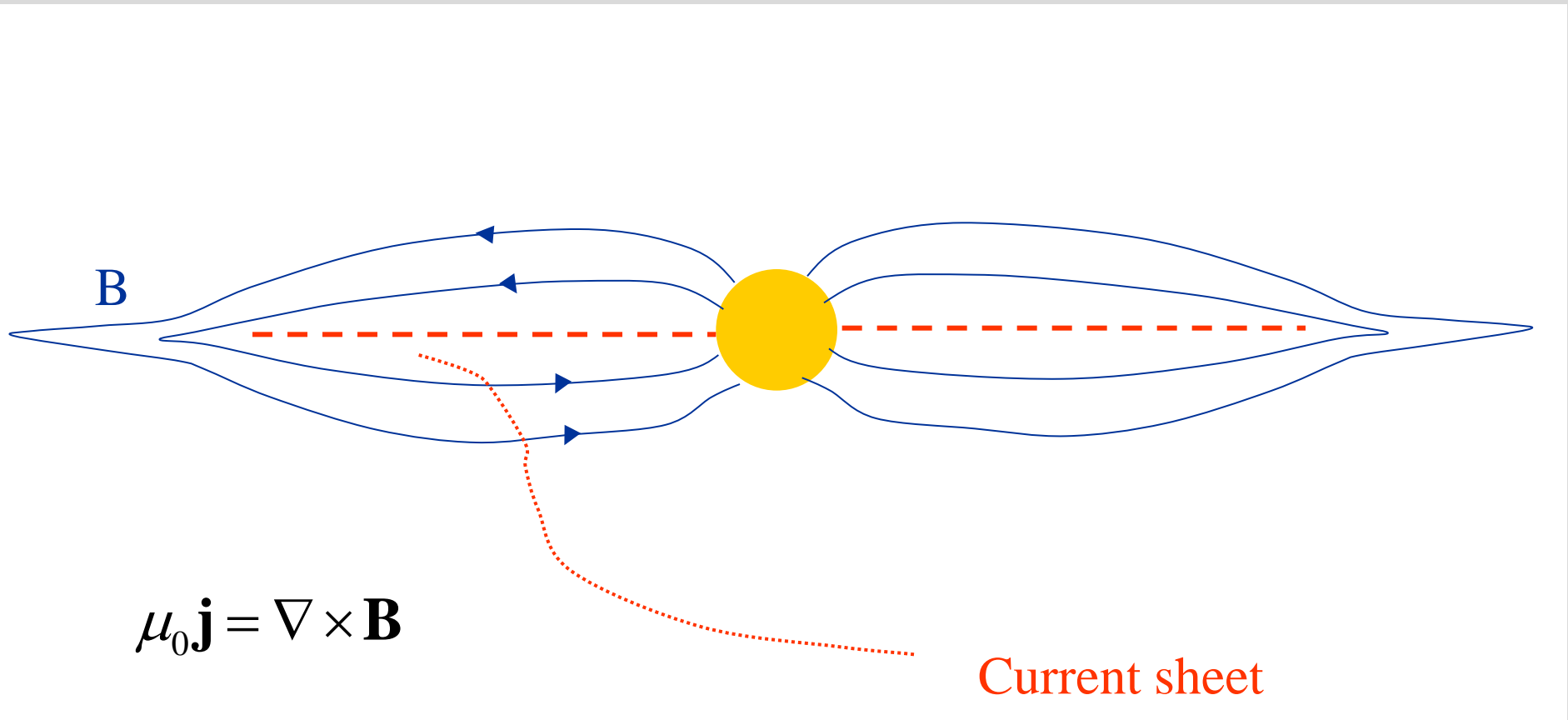
Solar and Heliospheric Observatory (SOHO)
LASCO C3 Coronagraph Movie

Solar wind



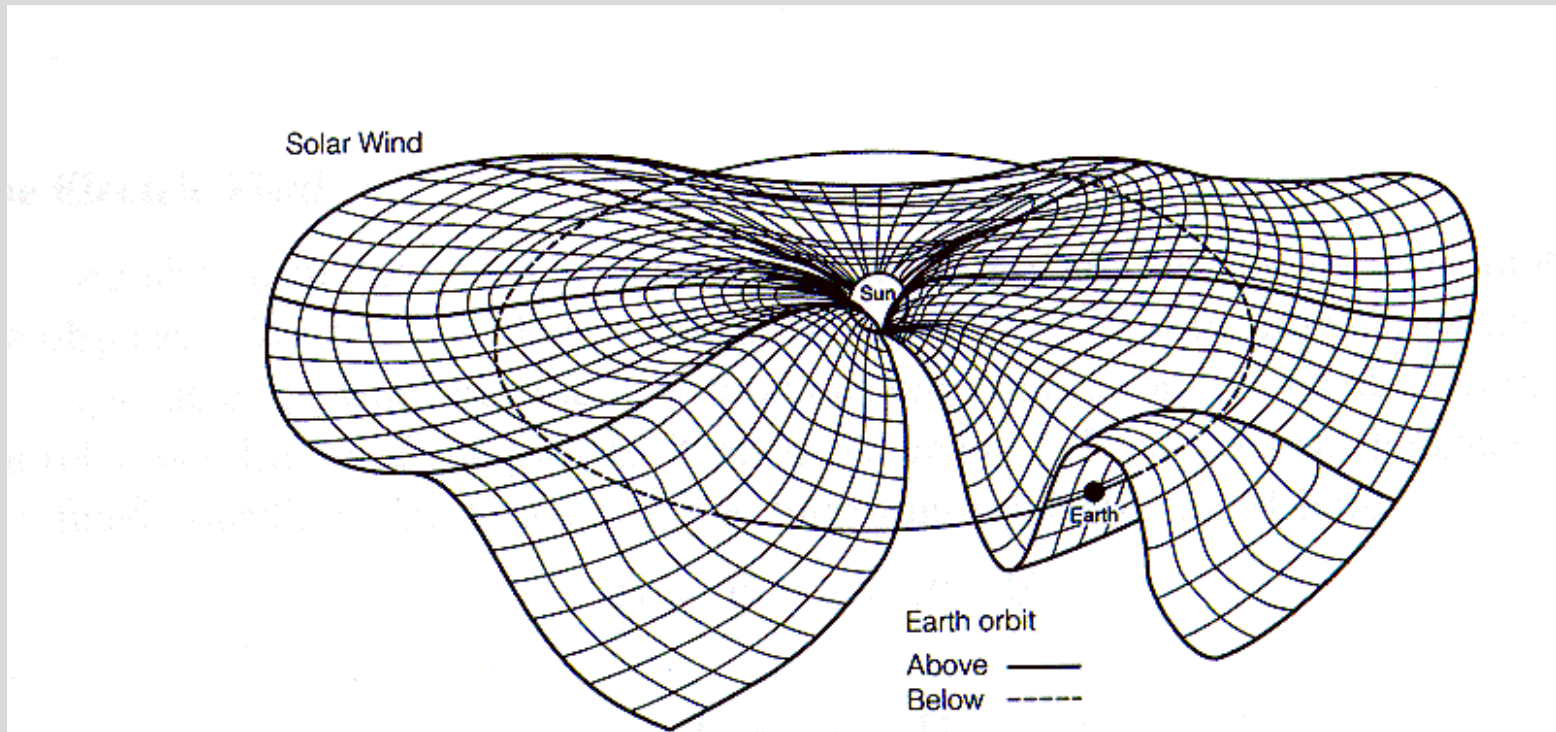
Solar wind

Interplanetary current sheet



Solar wind

Interplanetary current sheet



Later we will see that the N-S component of the interplanetary magnetic field (IMF) is important for the coupling between solar wind and magnetosphere)

Solar wind

Some basic facts

Average values

$$n_p = 8 \text{ cm}^{-3}$$

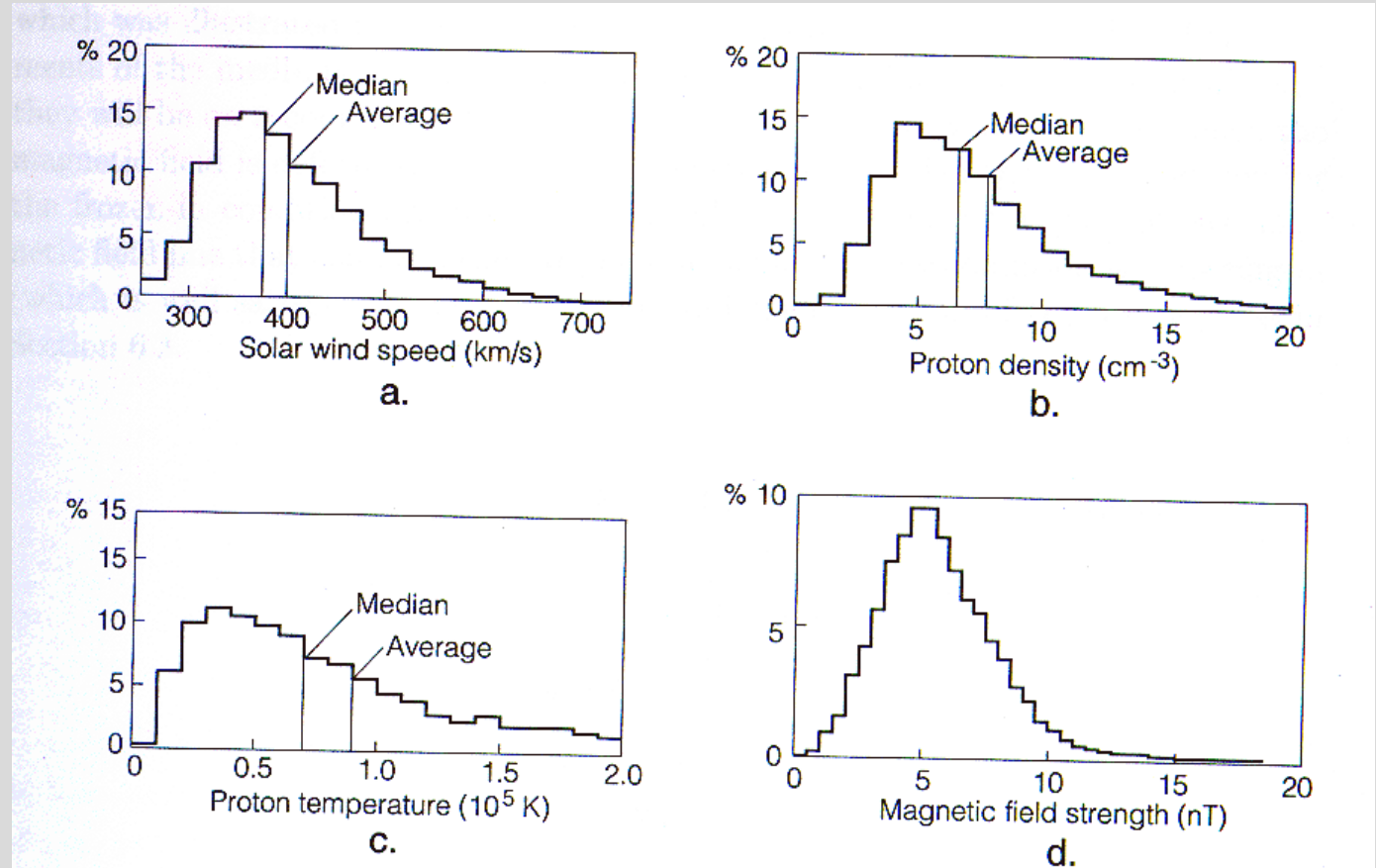
$$v = 320 \text{ km/s}$$

$$T_p = 4 \cdot 10^4 \text{ K}$$

$$T_e = 10^5 \text{ K}$$

$$B = 5 \text{ nT}$$

$$\Phi_K = \rho v^3 / 2 = 0.22 \text{ mW/m}^2$$

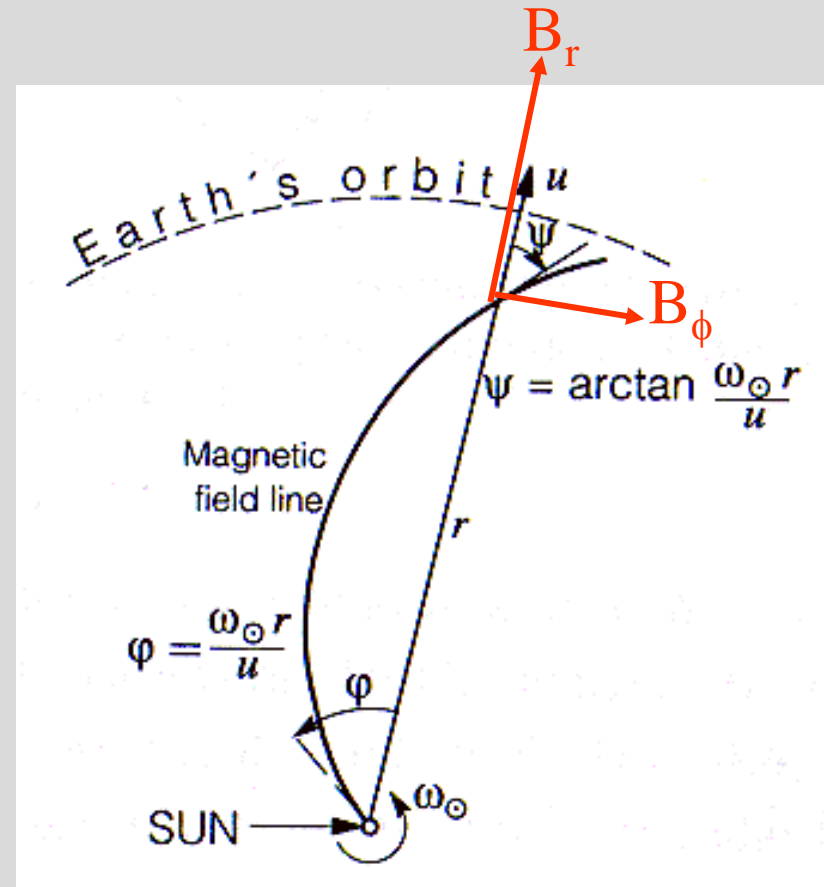


Solar wind

Parker spiral

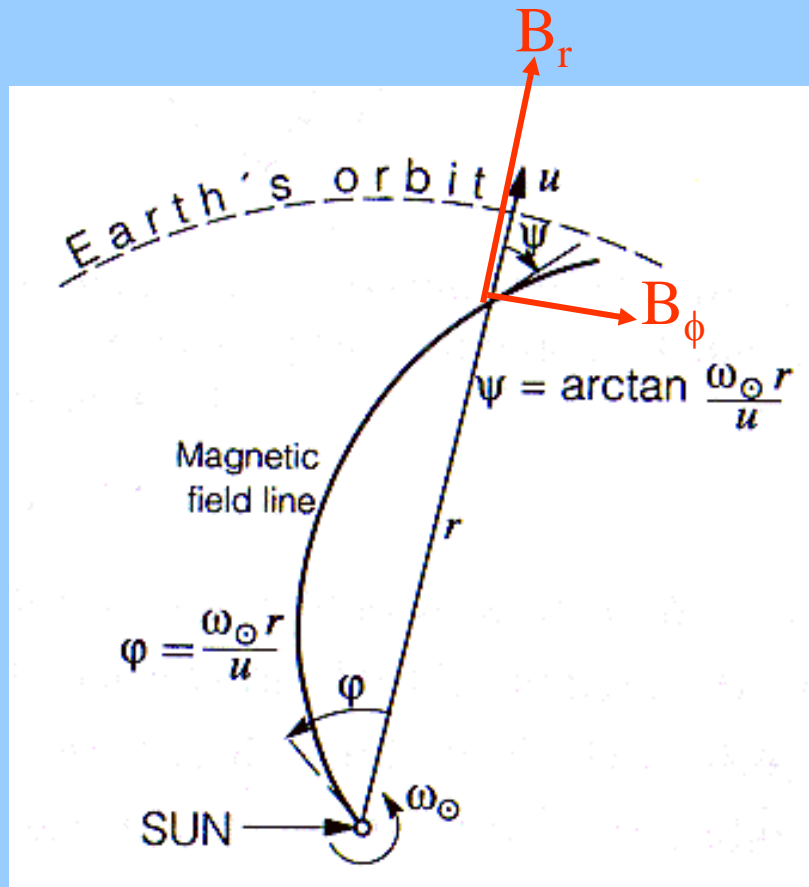
Archimedean spiral:

$$\frac{B_{\phi}}{B_r} = \tan \psi = \left(\frac{\omega r}{u_{SW}} \right)$$



Use rotation period
 T of sun: $T = 27$ days

What is the angle Ψ
 at Earth's orbit for a
 typical solar wind
 speed?



$$r = 1 \text{ A.U.}$$

Yellow $\approx 50^\circ$

Red $\approx 80^\circ$

Blue $\approx 1^\circ$

Green $\approx 10^\circ$

$$\Psi = \arctan\left(\frac{\omega r}{u}\right)$$

What is ω ? $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{27 \cdot 24 \cdot 60 \cdot 60} = 2.7 \cdot 10^{-6} \text{ s}^{-1}$

$$\Psi = \arctan\left(\frac{\omega r}{u}\right) = \arctan\left(\frac{2.7 \cdot 10^{-6} \cdot 1.5 \cdot 10^{11}}{320 \cdot 10^3}\right) = \arctan(1.27) = 52^\circ$$

Yellow



Classification of plasmas

- **High density plasmas**

- $\lambda \ll \rho$
- *magnetic field not important, collisions dominate, isotropic.*

- **Medium density plasmas**

- $\rho \ll \lambda \ll l_c$
- *magnetic field important, collisions important, anisotropies.*

- **Low density plasmas**

- $l_c \ll \lambda$
- *magnetic field important, anisotropies, uninhibited motion along magnetic field*

ρ : gyro radius

λ : mean free path

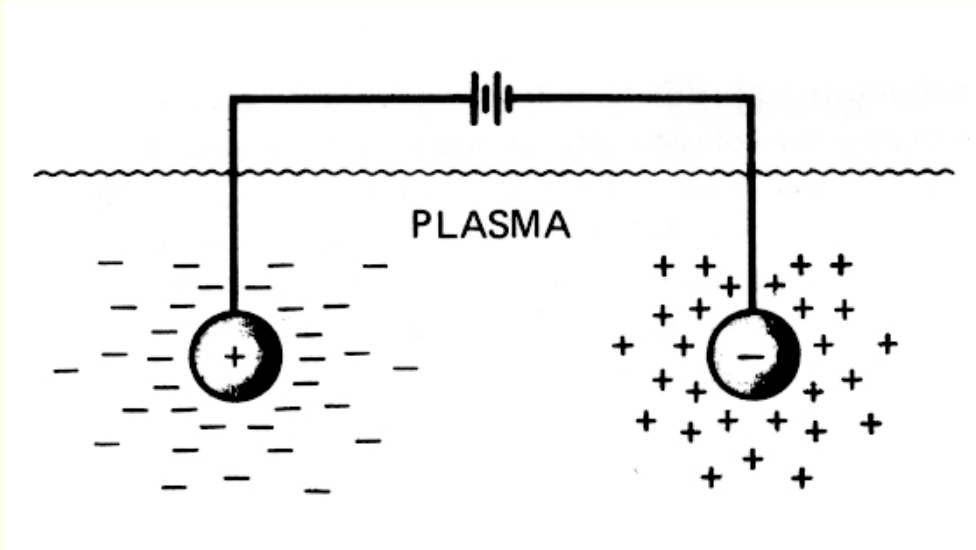
l_c : dimension of the plasma



Plasma models/descriptions

- Single particle motion
- Computer simulations of many-particle dynamics
- Generalization of statistical mechanics (kinetic theory)
- Generalization of fluid mechanics:
Magneto-hydrodynamics (MHD)

Quasineutrality



$$\Phi = \Phi_0 e^{-x/\lambda_D}$$

Debye length

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_e e^2}}$$

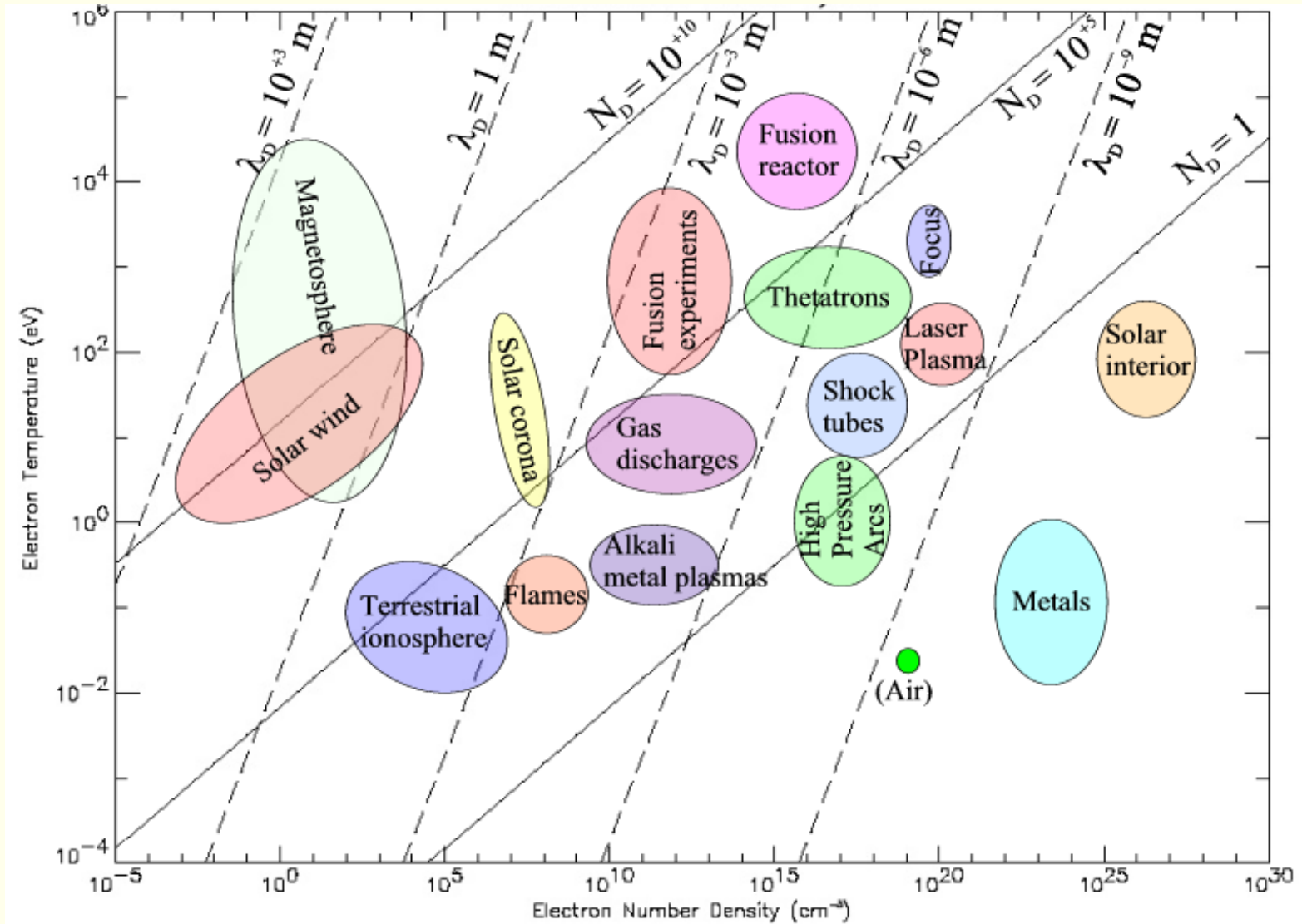
$$\frac{\Delta n}{n} = \frac{(n_e - n_i)}{n_e} < \left(\frac{\lambda_D}{l_c} \right)^2$$

$$l_C \gg \lambda_D \Rightarrow$$

Plasma close to neutral:

$$n_e \approx n_i$$

Debye lengths



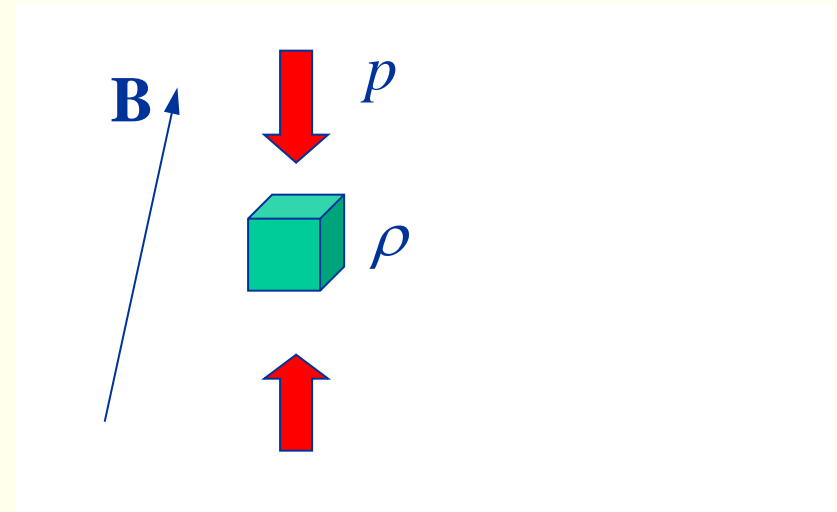
Plasma physics

Magnetohydrodynamics (MHD)

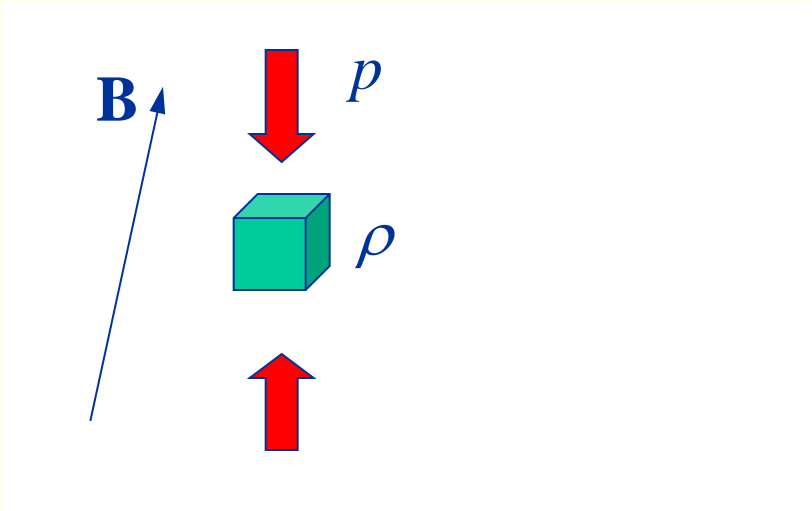
MHD is a combination of

- *fluid-/hydrodynamics* (which is based on Newton's laws of motion)
- *Maxwell's equations* (electrodynamics)

applied on a plasma volume element.



Magnetohydrodynamics (MHD)



For a volume element of plasma:

$$\mathbf{F} = m\mathbf{a} \quad \Rightarrow$$

$$-\nabla p + n_e q \mathbf{v}_e \times \mathbf{B} + \cancel{\rho_e \mathbf{E}} = \rho \frac{d\mathbf{v}}{dt} \quad \Rightarrow$$

quasineutrality

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

Magnetohydrodynamics (MHD)

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

This together with two of Maxwell's equations and Ohm's law make up the most common MHD equations:

$$(2) \quad \mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$(3) \quad \nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Only consider slow variations

$$(4) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Magnetohydrodynamics (MHD)

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

In equilibrium:

$$0 = -\nabla p + \mathbf{j} \times \mathbf{B} \quad \longleftrightarrow$$

$$-\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} = 0$$

$$-\nabla p - \nabla \left(\frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} = 0$$

Represents tension in magnetic field

If magnetic tension = 0

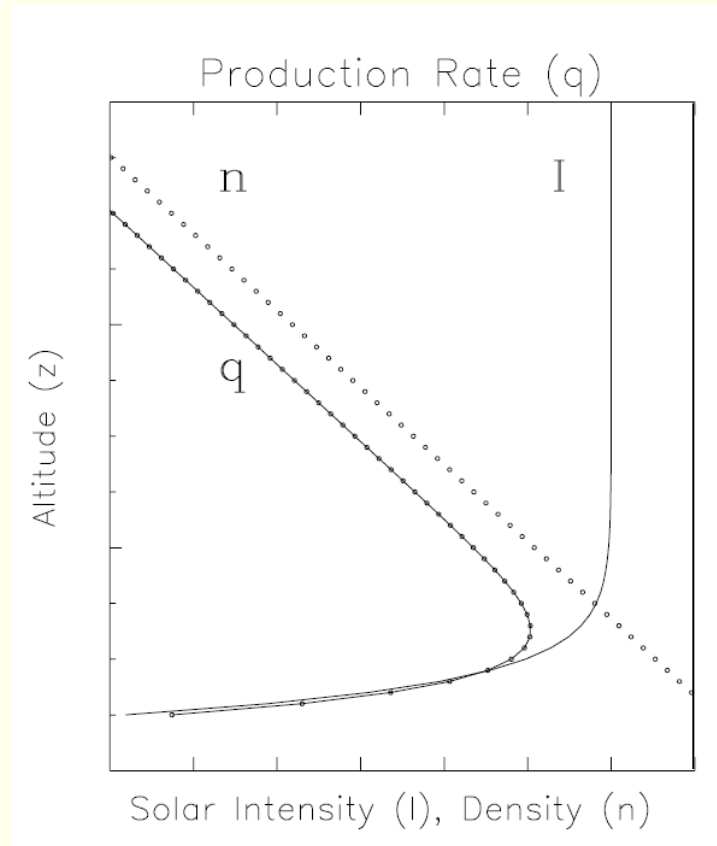
$$p + \frac{B^2}{2\mu_0} = \textit{konst}$$

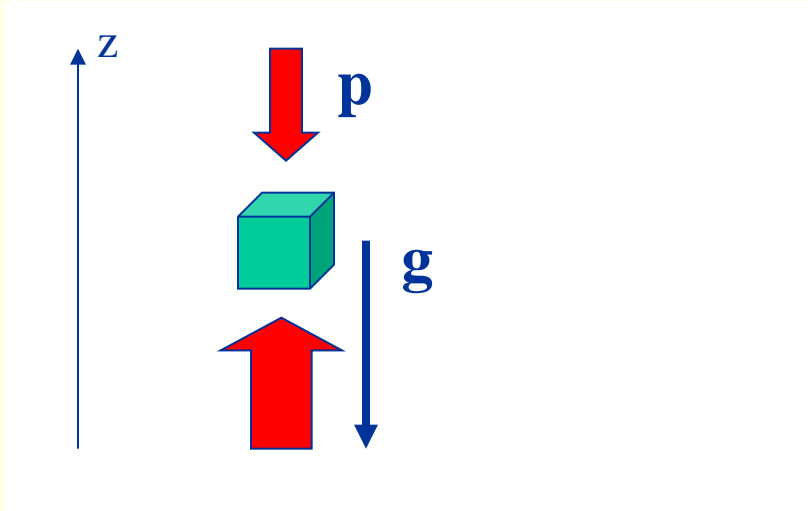
Magnetic pressure



The ionosphere

Basic principle for creation of ionospheric layer





Atmospheric scale height

$$-\frac{dp}{dz} = g\rho_m \quad \text{hydrostatic equilibrium for a volume element}$$

$$p = nk_B T = \frac{\rho k_B T}{m} \quad \text{ideal gas law}$$

$$-\frac{k_B T}{m} \frac{d\rho_m}{dz} = g\rho_m \quad \text{if } T \text{ is constant}$$

$$\rho_m = \text{const} \cdot e^{-z/(k_B T / gm)} = \text{const} \cdot e^{-z/H}$$

Scale height

$$H = k_B T / gm$$

Scale height

$$H = k_B T / gm$$

What is the approximate scale height in the atmosphere right here, right now?

(0° C = 273 K)

Blue

1 km

Yellow

30 km

Green

9 km

Red

100 km



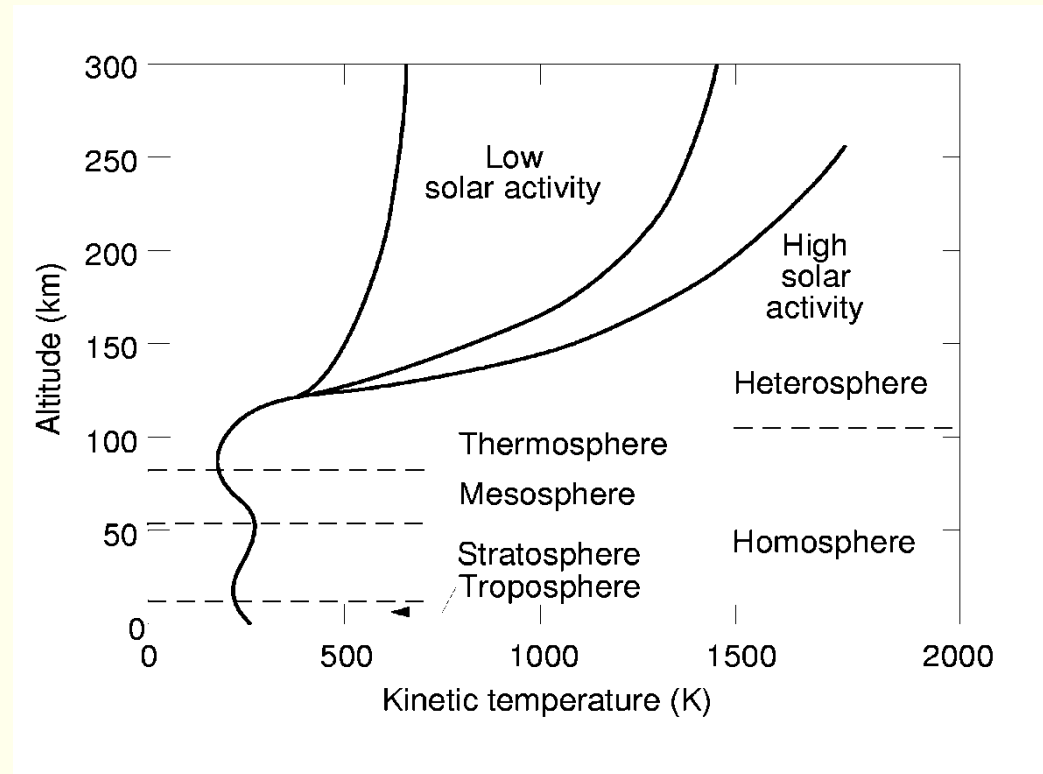
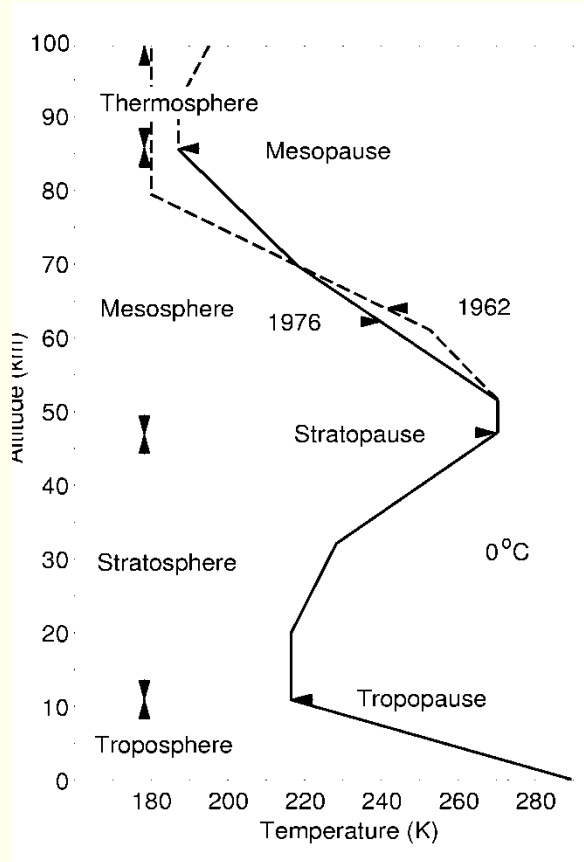
$$H = k_B T / gm = (1.38 \cdot 10^{-23} \cdot 290) / (9.81 \cdot 14 \cdot 2 \cdot 1.67 \cdot 10^{-27}) =$$
$$= 8724 \text{ m} \approx 9 \text{ km}$$

Green



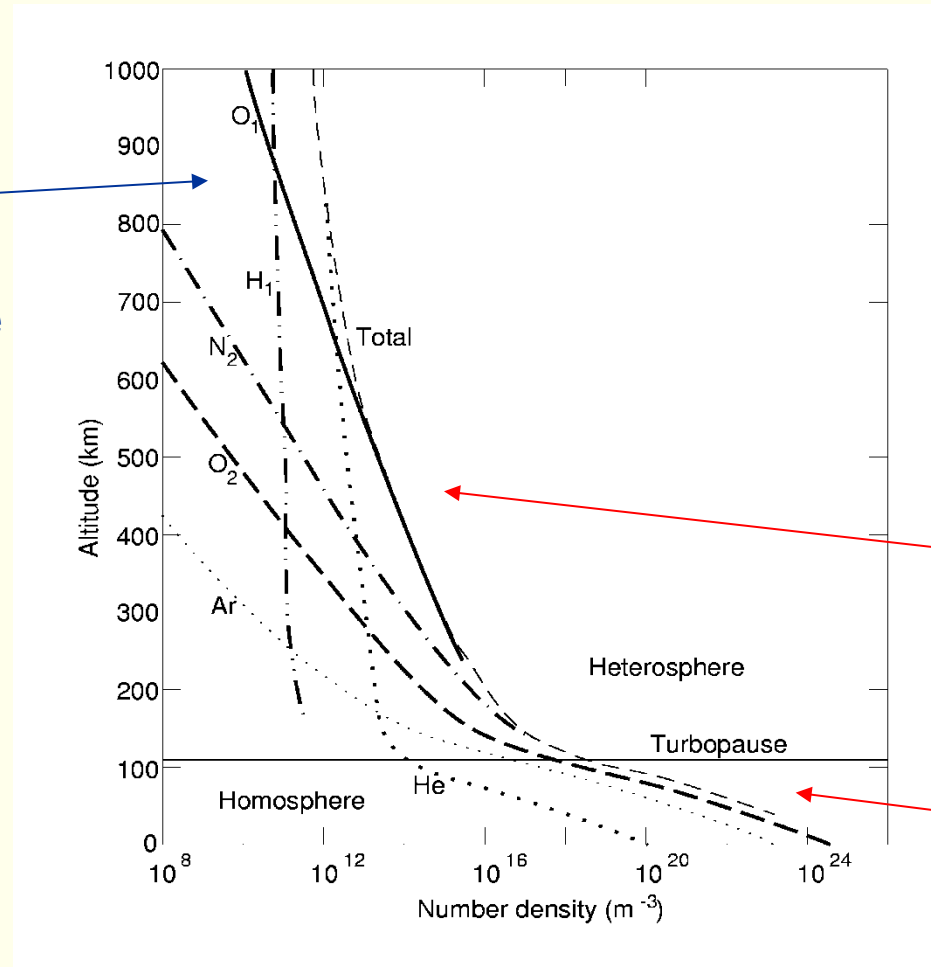
What did we neglect
when we derived the
scale height?

Temperature profile



Atmospheric composition

Longer scale height due to higher temperature



Separate scale heights for different components

Turbulent mixing – one scale height



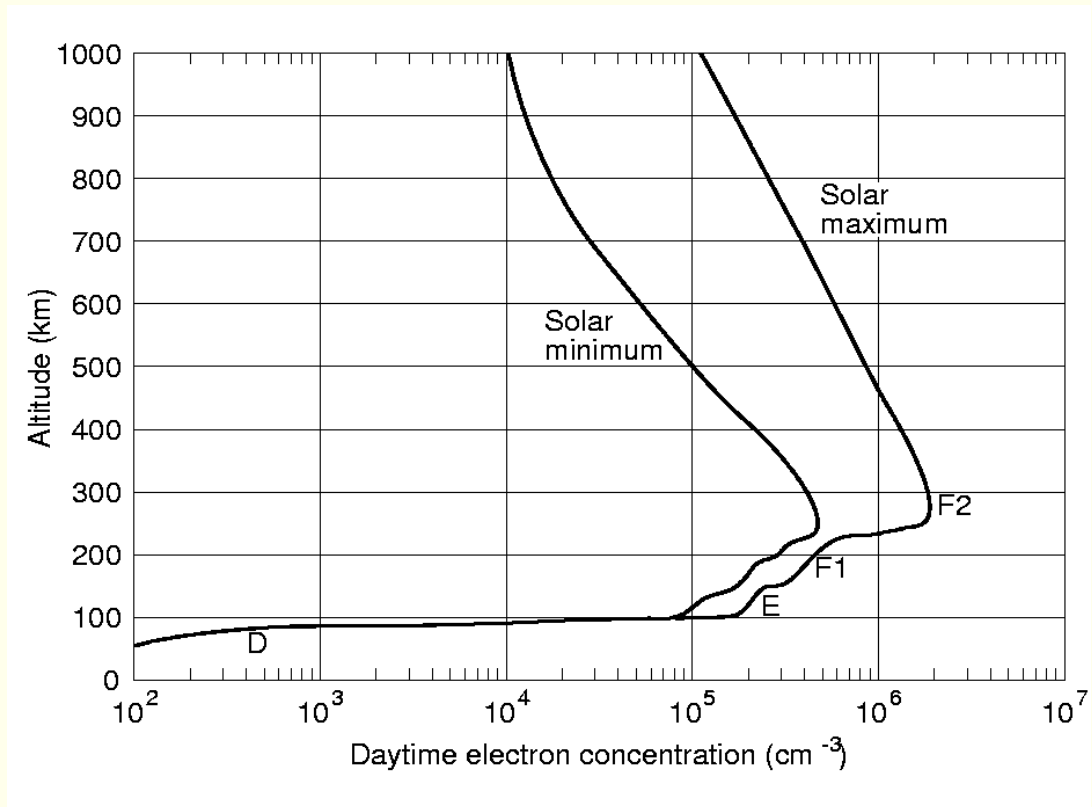
Ionosphere

- The ionized, electrically conducting part of the upper atmosphere
- The ionosphere is a **plasma**

History

- Stewart, 1882: Explained variations in the geomagnetic field
- Kenelly & Heavyside, 1902: explained Marconis transatlantic radio communication experiments
- Appleton & Barnett: experimental proof

Altitude distribution of electron density (n_e)



Continuity equation = conservation of ?

$$\frac{\partial n_e}{\partial t} = q - r - \nabla \cdot (n_e \mathbf{v}_e)$$

Ionization ($\text{m}^{-3}\text{s}^{-1}$)

Recombination ($\text{m}^{-3}\text{s}^{-1}$)

Flow ($\text{m}^{-3}\text{s}^{-1}$)



Ionization and recombination

Continuity equation

$$\frac{dn_e}{dt} = q - r$$

$$q = a_i I n_n$$

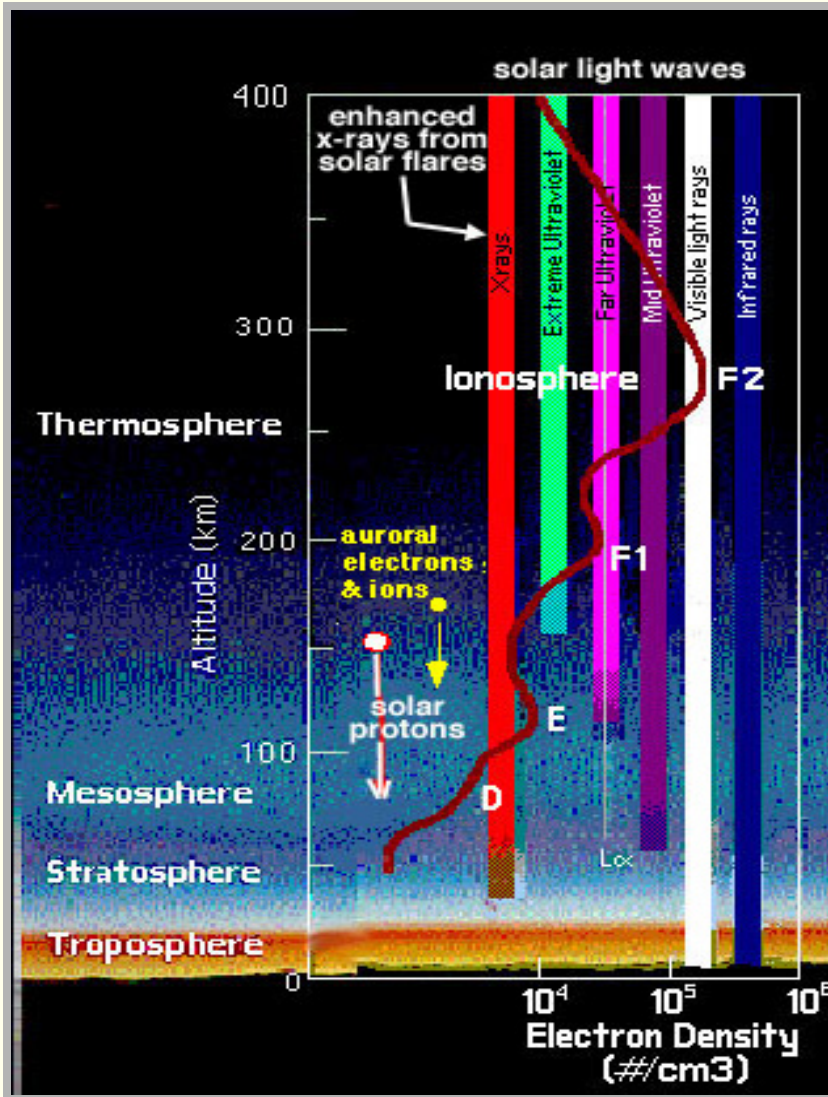
Ionization ($\text{m}^{-3}\text{s}^{-1}$)

Recombination ($\text{m}^{-3}\text{s}^{-1}$)

$$r = a_r n_e n_i = a_r n_e^2$$

Example: $e + \text{O}_2^+ \rightarrow \text{O} + \text{O}$ (dissociative recombination)

UV and X-ray radiation



$$\frac{dI}{dz} = In_n a_a$$



Derive Chapman layer

Electron density in Chapman layer

$$n_e = \left\{ \frac{a_i}{a_r} I_0 n_0 e^{-\left(H a_a n_0 e^{-z/H} + z/H \right)} \right\}^{1/2}$$



What does it look like in reality?

- Temperature not constant
- Many different wavelengths in solar radiation
- Several different molecules and atoms in neutral atmosphere. Composition also depends on altitude.

"E-region" - simple model calculation

O₂ dominating species, 10 nm X-ray radiation

$$a_a = 9.3 \times 10^{-23} \text{ m}^2$$

$$a_i = 9.3 \times 10^{-23} \text{ m}^2$$

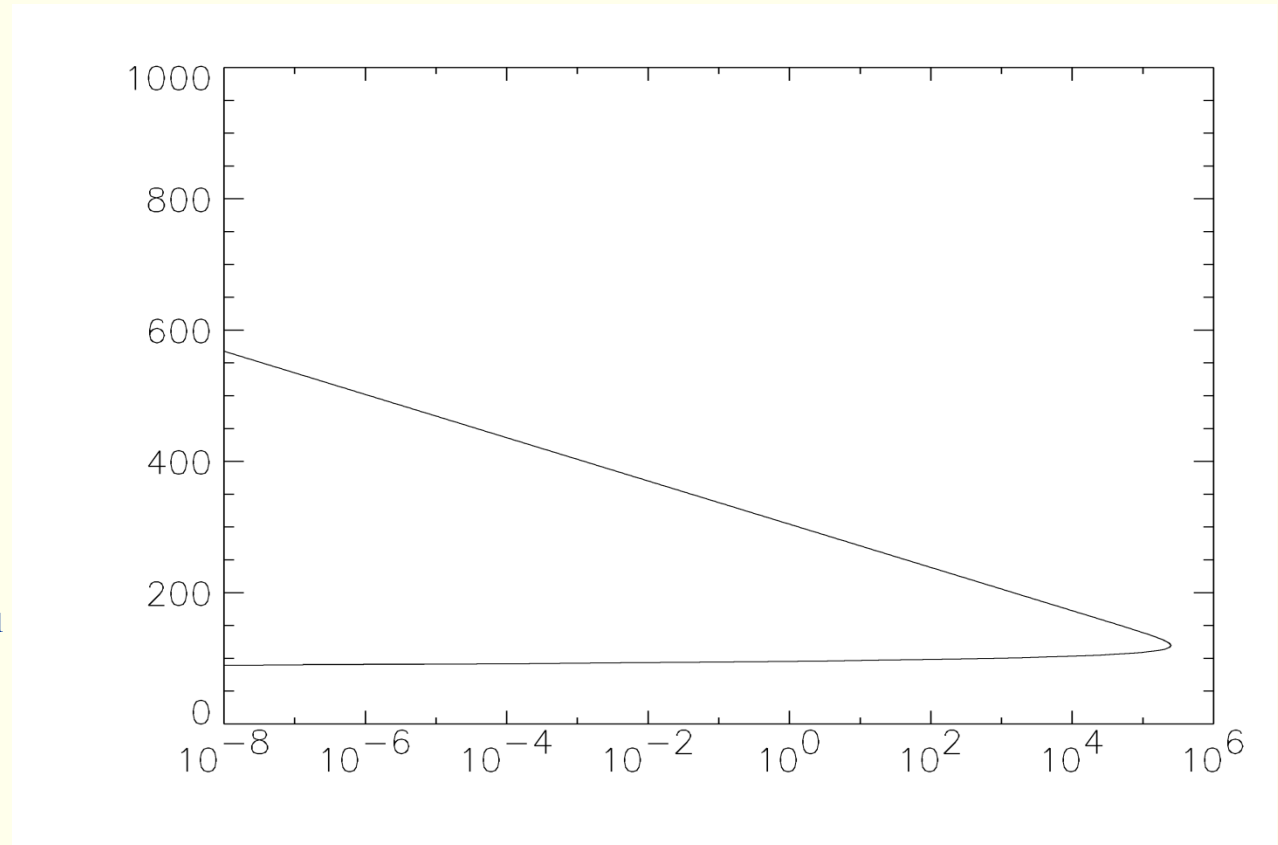
$$a_r = 3.0 \times 10^{-14} \text{ m}^3 \text{ s}^{-1}$$

$$T = 270 \text{ K}$$

$$m = 16 \cdot 2 \cdot m_p$$

$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 3.6 \times 10^{13} \text{ photons m}^2 \text{ s}^{-1}$$



N₂⁺ produced, but rapidly lost through charge exchange: $N_2^+ + O_2 \rightarrow N_2 + O_2^+$

"F1-region" - simple model calculation

O₂ dominating species, 30 nm UV radiation

$$a_a = 9.3 \times 10^{-23} \text{ m}^2$$

$$a_i = 9.3 \times 10^{-23} \text{ m}^2$$

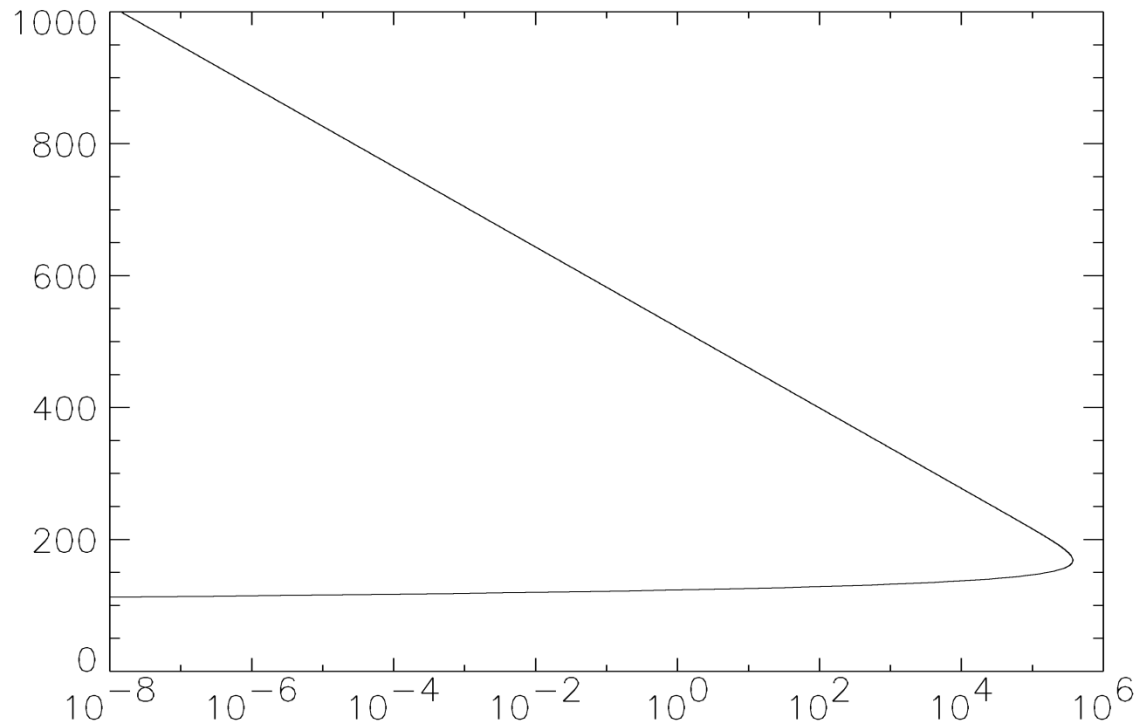
$$a_r = 3.0 \times 10^{-14} \text{ m}^3 \text{ s}^{-1}$$

$$T = 500 \text{ K}$$

$$m = 16 \cdot 2 \cdot m_p$$

$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 1.5 \times 10^{14} \text{ photons/m}^2/\text{s}$$



"F2-region" - simple model calculation

O dominating species, 30 nm UV radiation

$$a_a = 9.3 \times 10^{-23} \text{ m}^2$$

$$a_i = 9.3 \times 10^{-23} \text{ m}^2$$

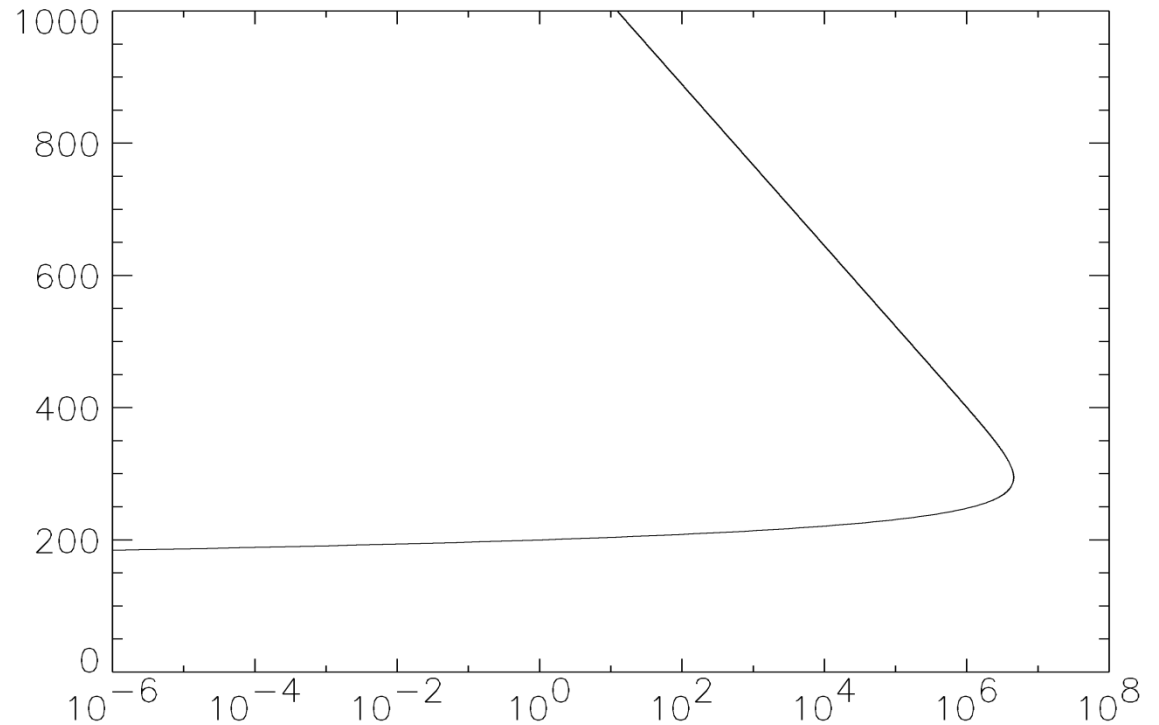
$$a_r = 1.0 \times 10^{-16} \text{ m}^3 \text{ s}^{-1}$$

$$T = 500 \text{ K}$$

$$m = 16 * m_p$$

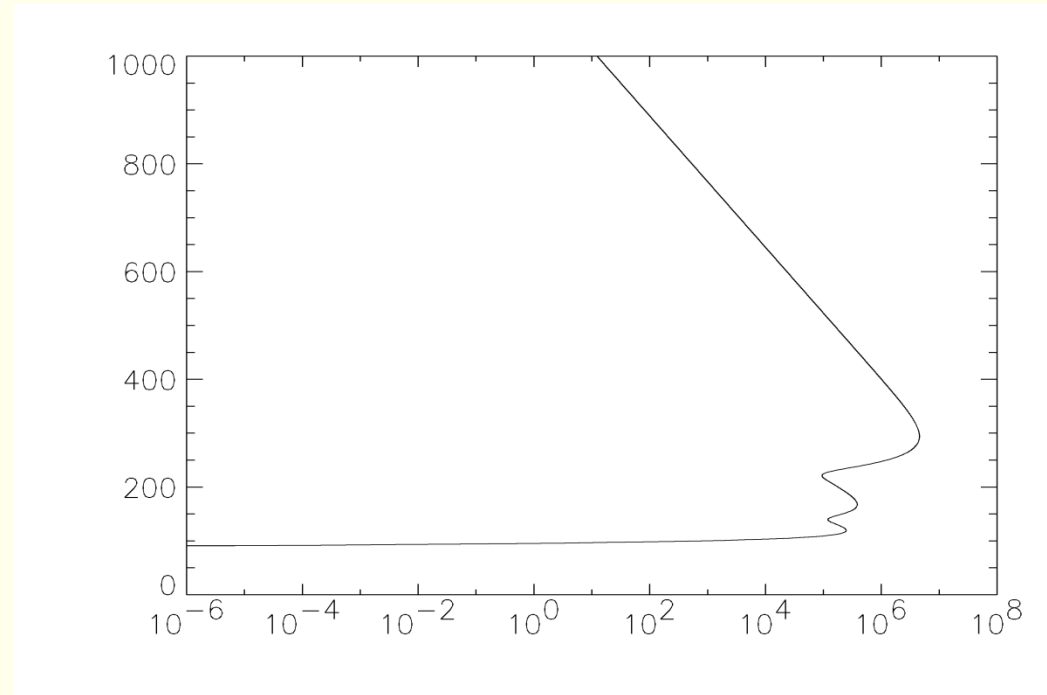
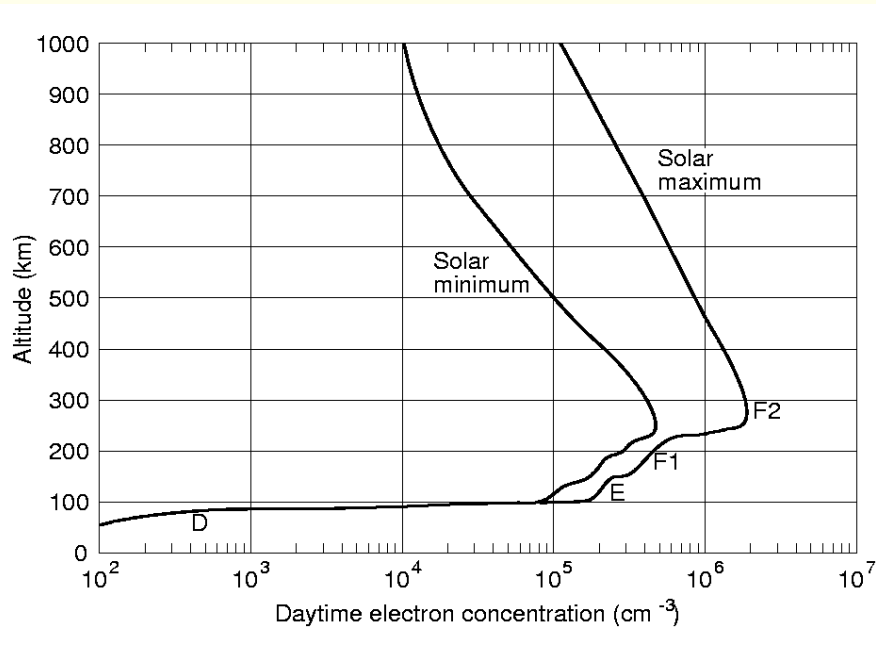
$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 1.5 \times 10^{14} \text{ photons/m}^2/\text{s}$$



Measurements

"E" + "F1" + "F2"



Ionospheric layers

Layer	D	E	F ₁	F ₂
Altitude (km)	60-85	85-140	140-200	200 - ca 1500
Nighttime electron density (cm ⁻³)	<10 ²	2 · 10 ³	—	2 - 5 · 10 ⁵
Daytime electron density (cm ⁻³)	10 ³	1 - 2 · 10 ⁵	2 - 5 · 10 ⁵	0.5 - 2 · 10 ⁶
Ion species	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺ O ⁺	O ⁺ He ⁺ H ⁺
Cause of ionization	Lyman α (1215 Å) + cosmic radiation	Lyman β (1025 Å) X-rays	UV	UV

NO⁺ created by chemical reaction $N_2^+ + O \rightarrow NO^+ + N$



Last Minute!