Last lecture (4)

- Solar wind
  - magnetic structure
- Ionosphere
  - ionospheric layers

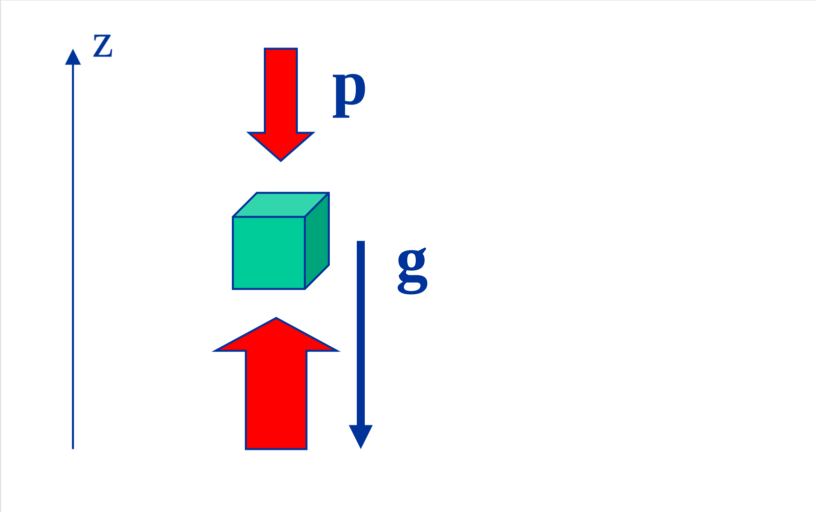
Today’s lecture (5)

- Ionosphere
  - index of refraction of a plasma
  - reflection of radio waves
  - particle drift motion in magnetized plasma
  - electrical conductivity in magnetized plasma
- Magnetosphere?

Today

Activity	Date	Time	Room	Subject	Litterature
L1	29/8	13-15	E52	Course description, Introduction, The Sun 1, Plasma physics 1	**CGF** Ch 1, 5, (p 110-113)
L2	1/9	15-17	L52	The Sun 2, Plasma physics 2	**CGF** Ch 5 (p 114-121), 6.3
L3	5/9	13-15	E51	Solar wind, **The ionosphere and atmosphere 1, Plasma physics 3**	**CGF** Ch 6.1, 2.1-2.6, 3.1-3.2, 3.5, **LL** Ch III, Extra material
T1	8/9	15-17	D41	Mini-group work 1	
L4	12/9	13-15	E35	The ionosphere 2, Plasma physics 4	**CGF** Ch 3.4, 3.7, 3.8
L5	14/9	10-12	V32	The Earth's magnetosphere 1, Plasma physics 5	**CGF** 4.1-4.3, **LL** Ch I, II, IV.A
T2	15/9	15-17	E51	Mini-group work 2	
L6	19/9	13-15	M33	The Earth's magnetosphere 2, Other magnetospheres	**CGF** Ch 4.6-4.9, **LL** Ch V.
T3	22/9	15-17	E51	Mini-group work 3	
L7	26/9	13-15	E31	Aurora, Measurement methods in space plasmas and data analysis 1	**CGF** Ch 4.5, 10**, LL** Ch VI, Extra material
L8	28/9	10-12	L52	Space weather and geomagnetic storms	**CGF** Ch 4.4, **LL** Ch IV.B-C, VII.A-C
T4	29/9	15-17	M31	Mini-group work 4	
L9	3/10	13-15	E52	Interstellar and intergalactic plasma, Cosmic radiation,	**CGF** Ch 7-9
T5	6/10	15-17	E31	Mini-group work 5	
L10	10/10	13-15	E52	Swedish and international space physics research.	
T6	13/10	15-17	E31	Round-up, old exams.	
Written examination	26/10	8-13	F2		

Atmospheric scale height



$$-\frac{dp}{dz} = g\rho_m$$ hydrostatic equilibrium for a volume element

$$p = nk_B T = \frac{\rho k_B T}{m}$$ ideal gas law

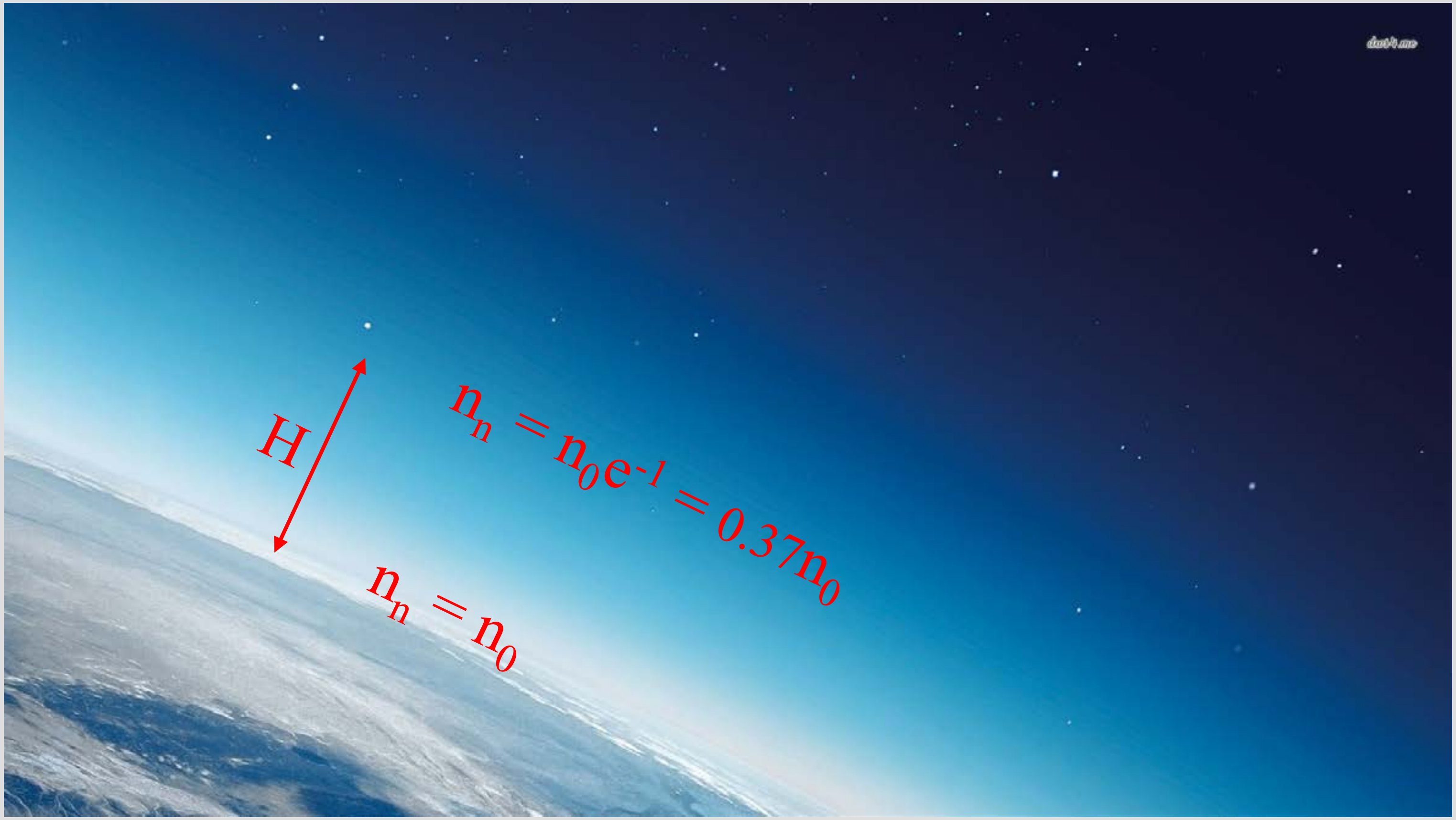
$$-\frac{k_B T}{m}\frac{d\rho_m}{dz} = g\rho_m$$ if T is constant

$$\rho_m = const \cdot e^{-z/(k_B T/gm)} = const \cdot e^{-z/H}$$

Scale height

$$H = k_B T/gm$$

Atmospheric scale height



$$n_n = const \cdot e^{-z/(k_B T/gm)} = n_0 \cdot e^{-z/H}$$

Scale height

$H = k_B T/gm$

Ionization and recombination

Continuity equation

$$\frac{dn_e}{dt} = q - r$$

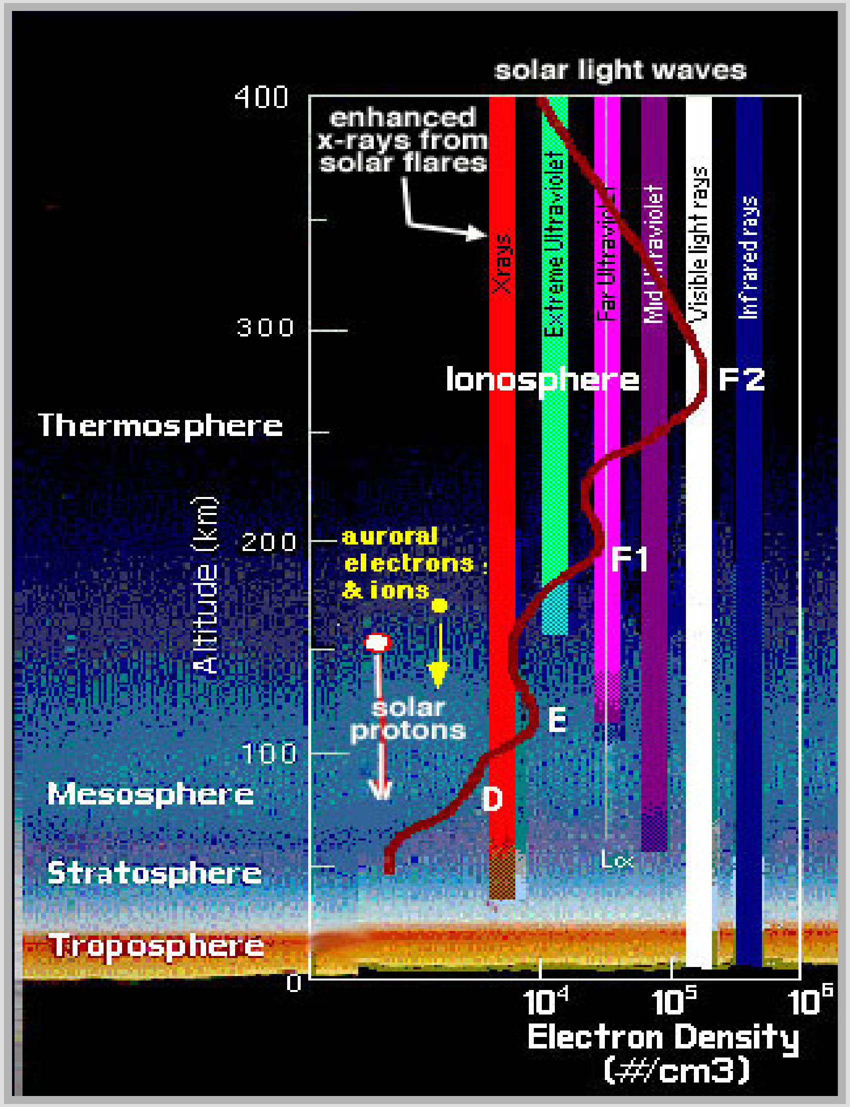
$$q = a_i I n_n$$

Ionization ($m^{-3}s^{-1}$)

Recombination ($m^{-3}s^{-1}$)

$$r = a_r n_e n_i = a_r n_e^2$$

Example: $e + O_2^+ \rightarrow O + O$ (dissociative recombination)



UV and X–ray radiation

$$\frac{dI}{dz} = I n_n a_a$$

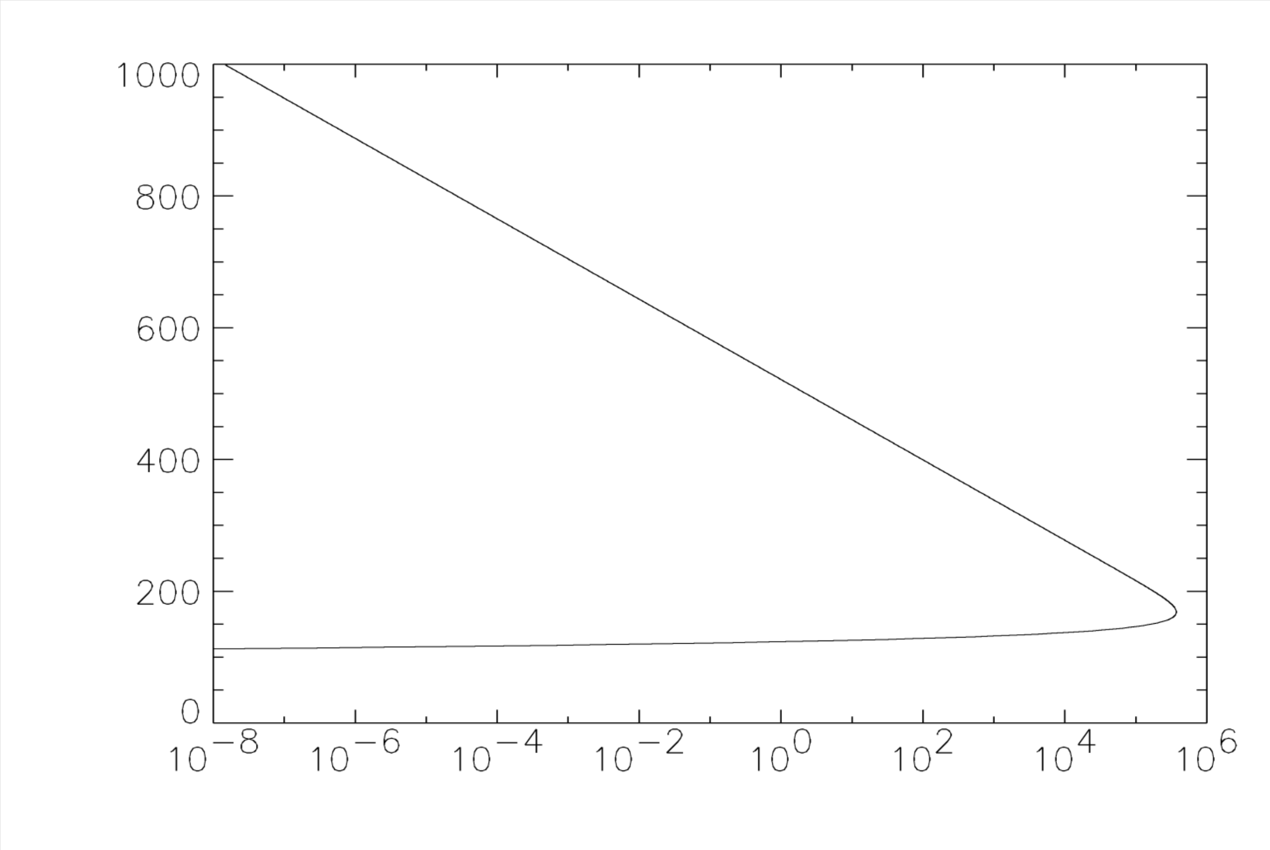
Electron density in Chapman layer

$$n_e = \left\{ \frac{a_i}{a_r} I_0 n_0 e^{-\left(H a_a n_0 e^{-z/H} + z/H\right)} \right\}^{1/2}$$

"F1-region" - simple model calculation

O_2 dominating species, 30 nm UV radiation

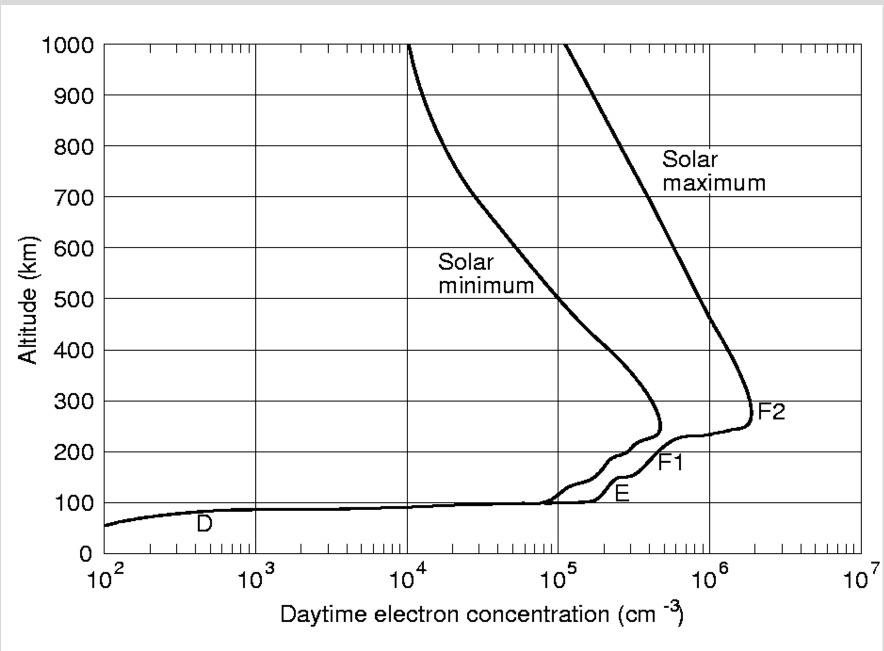
$a_a = 9.3\times10^{-23}$
$a_j = 9.3\times10^{-23}$
$a_r = 3.0\times10^{-14}$
$T = 500$
$m = 16*2*amu$
$n_0 = 2.7\times10^{25}\ m^{-3}$
$I_0 = 1.5\times10^{14}$ photons/m^2/s

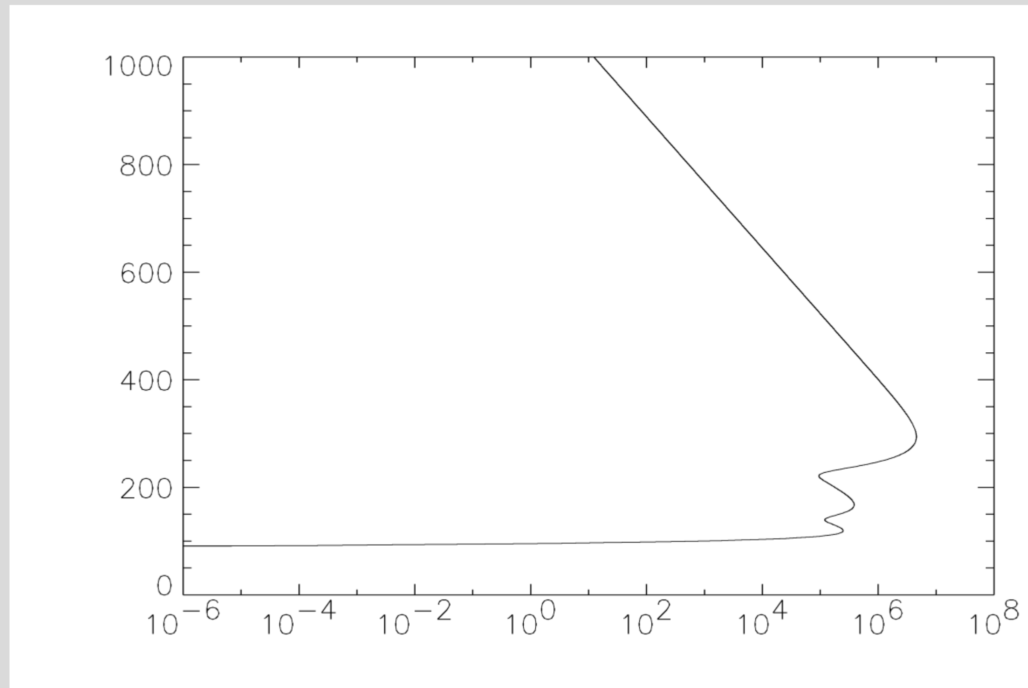


What does it look like in reality?

- Temperature not constant
- Many different wavelengths in solar radiation
- Several different molecules and atoms in neutral atmosphere. Composition also depends on altitude.

Measurements ”E” + ”F1” + ”F2”





Ionospheric layers

Layer	D	E	F_1	F_2
Altitude (km)	60-85	85-140	140-200	200 - ca 1500
Nighttime electron density (cm^{-3})	$<10^2$	$2 \cdot 10^3$	—	$2 - 5 \cdot 10^5$
Daytime electron density (cm^{-3})	10^3	$1 - 2 \cdot 10^5$	$2 - 5 \cdot 10^5$	$0.5 - 2 \cdot 10^6$
Ion species	NO^+ O_2^+	NO^+ O_2^+	NO^+ O_2^+ O^+	O^+ He^+ H^+
Cause of ionization	Lyman α (1215 Å) + cosmic radiation	Lyman β (1025 Å) X-rays	UV	UV

NO^+ created by chemical reaction $N_2^+ + O \rightarrow NO^+ + N$

Propation of radio waves in the ionosphere

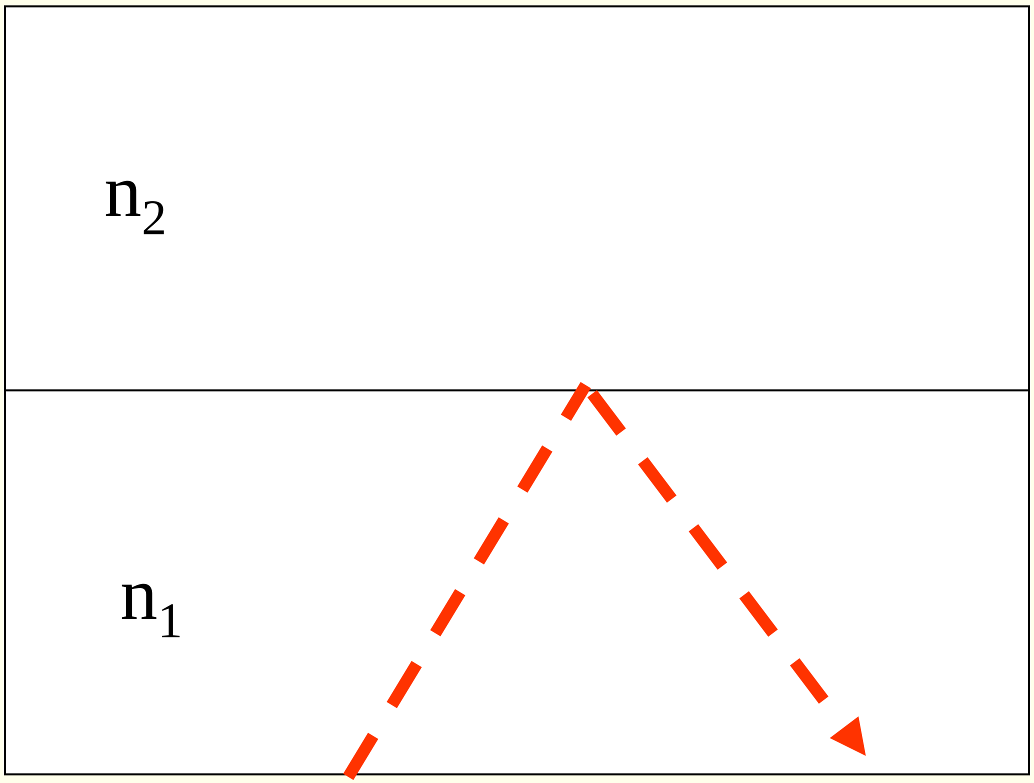
1. Absorption/damping

Takes place in the D-region due to high collision frequency. (Collisions with neutral atoms.)

2. Reflection

takes place in the F-region due to large gradients in the refraction index.

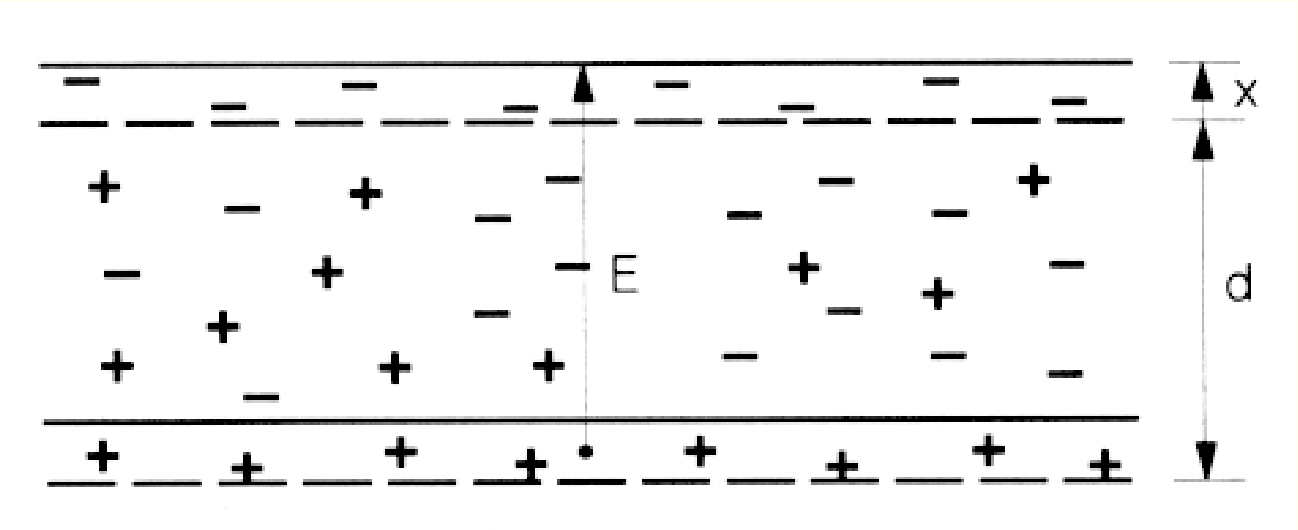
Reflection of radio waves



Total reflection at a sharp boundary (or large gradient) if

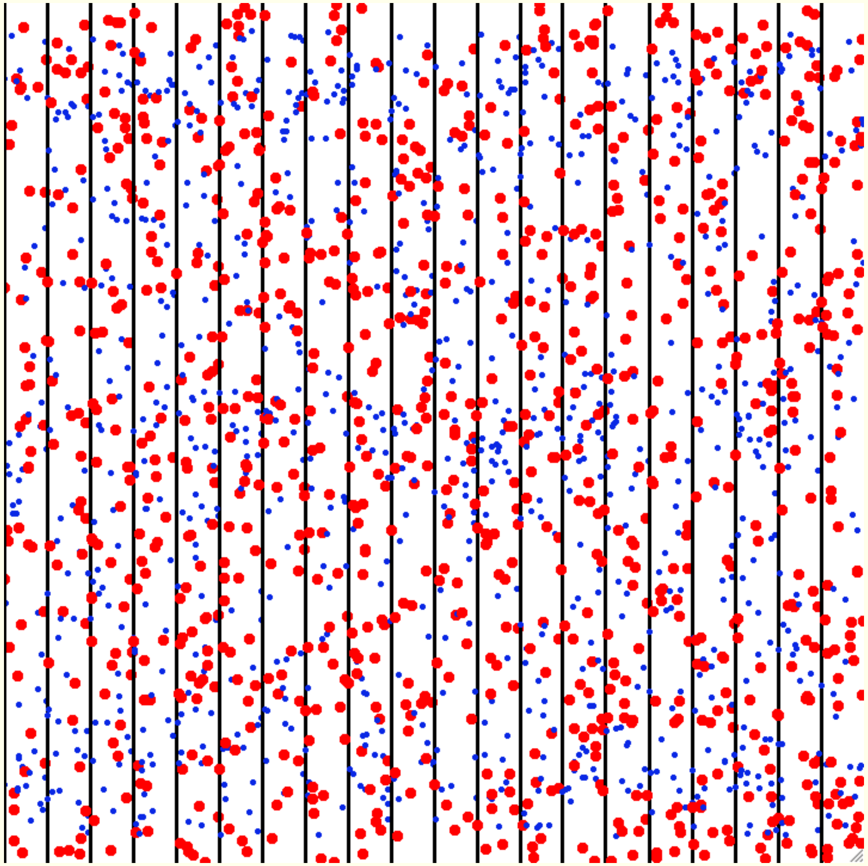
$$n_2 < n_1$$

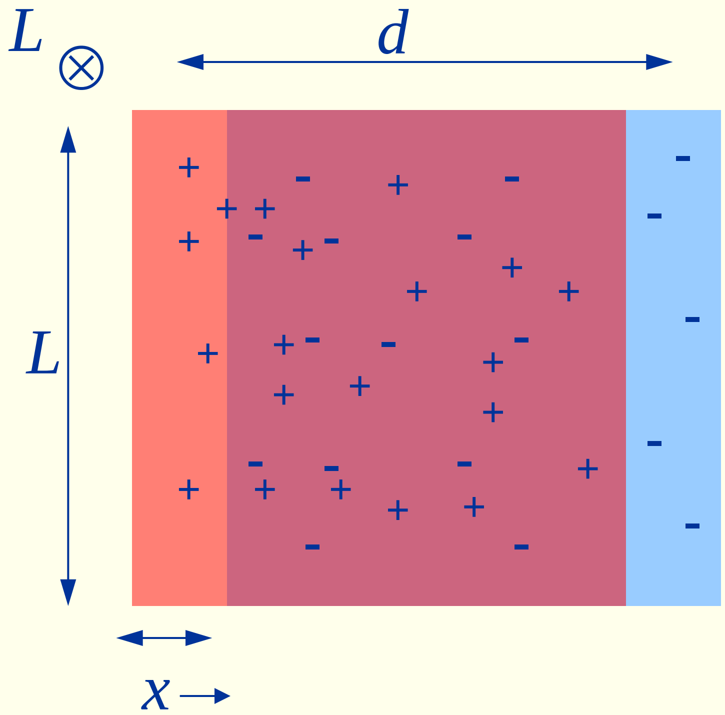
Plasma frequency



Charge imbalance creates an electric field which tends to even out the imbalance.

Plasma oscillations parallel to B





Newtons law on an individual electron inside the slab:

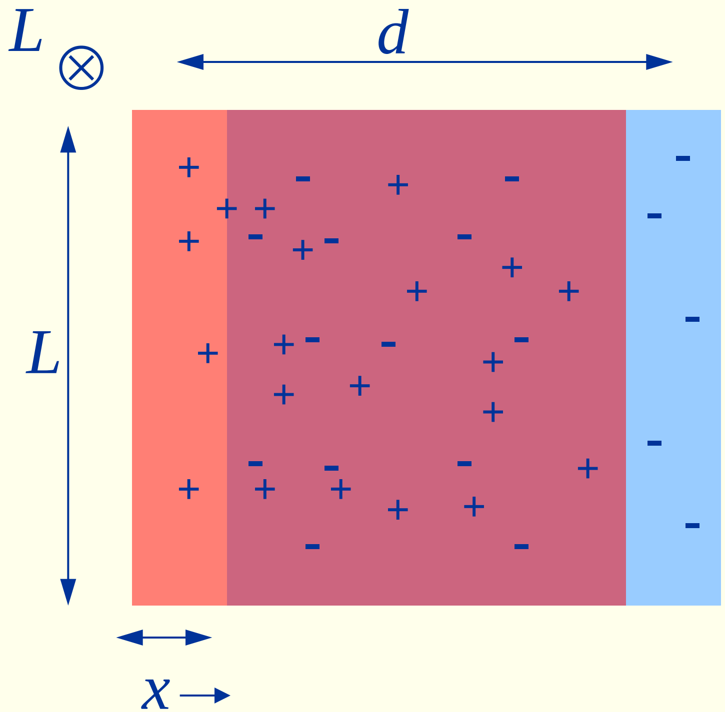
$$F = m_e a$$

$$F = -eE$$

Surface charge density

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\sigma = -e n_e x$$



$$-\frac{n_e e^2 x}{\varepsilon_0 m_e} = \frac{d^2 x}{dt^2}$$

$$x = \sin(\omega_{pe} t)$$

$$F = m_e a \qquad E = \frac{\sigma}{\varepsilon_0}$$

$$F = -eE \qquad \sigma = e n_e x$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

What is the plasma frequency f_{pe} at the daytime E-region, close to solar minimum? (see Fälthammar p 28)

$$f_{pe} = \frac{\omega_{pe}}{2\pi} \equiv \frac{1}{2\pi}\sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

Blue	7 kHz	Yellow	400 MHz
Green	3 MHz	Red	2 GHz

$$f = \frac{\omega_{pe}}{2\pi} \equiv \frac{1}{2\pi}\sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}} = \frac{1}{2\pi}\sqrt{\frac{(1.6\cdot 10^{-19})^2}{8.854\cdot 10^{-12} 0.91\cdot 10^{-30}}}\sqrt{n_e} =$$

$$8.97\sqrt{n_e} = 8.97\sqrt{10^5\cdot 10^6} = 2.8\cdot 10^6\, Hz = 2.8\, MHz$$

Green

Index of refraction for electromagnetic waves in a plasma

(1) $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

(2) $$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

(3) $$\mathbf{j} = -e n_e \mathbf{v}_e$$

(4) $$m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\mathbf{E}$$

Assume all quantities vary sinusoidally, with frequency ω, *e.g.*:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

(1) $\Rightarrow$ $$\nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \frac{\partial \mathbf{B}}{\partial t}$$

(2) $\Rightarrow$ $$\nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\therefore$ $$\nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial \mathbf{j}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \mu_0 e n_e \frac{\partial \mathbf{v}_e}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

Index of refraction for electromagnetic waves in a plasma

$$i\mathbf{k}(\mathbf{k}\cdot\mathbf{E})+k^2\mathbf{E}=\mu_0 e n_e(-i\omega\mathbf{v}_e)-\frac{1}{c^2}(-\omega^2)\mathbf{E}$$

Does not represent E.M. wave

(4) $\Rightarrow$

$$k^2\mathbf{E}=-i\mu_0 e n_e\omega\left(-\frac{ie\mathbf{E}}{\omega m_e}\right)-\frac{1}{c^2}(-\omega^2)\mathbf{E}$$

$\Rightarrow$

$$c^2k^2=-c^2\frac{\mu_0 n_e e^2}{m_e}+\omega^2=\frac{-1}{\mu_0\varepsilon_0}\frac{\mu_0 n_e e^2}{m_e}+\omega^2$$

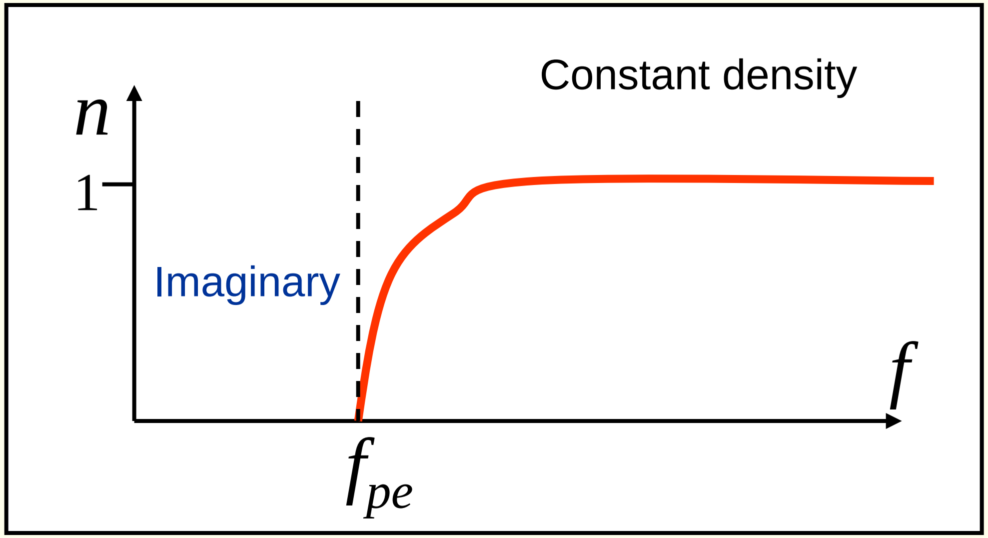
$$\therefore\quad \omega^2=c^2k^2+\omega_p^2$$

$$n^2=\frac{c^2}{v_{ph}^2}=\frac{c^2k^2}{\omega^2}=\frac{\omega^2-\omega_p^2}{\omega^2}=1-\frac{\omega_p^2}{\omega^2}$$

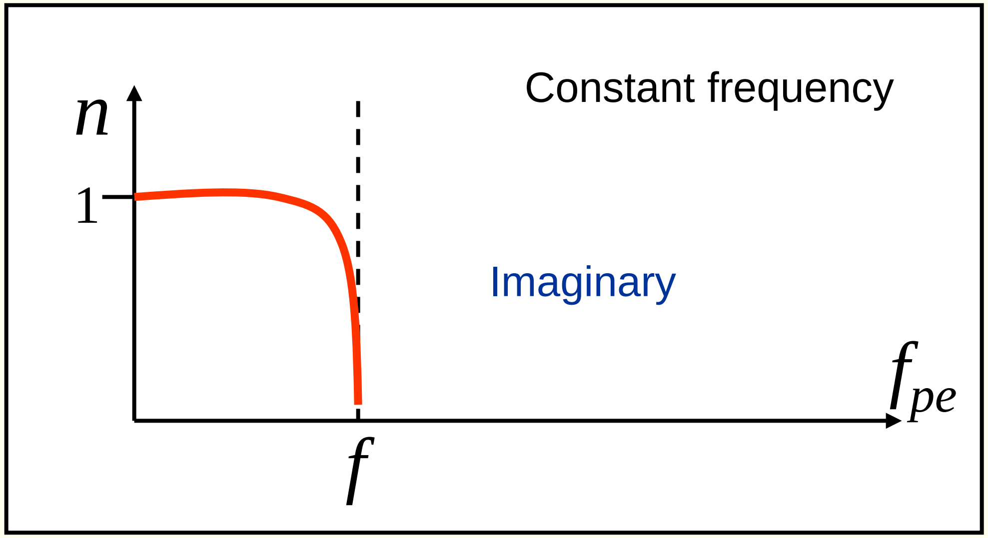
$\therefore$

$$n=\sqrt{1-\frac{\omega_p^2}{\omega^2}}=\sqrt{1-\frac{f_p^2}{f^2}}$$

Refraction index for plasma

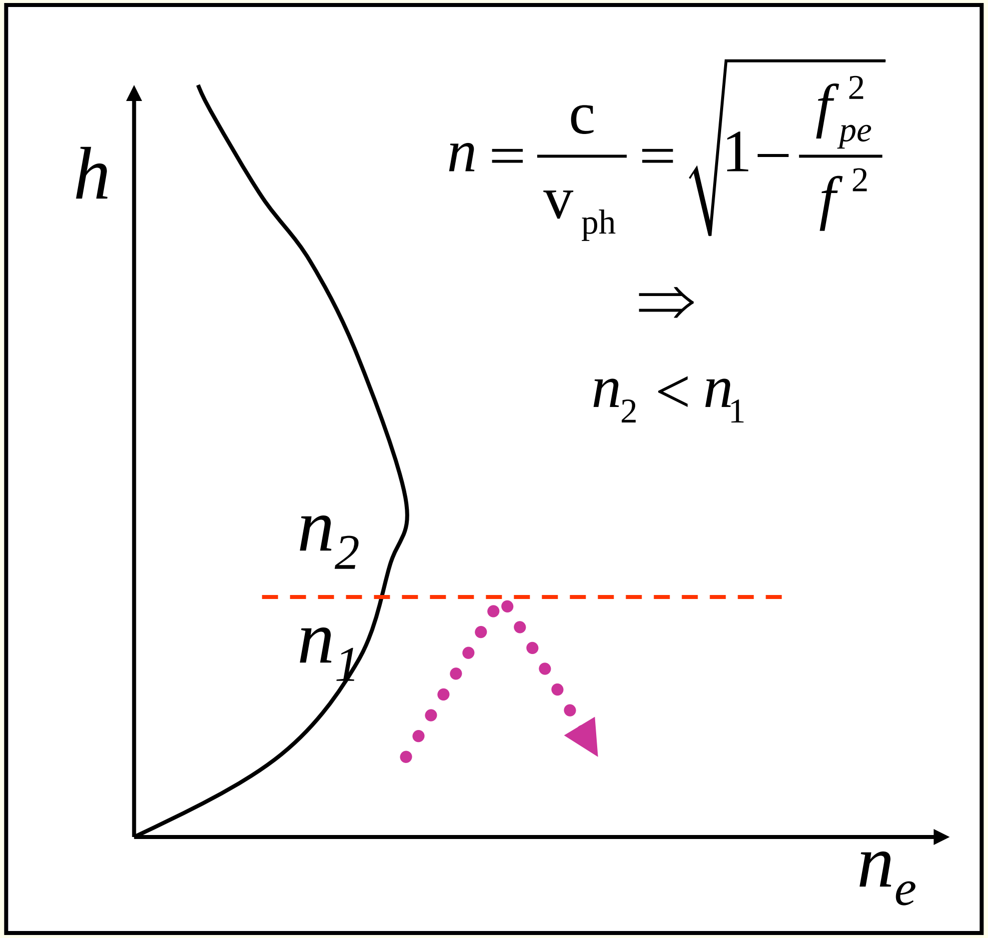


$$n = \frac{c}{v_{ph}} = \sqrt{1 - \frac{f_{pe}^2}{f^2}}$$



$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

Where does the total reflection take place?

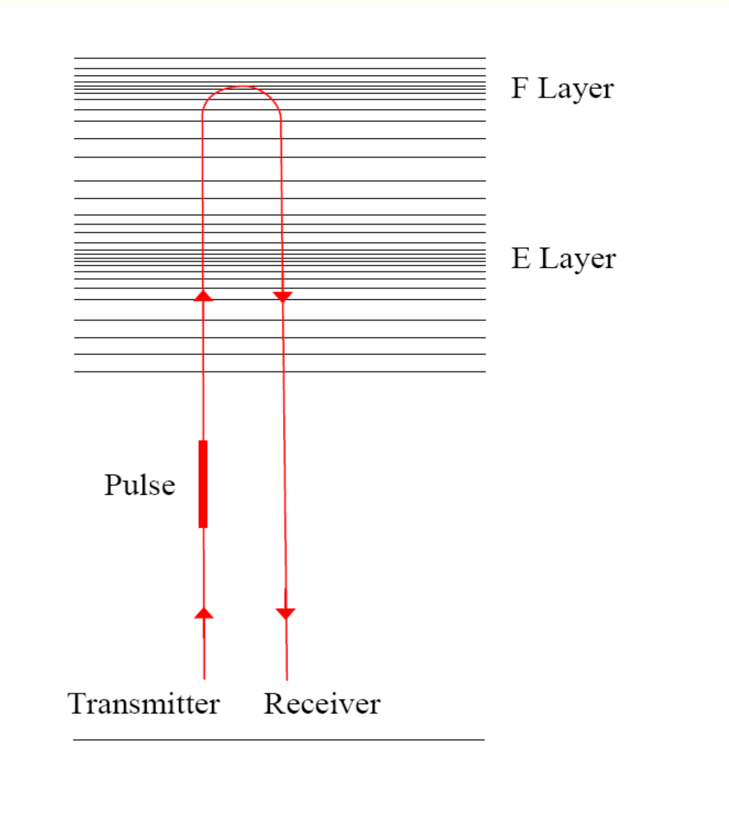


Large gradient when

$$f \approx f_{pe}$$

Higher frequencies $\rightarrow$ higher f_{pe} (n_e)

Ionosonde



The pulse will be reflected where

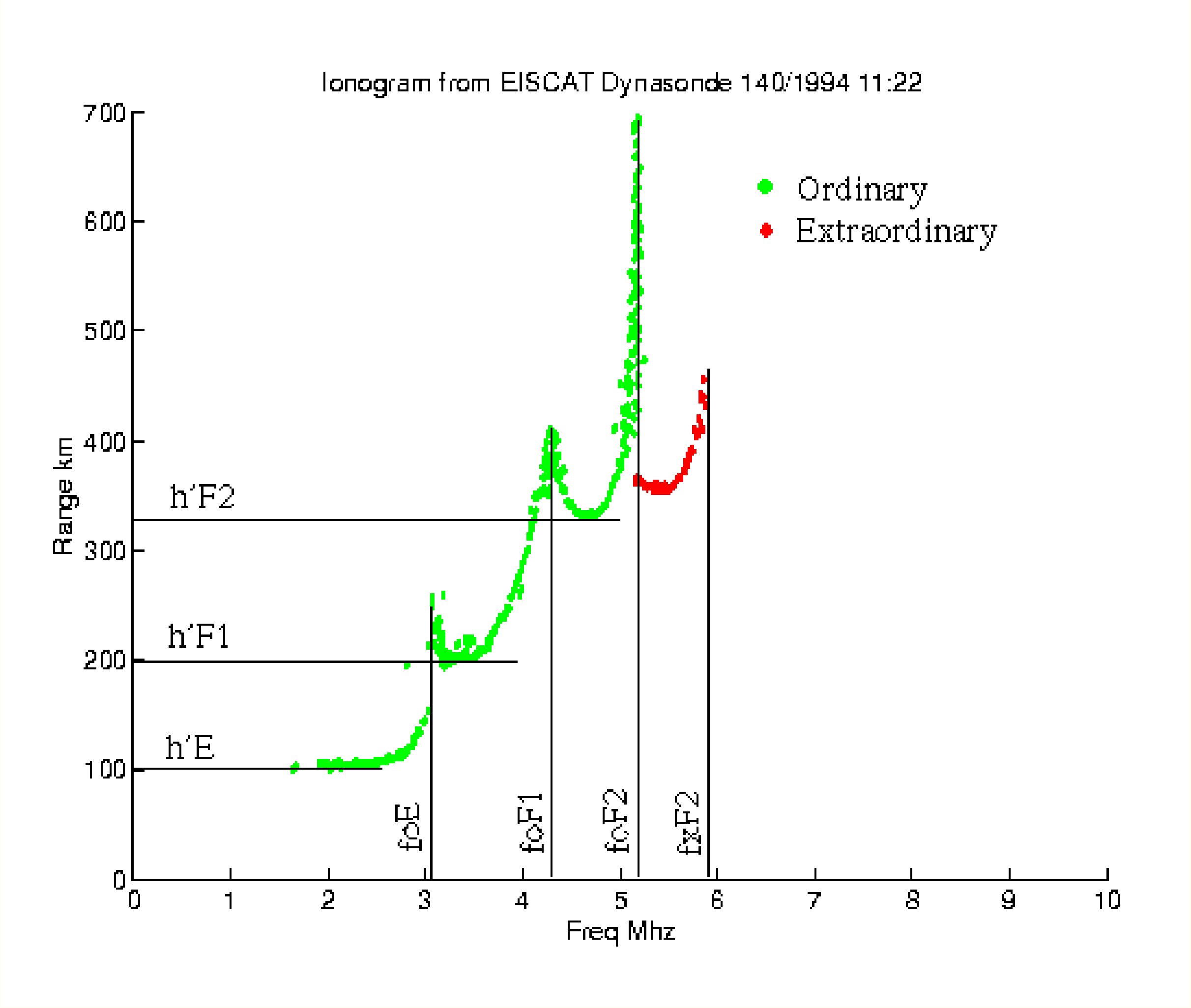
$$f = f_{pe}$$

The altitude will be determined by

$$2h = ct$$

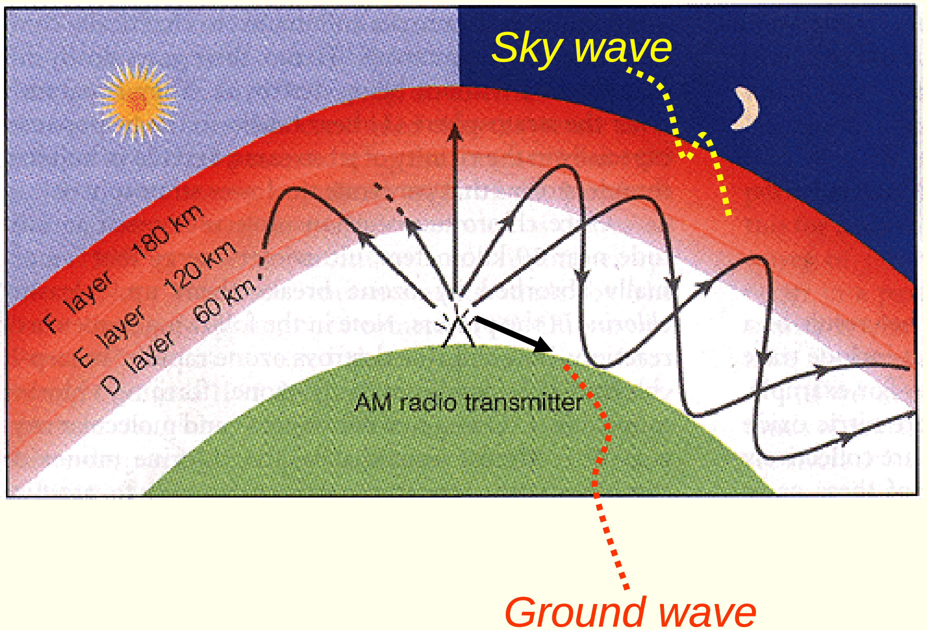
Where t is the time between when the pulse is sent out and the registered again.

Ionogram



Ionogram from EISCAT Dynasonde 140/1994 11:22
Ordinary
Extraordinary
Range km
Freq Mhz
h'F2
h'F1
h'E
foE
foF1
foF2
fxF2
0
100
200
300
400
500
600
700
1
2
3
4
5
6
7
8
9
10

Reflection of radio waves



F2-layer during night:

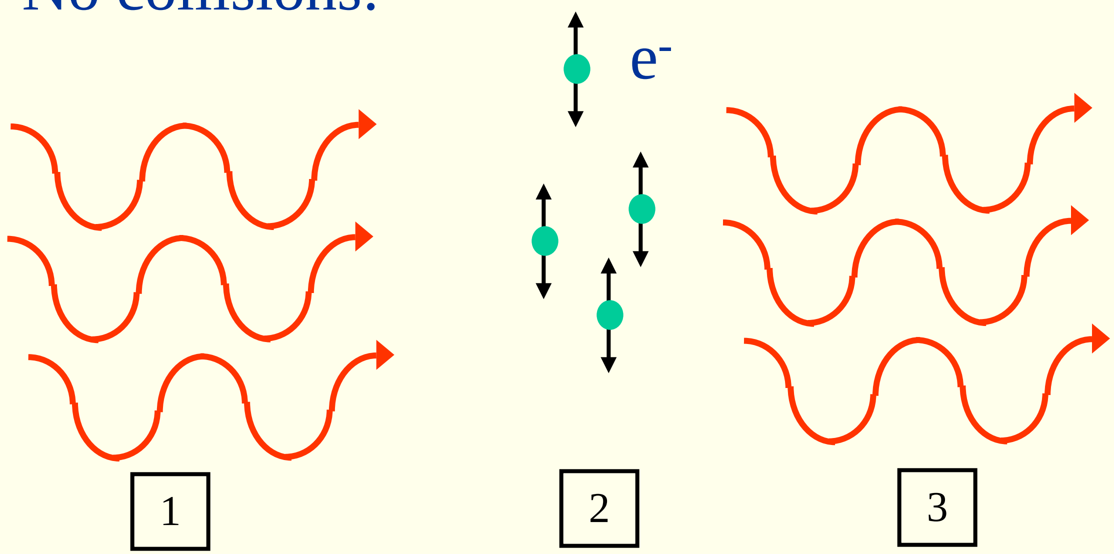
$$n_e = 5 \cdot 10^{11}\ \text{m}^{-3} \quad \Rightarrow$$

$$f_{pe} = 10^7\ \text{Hz} = 10\ \text{MHz}$$

= HF/short wave

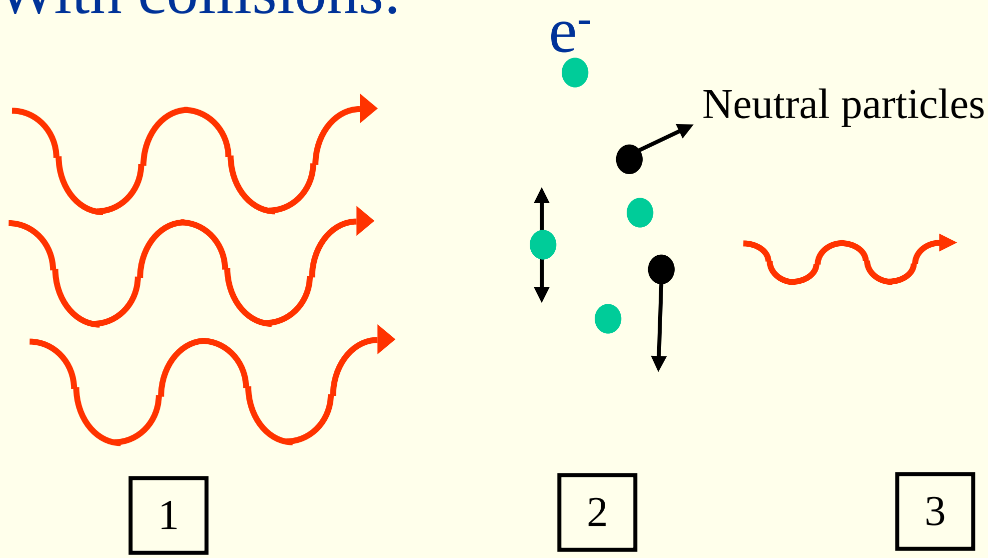
Absorption of radio waves

No collisions:



Absorption of radio waves

With collisions:



Index of refraction for electromagnetic waves in a plasma

(1) $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

(2) $\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$

(3) $\mathbf{j} = -en_e \mathbf{v}_e$

(4) $m_e \dfrac{\partial \mathbf{v}_e}{\partial t} = -e\mathbf{E}$

Assume all quantities vary sinusoidally, with frequency ω, *e.g.*:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

(1) $\Rightarrow$ $\nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \dfrac{\partial \mathbf{B}}{\partial t}$

(2) $\Rightarrow$ $\nabla \times \dfrac{\partial \mathbf{B}}{\partial t} = \mu_0 \dfrac{\partial \mathbf{j}}{\partial t} + \dfrac{1}{c^2} \dfrac{\partial^2 \mathbf{E}}{\partial t^2}$

$$\therefore \; \nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial}{\partial t}(en_e \mathbf{v}_e) + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 e n_e \frac{\partial \mathbf{v}_e}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

Index of refraction for electromagnetic waves in a plasma

$$i\mathbf{k}\left(i\mathbf{k}\cdot\mathbf{E}\right)-k^2\mathbf{E}=\mu_0\left(-i\omega\right)en_e\mathbf{v}_e+\frac{1}{c^2}\left(-i\omega\right)^2\mathbf{E}$$

Does not represent E.M. wave

(4) ⇨

$$-k^2\mathbf{E}=\mu_0\left(-i\omega\right)en_e\frac{ie\mathbf{E}}{\omega m_e}+\frac{1}{c^2}\left(-i\omega\right)^2\mathbf{E}$$

⇨

$$c^2k^2=-c^2\frac{\mu_0 n_e e^2}{m_e}+\omega^2=\frac{-1}{\mu_0\varepsilon_0}\frac{\mu_0 n_e e^2}{m_e}+\omega^2$$

$$\therefore\quad \omega^2=c^2k^2+\omega_p^2$$

$$n^2=\frac{c^2}{v_{ph}^2}=\frac{c^2k^2}{\omega^2}=\frac{\omega^2-\omega_p^2}{\omega^2}=1-\frac{\omega_p^2}{\omega^2}$$

$\therefore$

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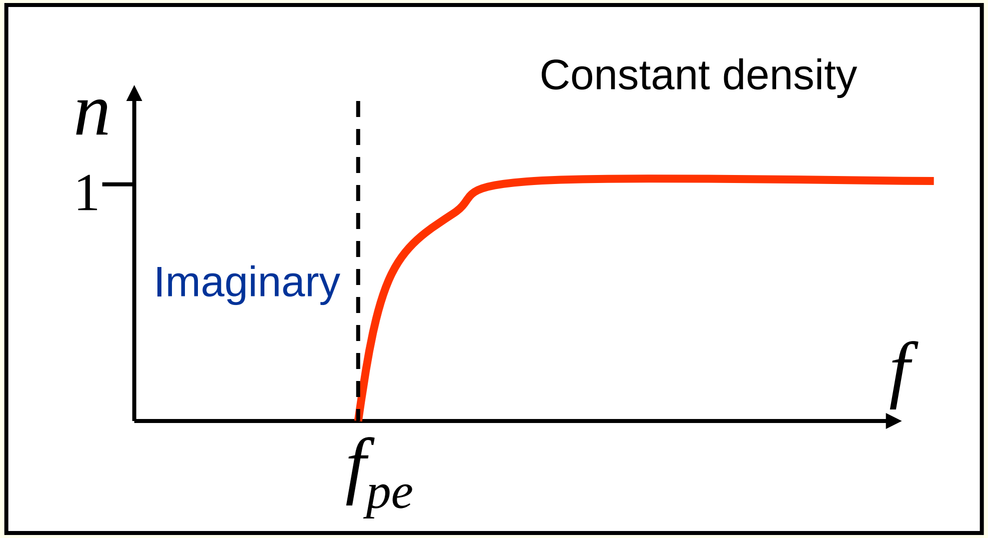
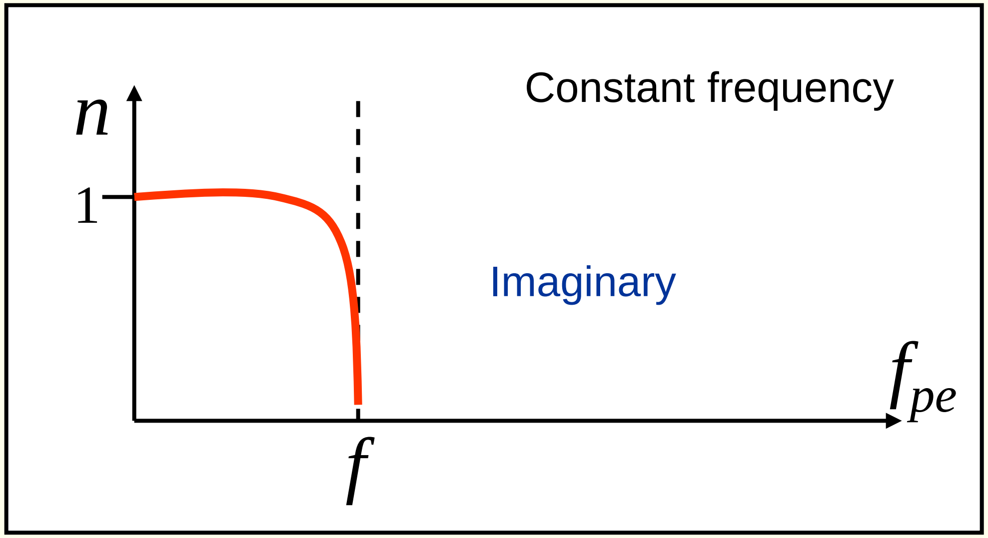
Refraction index for plasma

$$n = \frac{c}{v_{ph}} = \sqrt{1 - \frac{f_{pe}^2}{f^2}}$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$





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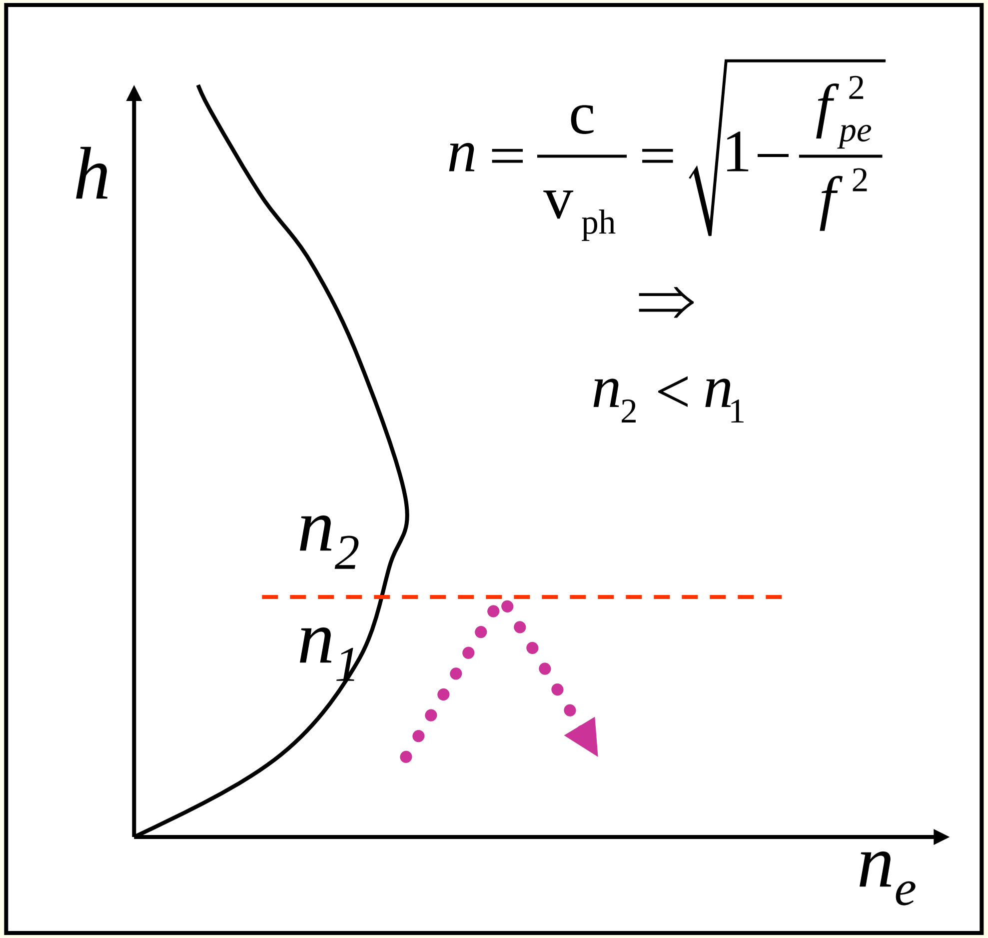
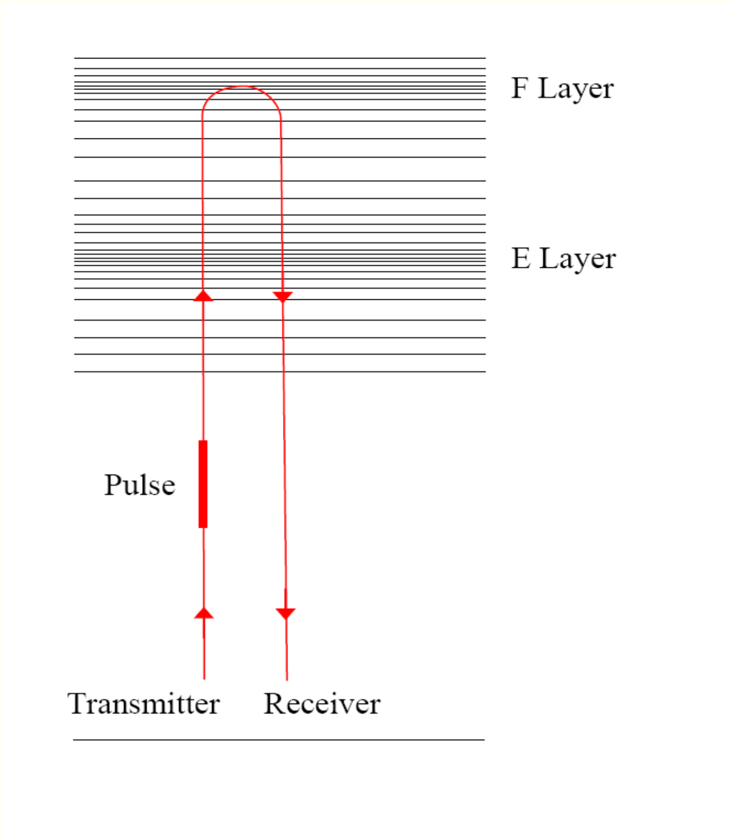


Large gradient when

$$f \approx f_{pe}$$

Higher frequencies → higher f_{pe} (n_e)

Ionosonde



The pulse will be reflected where

$$f = f_{pe}$$

The altitude will be determined by

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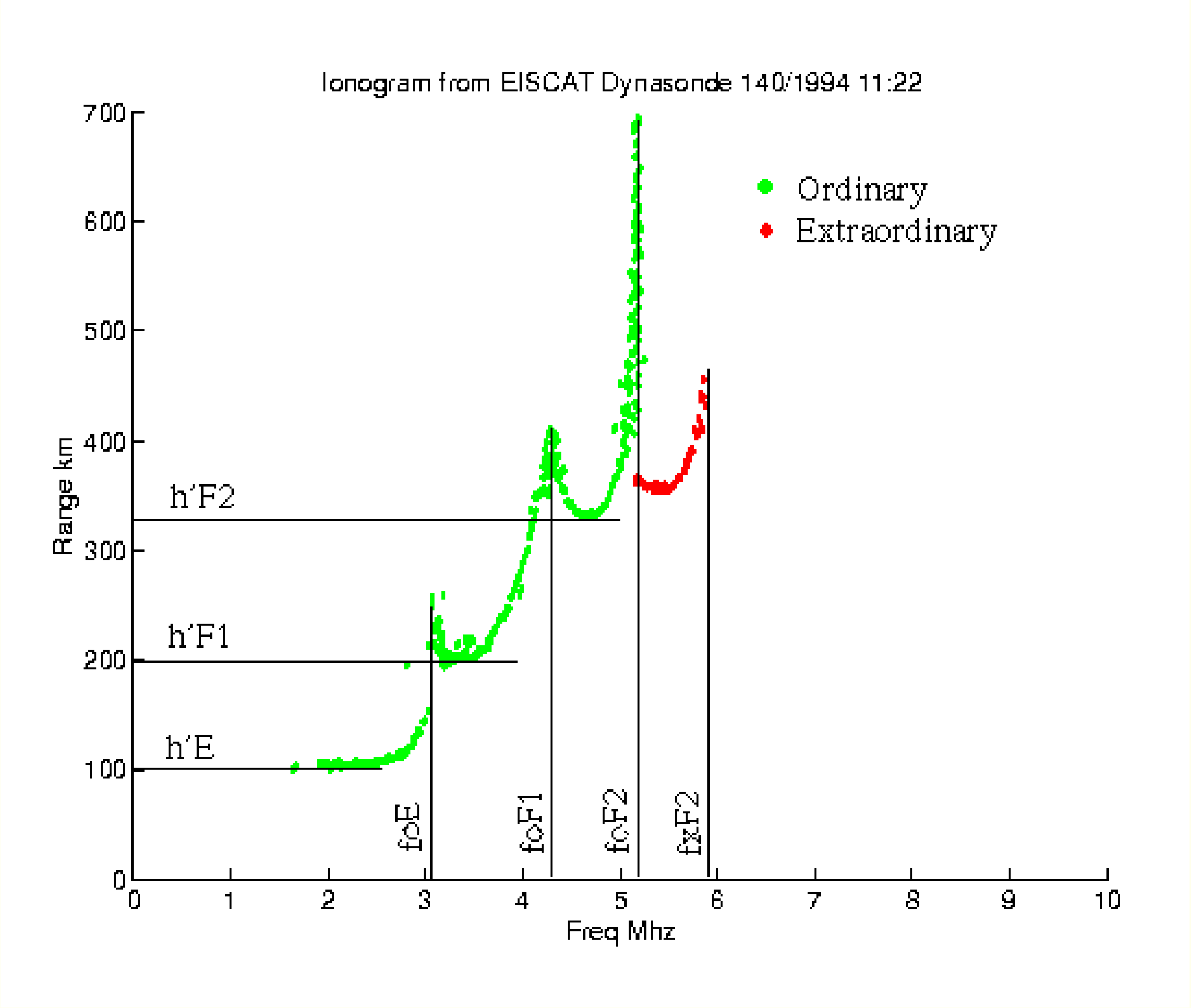
Where t is the time between when the pulse is sent out and the registered again.

Ionogram


Ionogram from EISCAT Dynasonde 140/1994 11:22
Ordinary
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Range km
Freq Mhz
h'F2
h'F1
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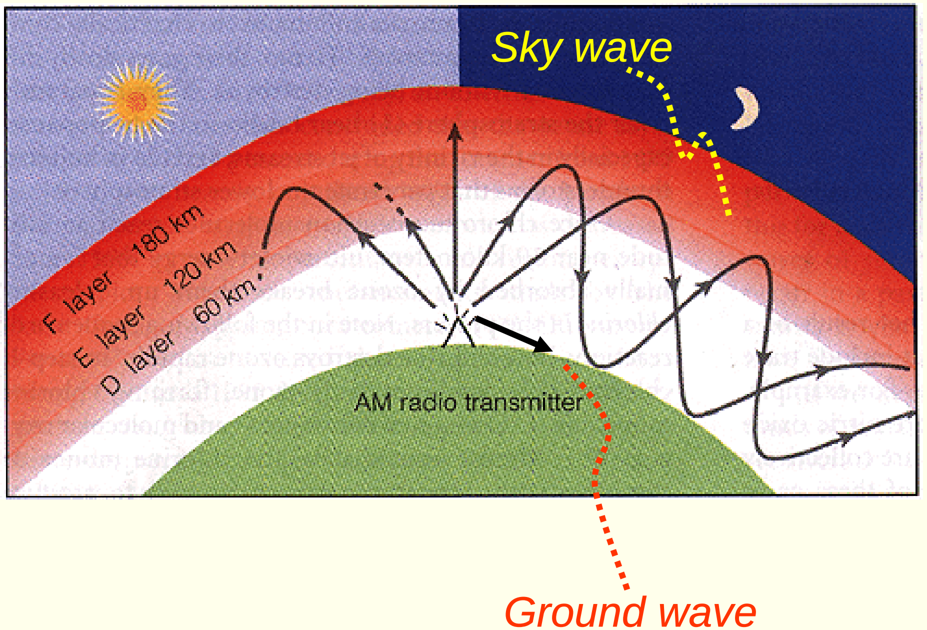
foE
foF1
foF2
fxF2
0
100
200
300
400
500
600
700
0
1
2
3
4
5
6
7
8
9
10

Reflection of radio waves



F2-layer during night:

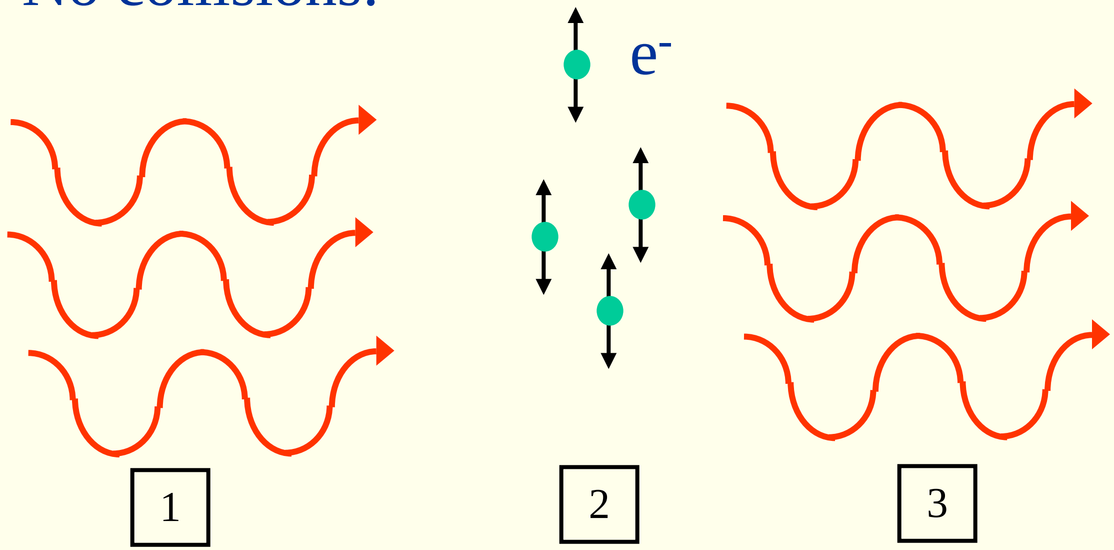
$$n_e = 5 \cdot 10^{11}\ \mathrm{m}^{-3} \quad \Rightarrow$$

$$f_{pe} = 10^7\ \mathrm{Hz} = 10\ \mathrm{MHz}$$

= HF/short wave

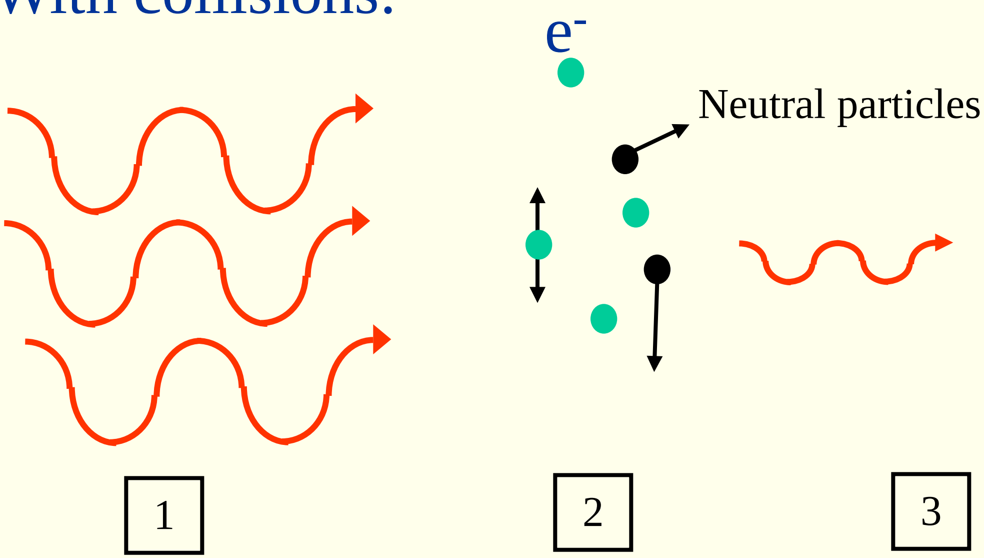
Absorption of radio waves

No collisions:



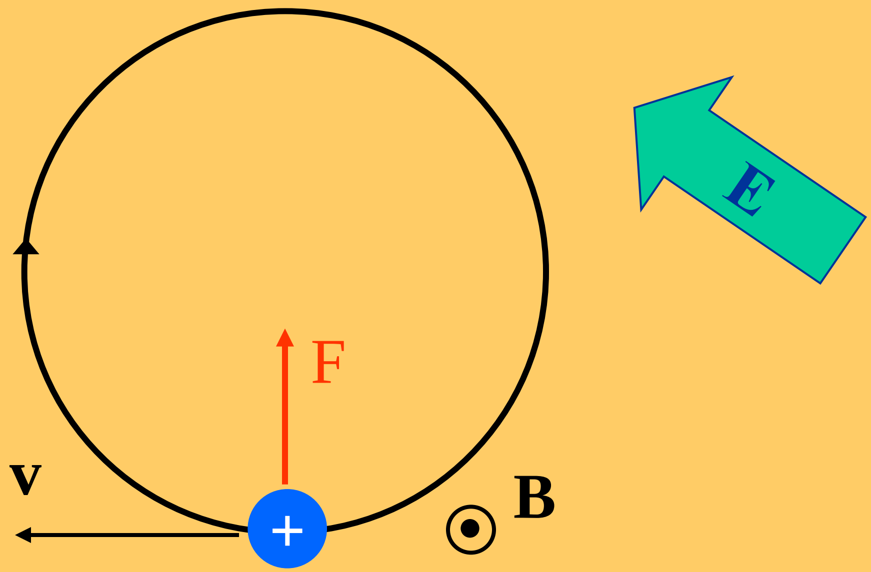
Absorption of radio waves

With collisions:



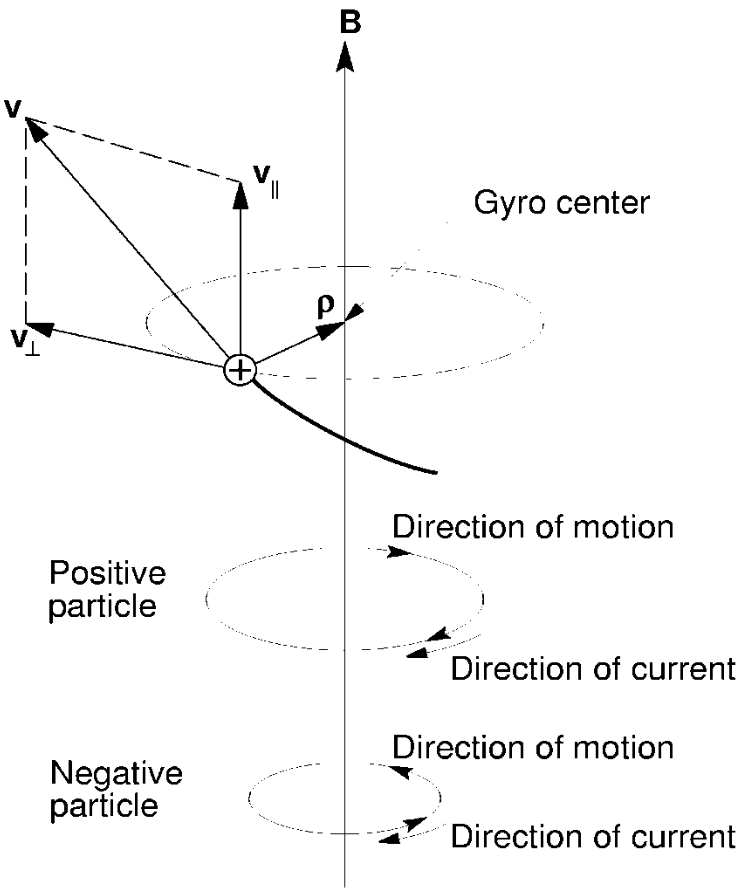
Think about this:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$



What happens if you add an electric field $\mathbf{E}$?

Particle motion in magnetic field



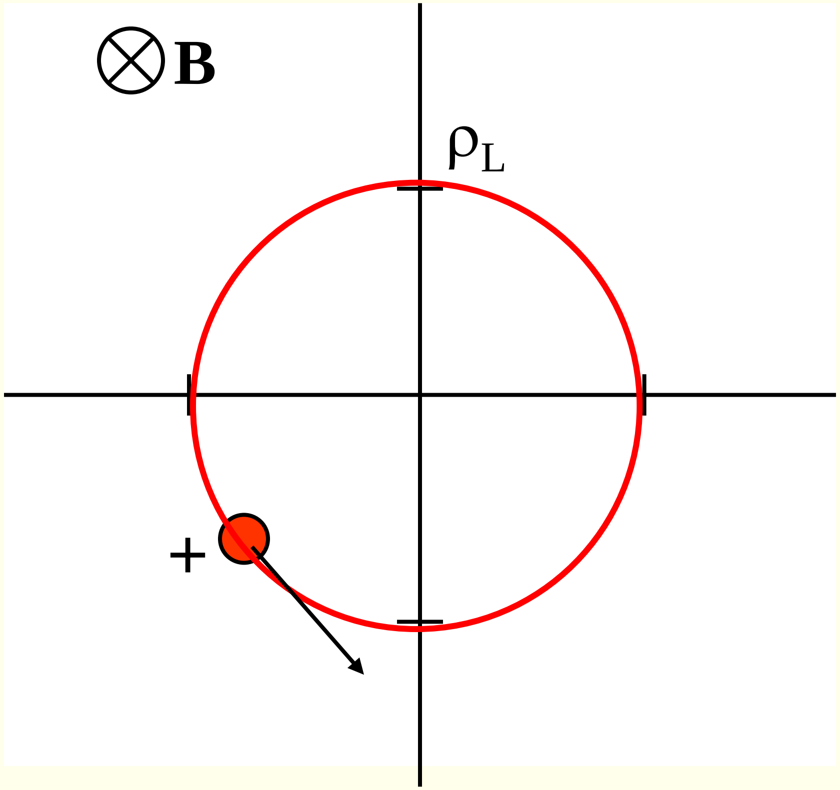
gyro radius

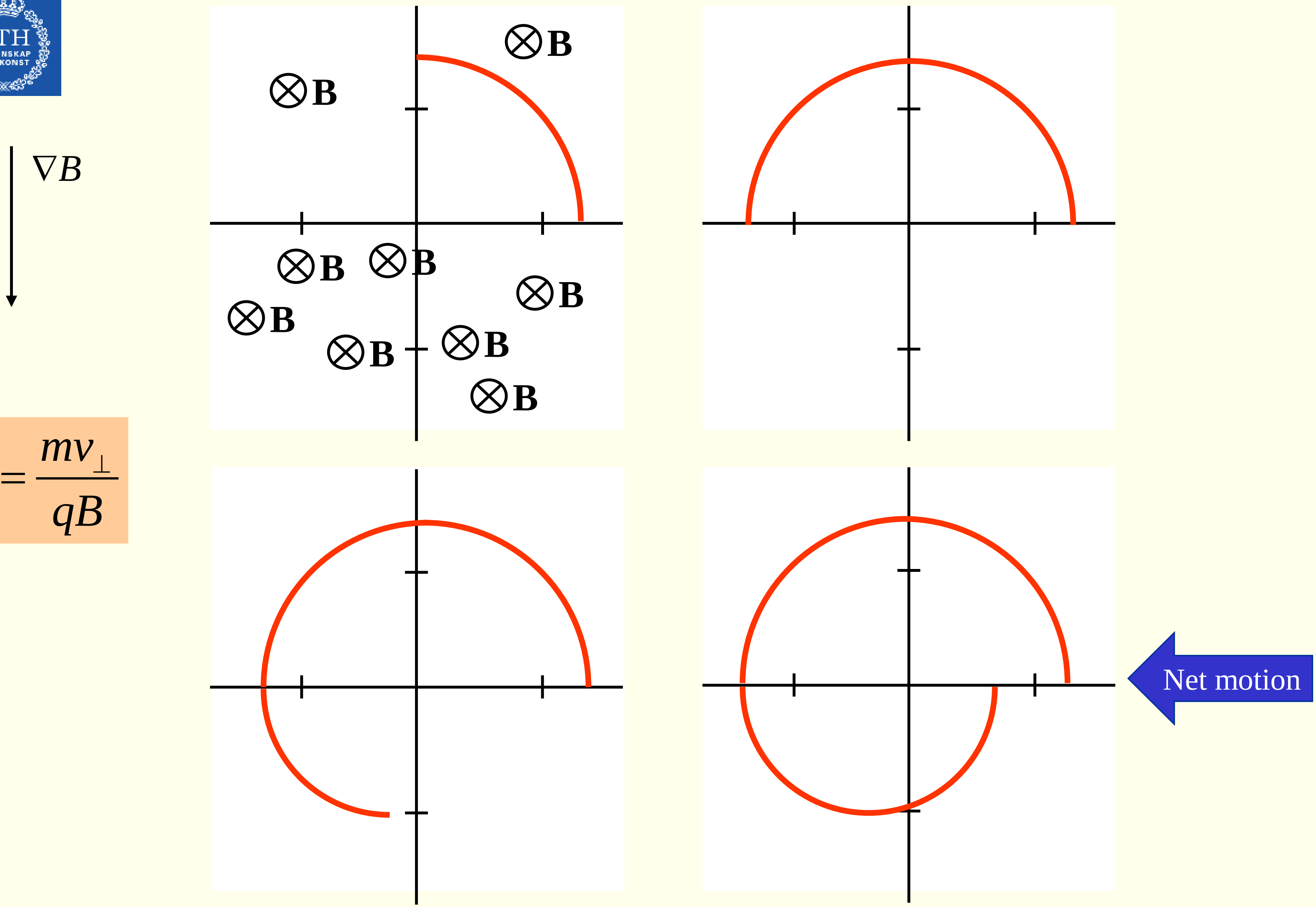
$$\rho = \frac{mv_\perp}{qB}$$

gyro frequency

$$\omega_g = \frac{qB}{m}$$

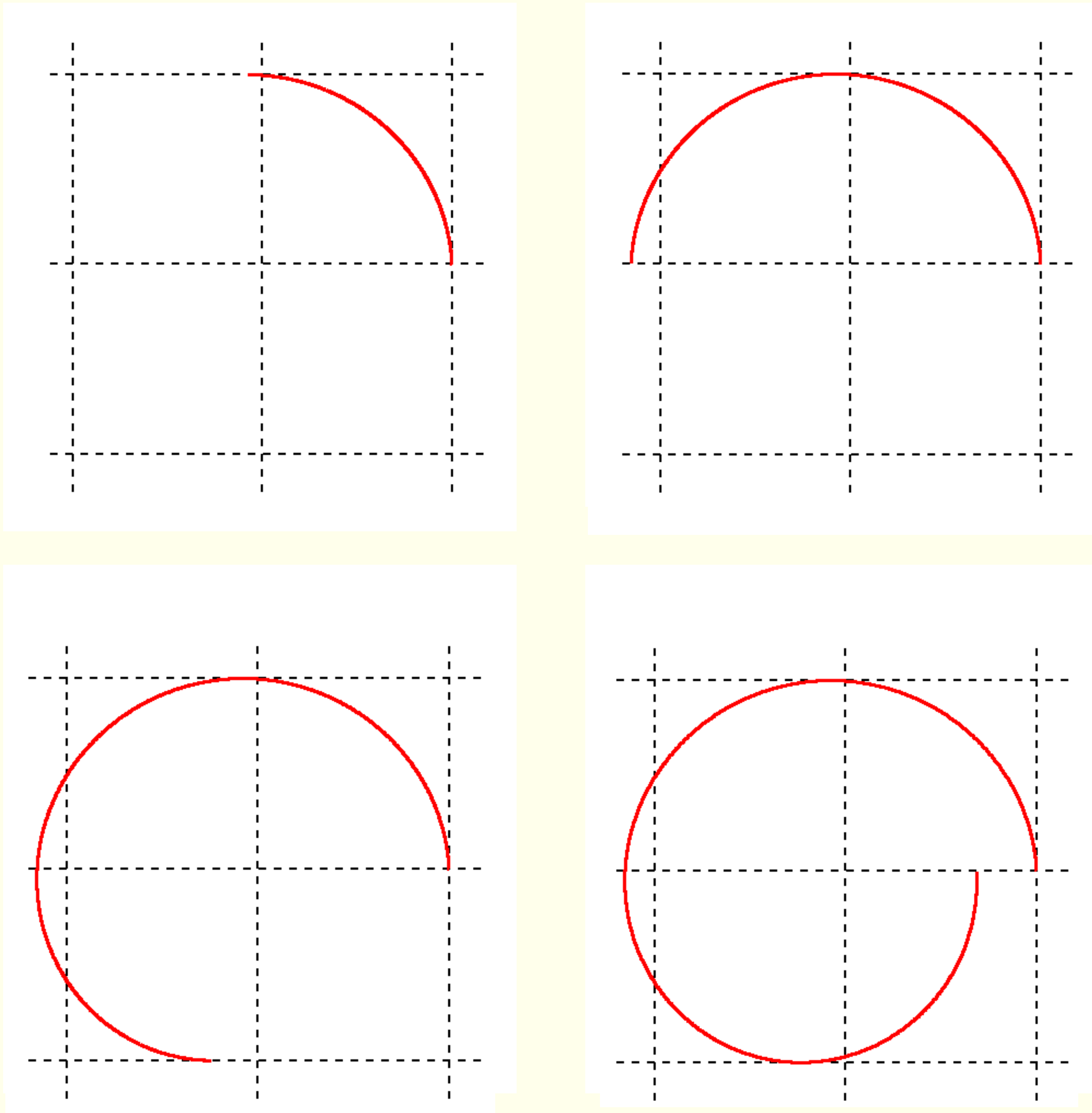
Drift motion





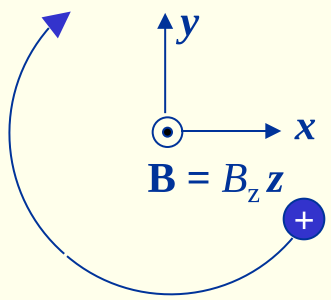
$$\rho = \frac{m v_{\perp}}{qB}$$


E
B



Drift motion

Consider a charged particle in a magnetic field.



Assume an electric field in the x-z plane:

$$\mathbf{E} = \left(E_x, 0, E_z\right)$$

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times\mathbf{B}+\mathbf{E}\right) \Rightarrow$$

$$\begin{cases} m\dfrac{dv_x}{dt} = qv_yB + qE_x \\ m\dfrac{dv_y}{dt} = -qv_xB \\ m\dfrac{dv_z}{dt} = qE_z \end{cases}$$

Constant acceleration along z

$$\Rightarrow$$

$$\begin{cases} \dfrac{d^2v_x}{dt^2} = \dfrac{qB}{m}\dfrac{dv_y}{dt} = \omega_g\dfrac{dv_y}{dt} = -\omega_g^2 v_x \\ \dfrac{d^2v_y}{dt^2} = -\dfrac{qB}{m}\dfrac{dv_x}{dt} = -\omega_g\dfrac{dv_x}{dt} = -\omega_g^2 v_y - \dfrac{q^2B}{m^2}E_x \end{cases}$$

$$\Rightarrow$$

Drift motion

$$\begin{cases} \dfrac{d^2 v_x}{dt^2} = \dfrac{qB}{m}\dfrac{dv_y}{dt} = \omega_g \dfrac{dv_y}{dt} = -\omega_g^2 v_x \\ \dfrac{d^2 v_y}{dt^2} = -\dfrac{qB}{m}\dfrac{dv_x}{dt} = -\omega_g \dfrac{dv_x}{dt} = -\omega_g^2 v_y - \dfrac{q^2 B}{m^2} E_x \end{cases}$$

$$\therefore$$

$$\begin{cases} \dfrac{d^2 v_x}{dt^2} - \omega_g^2 v_x \\ \dfrac{d^2\left(v_y + \dfrac{E_x}{B}\right)}{dt^2} = -\omega_g^2 \left(v_y + \dfrac{E_x}{B}\right) \end{cases}$$

$$\Rightarrow$$

$$\begin{cases} v_x = v_\perp e^{i\omega_g t + \delta_x} \\ v_y = -\dfrac{E_x}{B} + v_\perp e^{i\omega_g t + \delta_y} \end{cases}$$

Average over a gyro period:

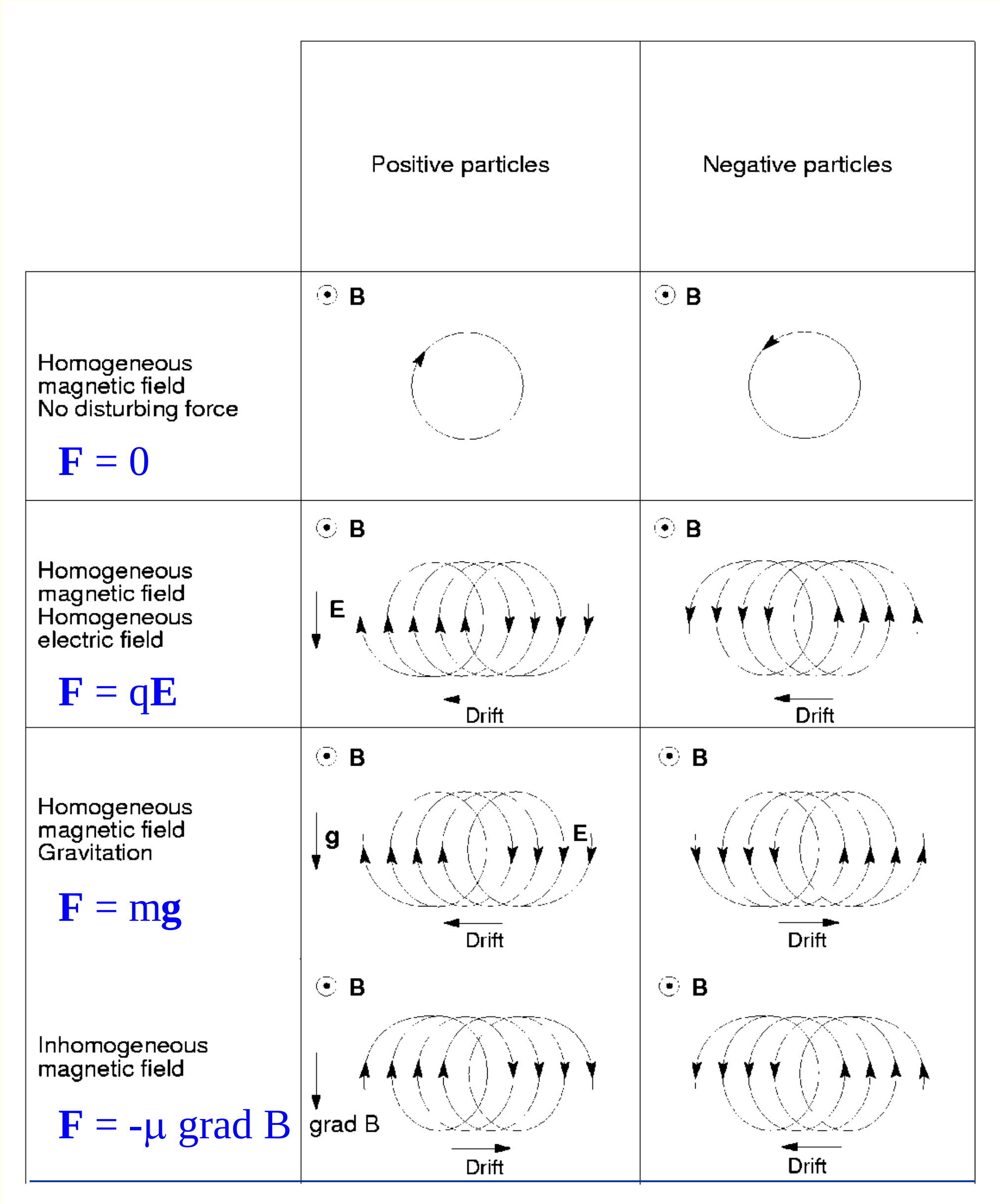
$$v_{drift,y} = -\frac{E_x}{B} = -\frac{E_x B_z}{B^2} = \frac{(\mathbf{E}\times\mathbf{B})_y}{B^2}$$

In general:

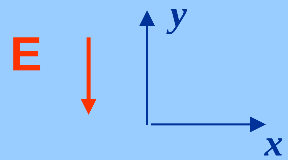
$$\mathbf{v}_{drift} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} = \frac{q\mathbf{E}\times\mathbf{B}}{qB^2} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Drift motion

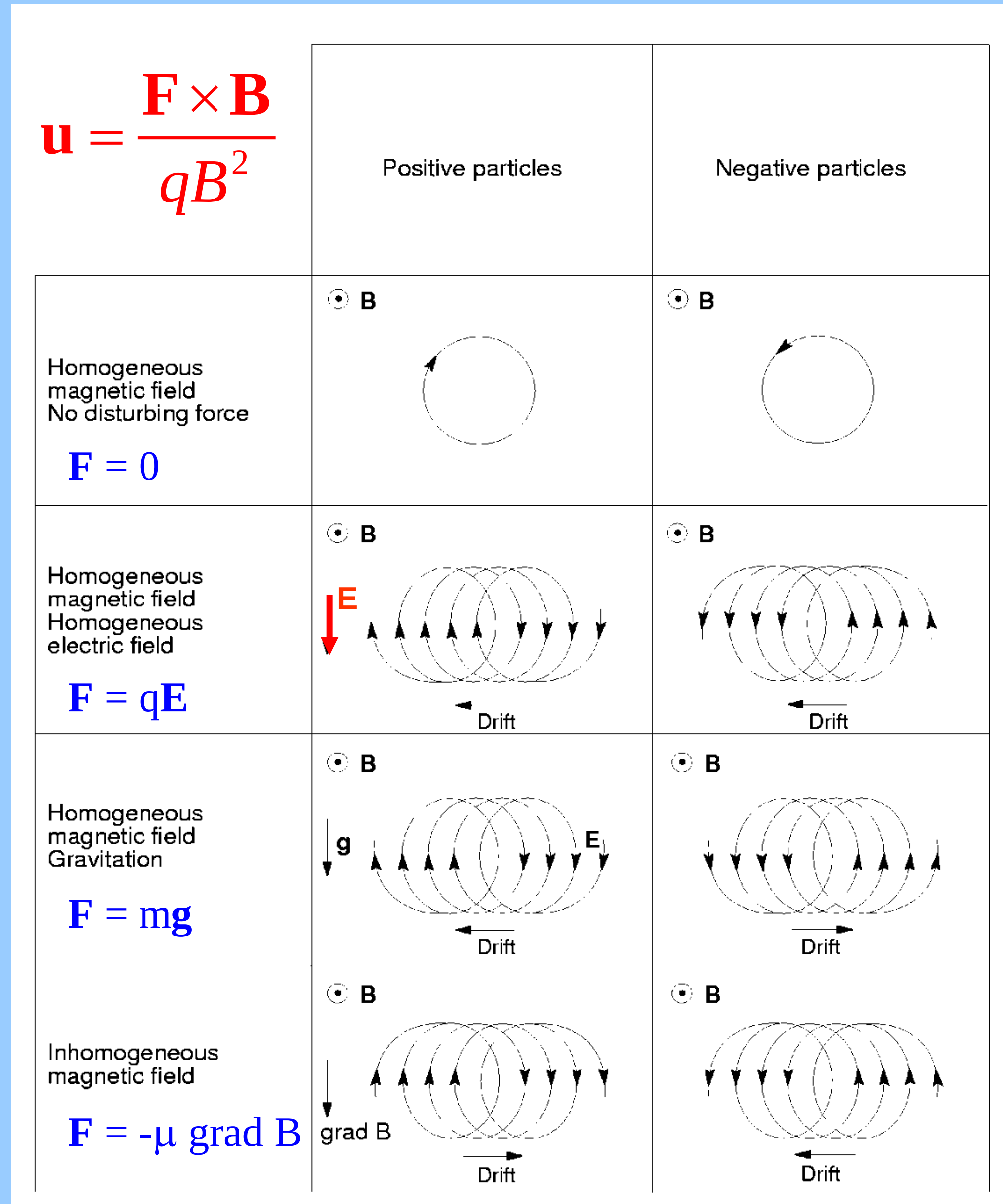
$$\mathbf{u}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$



Suppose you apply an electric field **E** in the direction showed in the figure, and that one electron and one ion (charge –*e* and *e*) is present. What will the resulting current be?



$$\mathbf{I} \equiv e\mathbf{u}_\mathrm{i} - e\mathbf{u}_\mathrm{e}$$



Yellow $\mathbf{I} = -e\dfrac{E}{B}\hat{\mathbf{x}}$

Blue $\mathbf{I} = 0$

Red $\mathbf{I} = \dfrac{1}{2}e\dfrac{E}{B}\hat{\mathbf{x}} - \dfrac{1}{2}e\dfrac{E}{B}\hat{\mathbf{y}}$

Green $\mathbf{I} = e\dfrac{E}{B}\hat{\mathbf{y}}$

$$\mathbf{I} \equiv e\mathbf{u_i} - e\mathbf{u_e}$$

$$\mathbf{u_i} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2} = \frac{e\mathbf{E}\times\mathbf{B}}{eB^2} = -\hat{\mathbf{x}}\frac{EB}{B^2} = -\hat{\mathbf{x}}\frac{E}{B}$$

$$\mathbf{u_e} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2} = \frac{-e\mathbf{E}\times\mathbf{B}}{-eB^2} = -\hat{\mathbf{x}}\frac{EB}{B^2} = -\hat{\mathbf{x}}\frac{E}{B}$$

$$\mathbf{I} \equiv e\mathbf{u_i} - e\mathbf{u_e} = e\left(\mathbf{u_i} - \mathbf{u_e}\right) = 0$$

Blue

So, if there is no current when you apply an electric field, is the conductivity of the ionospheric plasma zero ?

Last Minute!