4.1)

$$r_Z(\tau) = E[z(t+\tau)z(t)] = E[x(t+\tau)y(t+\tau)x(t)y(t)] \overset{\text{independence}}{=} E[x(t+\tau)x(t)]E[y(t+\tau)y(t)]$$
$$= r_x(\tau)\,r_y(\tau)$$

$$\Rightarrow r_Z(\tau) = (\sigma_x^2 e^{-a|\tau|} + m_x^2)(\sigma_y^2 e^{-b|\tau|} + m_y^2) = \sigma_x^2\sigma_y^2 e^{-(a+b)|\tau|} + m_x^2\sigma_y^2 e^{-b|\tau|} + m_y^2\sigma_x^2 e^{-a|\tau|}$$
$$+ m_x^2 m_y^2$$

$$R_Z(f) = \underbrace{F\{r_Z(\tau)\}} = F\{r_x(\tau)r_y(\tau)\} = F\{r_x(\tau)\} * F\{r_y(\tau)\} \to \dots$$

$$\hookrightarrow F\{e^{-a|t|}\} = \frac{2a}{a^2+4\pi^2f^2} \Rightarrow R_Z(f) = \sigma_x^2\sigma_y^2\frac{2(a+b)}{(a+b)^2+4\pi^2f^2}$$

$$F\{a\} = a\,\delta(f)$$

$$+ m_x^2\sigma_y^2\frac{2b}{b^2+4\pi^2f^2} + m_y^2\sigma_x^2\frac{2a}{a^2+4\pi^2f^2}$$
$$+ m_x^2 m_y^2\,\delta(f)$$

4.8)

a)
b) $A(t) \to$ WSS

m_A : constant
$r_A(\tau)$: independent of time t

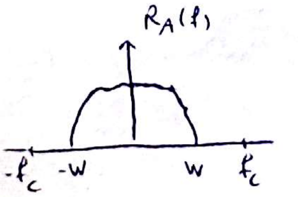
$$E[s(t)] \underset{\substack{\text{independence}\\ A(t),\ \Phi}}{=} C_0 m_A \underbrace{E[\cos(2\pi f_c t + \Phi)]}_{=0} = 0 \quad \text{constant} \quad (\mathrm{I})$$

$$r_s(\tau) = E[s(t+\tau)s(t)] = E[C_0^2 A(t+\tau)A(t)\cos(2\pi f_c(t+\tau)+\Phi)\cos(2\pi f_c t+\Phi)]$$
$$= C_0^2\, r_A(\tau)\frac{1}{2}\Big(\underbrace{E[\cos(2\pi f_c(2t+\tau)+\Phi)]}_{=0} + E[\cos(2\pi f_c\tau)]\Big)$$
$$= \frac{C_0^2 r_A(\tau)}{2}\cos(2\pi f_c\tau) \quad \text{only function of } \tau \quad (\mathrm{II})$$

$$(\mathrm{I}),(\mathrm{II}) \Rightarrow s(t) \ \text{W.S.S.}$$

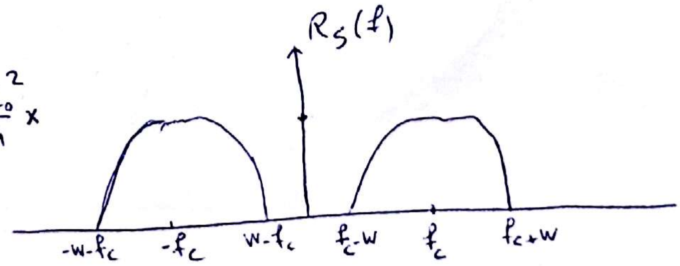
c)

$$R_s(f) = \frac{C_0^2}{2}R_A(f) * \left\{\frac{1}{2}\delta(f+f_c) + \frac{1}{2}\delta(f-f_c)\right\}$$
$$= \frac{C_0^2}{4}\left(R_A(f+f_c) + R_A(f-f_c)\right)$$



$\Longrightarrow$

$\frac{C_0^2}{4} \times$



4.9) b) $r(k) = \delta(k) + \frac{1}{|k|!}$

$r(0) = 2$

$r(1) = r(-1) = \frac{1}{1!} = 1$

$r(2) = r(-2) = \frac{1}{2!}$

$\vdots$

$|k| \geq 1 \quad r(k) = r(-k) = \frac{1}{k!}$

$\Rightarrow R(\nu) = 2 + (e^{j2\pi\nu} + e^{-j2\pi\nu}) + \frac{1}{2!}(e^{j4\pi\nu} + e^{-j4\pi\nu}) + \dots$

$$R(\nu) = 2 + \sum_{k=1}^{\infty} \frac{1}{k!}\left(e^{j2\pi k\nu} + e^{-j2\pi k\nu}\right) = \sum_{k=0}^{\infty} \frac{(e^{j2\pi\nu})^k}{k!} + \sum_{k=0}^{\infty} \frac{(e^{-j2\pi\nu})^k}{k!}$$

Taylor expansion

$$= e^{e^{j2\pi\nu}} + e^{e^{-j2\pi\nu}} = e^{\cos(2\pi\nu)}\left(e^{j\sin(2\pi\nu)} + e^{-j\sin(2\pi\nu)}\right)$$

$$= 2e^{\cos(2\pi\nu)} \cos(\sin(2\pi\nu))$$

4.10) b)

$F_d\{a^{|n|}\} = \frac{1-a^2}{1+a^2-2a\cos(2\pi\nu)}$ $\quad |a|<1$

$R(\nu) = \frac{9}{10-6\cos(2\pi\nu)}$

$r(k) = F^{-1}\{R(\nu)\}$

$$\begin{cases} ① \ k_1(1-a^2) = 9 \\ ② \ k_2(1+a^2) = 10 \\ ③ \ k_2 a = 3 \end{cases}$$

②,③ $3a^2 - 10a + 3 = 0$

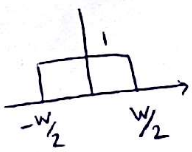
$a = \frac{1}{3}, \underbrace{3}_{\text{not acceptable}}$

$a = \frac{1}{3} \rightarrow k_1 = \frac{81}{8}$

$k_2 = 9$

$$r(k) = \frac{k_1}{k_2}\left(\frac{1}{3}\right)^{|k|} = \frac{9}{8}(3)^{-|k|}$$

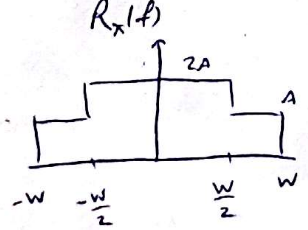
4.11) a)

$\Pi\left(\frac{f}{W}\right) \equiv$  $\xrightarrow{F^{-1}} W\,\mathrm{sinc}(Wt)$

Duality

$\Pi\left(\frac{t}{P}\right) \xrightarrow{F} P\,\mathrm{sinc}(Pf)$

$R_x(f)$



$R_x(f) = A\,\Pi\left(\frac{f}{W}\right) + A\,\Pi\left(\frac{f}{2W}\right)$

$r_x(\tau) = AW\,\mathrm{sinc}(W\tau) + 2AW\,\mathrm{sinc}(2Wt)$