Last lecture (5)

- Ionosphere
  - index of refraction
  - reflection of radio waves
  - particle drift motion in magnetized plasma

Today's lecture (6)

- Ionosphere
  - electrical conductivity in ionosphere
- Magnetosphere, introduction
- Magnetospheric size (standoff distance)
- Particle motion in the magnetosphere

Today

Activity	Date	Time	Room	Subject	Litterature
L1	29/8	13-15	E52	Course description, Introduction, The Sun 1, Plasma physics 1	**CGF** Ch 1, 5, (p 110-113)
L2	1/9	15-17	L52	The Sun 2, Plasma physics 2	**CGF** Ch 5 (p 114-121), 6.3
L3	5/9	13-15	E51	Solar wind, The ionosphere and atmosphere 1, Plasma physics 3	**CGF** Ch 6.1, 2.1-2.6, 3.1-3.2, 3.5, **LL** Ch III, Extra material
T1	8/9	15-17	D41	Mini-group work 1	
L4	12/9	13-15	E35	The ionosphere 2, Plasma physics 4	**CGF** Ch 3.4, 3.7, 3.8
L5	14/9	10-12	V32	The Earth's magnetosphere 1, Plasma physics 5	**CGF** 4.1-4.3, **LL** Ch I, II, IV.A
T2	15/9	15-17	E51	Mini-group work 2	
L6	19/9	13-15	M33	The Earth's magnetosphere 2, Other magnetospheres	**CGF** Ch 4.6-4.9, **LL** Ch V.
T3	22/9	15-17	E51	Mini-group work 3	
L7	26/9	13-15	E31	Aurora, Measurement methods in space plasmas and data analysis 1	**CGF** Ch 4.5, 10**, LL** Ch VI, Extra material
L8	28/9	10-12	L52	Space weather and geomagnetic storms	**CGF** Ch 4.4, **LL** Ch IV.B-C, VII.A-C
T4	29/9	15-17	M31	Mini-group work 4	
L9	3/10	13-15	E52	Interstellar and intergalactic plasma, Cosmic radiation,	**CGF** Ch 7-9
T5	6/10	15-17	E31	Mini-group work 5	
L10	10/10	13-15	E52	Swedish and international space physics research.	
T6	13/10	15-17	E31	Round-up, old exams.	
Written examination	26/10	8-13	F2		

EF22445 Space Physics II
7.5 ECTS credits, P2

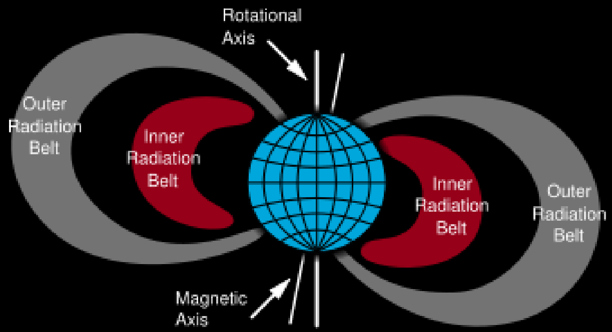
- shocks and boundaries in space
- solar wind interaction with magnetized and unmagnetized bodies
- reconnection
- sources of magnetospheric plasma
- magnetospheric and ionospheric convection
- auroral physics
- storms and substorms
- global oscillations of the magnetosphere

Courses at the Alfvén Laboratory

EF2260 SPACE ENVIRONMENT AND SPACECRAFT ENGINEERING , 6 ECTS credits, period 2

- environments spacecraft may encounter in various orbits around the Earth, and the constraints this places on spacecraft design
- basic operation principles underlying the thermal control system and the power systems in spacecraft
- measurements principles in space

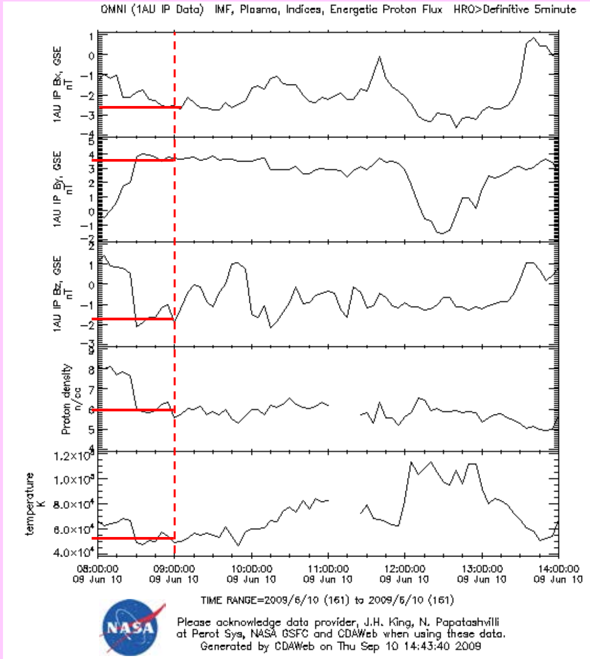
The Astrid-2 satellite



Radiation environment in near-earth space

Projects:

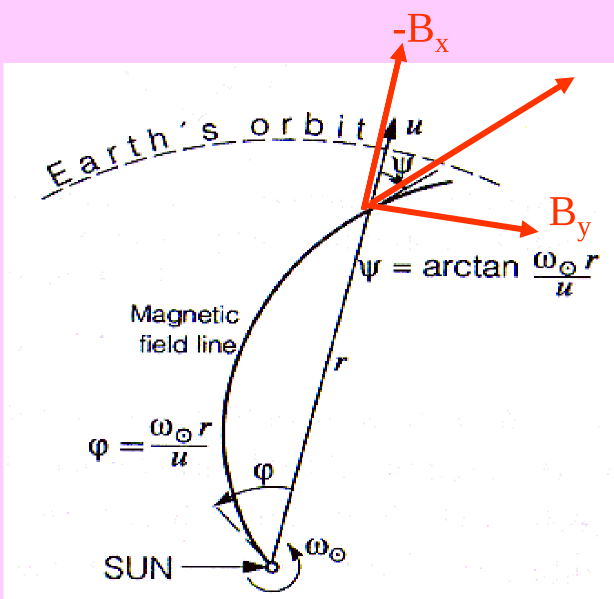
- Design power supply for spacecraft
- Study of radiation effects on electronics



Mini-groupwork 2

a)

$$\psi = \arctan \frac{\omega_{sun} r}{u_{sw}} \quad \Rightarrow \quad u_{sw} = \frac{\omega_{sun} r}{\tan \psi}$$



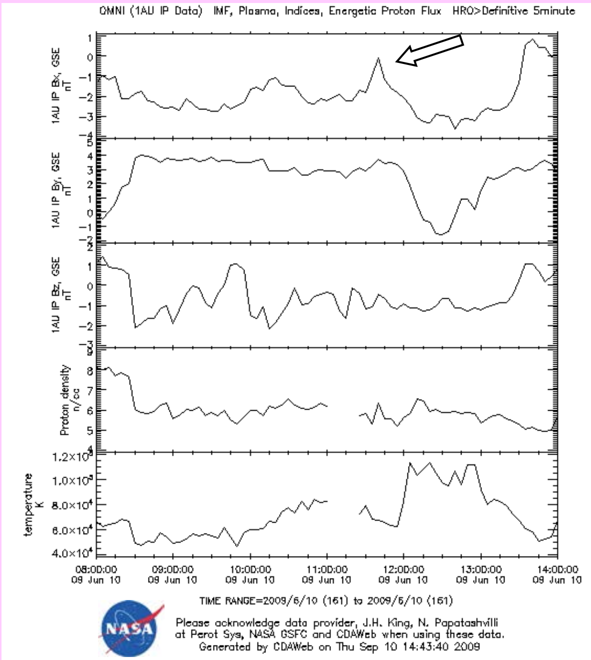
$\omega_{sun} = 2\pi/T = 2.9 \cdot 10^{-6}\ s^{-1}$ (T = 25 days at equator)

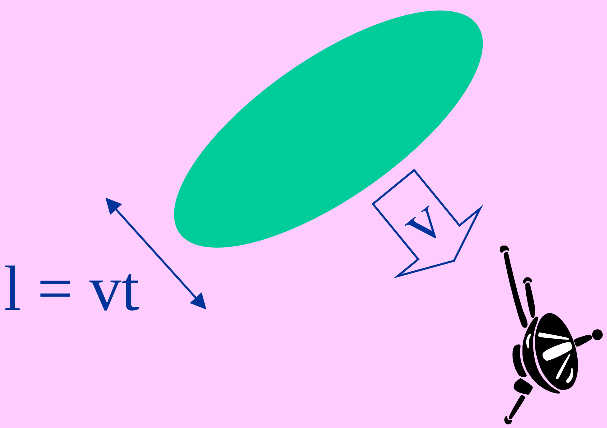
r = 1 A.U.

$\tan \psi = |B_y/B_x| \approx 3.6/2.6$ (from figure) ($\psi = 41°$)

With these figures I get $u_{sw} = 313$ km/s

Mini-groupwork 2





b)

The magnetic Reynolds number is calculated by using typical plasma flow velocities v_c and typical length scales of magnetic field variations l_c

Use solar wind velocity obtained in a) for typical flow velocity. To obtain l_c, multiply the time t it takes the magnetic field structure (indicated in the figure), to pass over the satellite and use l_c, = vt. I get l_c = $2.8 \cdot 10^8$ m.

Using a temperature of $5 \cdot 10^4$ K, we can evaluate the conductivity, remembering that the temperature should be given in eV. We get the conversion from

$$W = k_B T$$

which gives the result that 1 eV corresponds to a temperature of 7729 K. We then get T = 6.5 eV, and

$$\sigma = 3.1 \cdot 10^4 \text{ S/m}$$

Putting in the numbers I get

$$R_m = \mu_0 \, \sigma \, v_c \, l_c \approx 9.8 \cdot 10^{14} >> 1$$

So the solar wind magnetic field is frozen into the plasma to a very good approximation.

Frozen in magnetic flux *PROOF II*

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

A

B

Order of magnitude estimate:

$$\frac{A}{B} = \frac{\nabla \times (\mathbf{v} \times \mathbf{B})}{\frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}} \approx \frac{\frac{v \Delta B}{L}}{\frac{\Delta B}{\mu_0 \sigma L^2}} = v L \mu_0 \sigma \equiv R_m$$

Magnetic Reynolds number R_m:

$$R_m >> 1 \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Frozen-in fields!

$$R_m << 1 \Rightarrow \quad \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Diffusion equation!

Typical length scale L

$$\frac{dB}{dx} \sim \frac{B}{L}$$

Energy - temperature

Average energy of molecule/atom:

$$E = \frac{3}{2} k_B T \quad \Rightarrow$$

$$T = \frac{2E}{3k_B}$$

1 eV = 1.6·10^{-19} J ⇒

$$T = \frac{2E}{3k_B} = \frac{2 \cdot 1.6 \cdot 10^{-19}\,\mathrm{J}}{3 \cdot 1.38 \cdot 10^{-23}\,\dfrac{\mathrm{J}}{\mathrm{K}}} = 7729\,\mathrm{K}$$

But beware!

In plasma physics, usually:

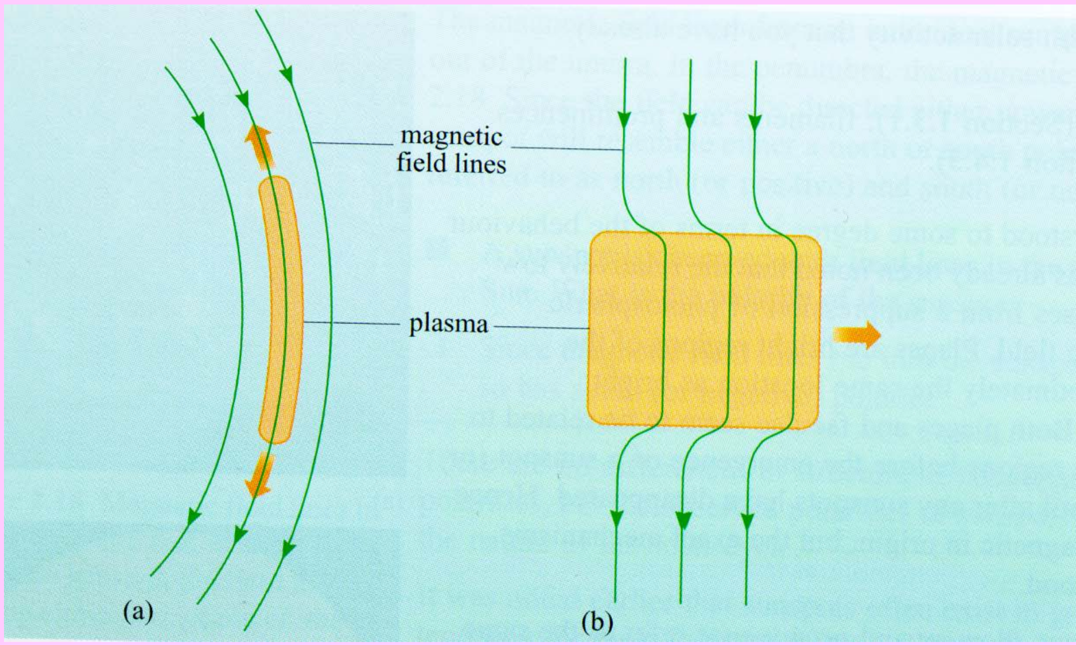
$$E = \cancel{\frac{3}{2}} k_B T \quad \Rightarrow$$

$$T = \frac{E}{k_B}$$

$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J} \Rightarrow$

$$E = k_B T = \frac{1.6 \cdot 10^{-19}\,\text{J}}{1.38 \cdot 10^{-23}\,\frac{\text{J}}{\text{K}}} = 11594\,\text{K}$$

Does the plasma follow the magnetic field (a) or the other way around (b)?



Depends on relative energy density (pressure)

$$p_{pl} = nk_B T$$

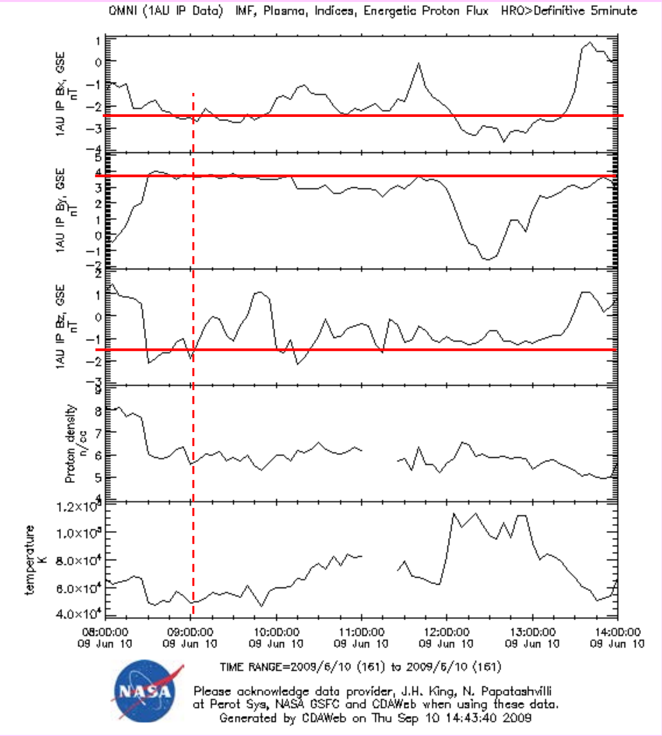
$$p_B = \frac{B^2}{2\mu_0}$$

$$\beta = \frac{p_{pl}}{p_B}$$

$\beta << 1$ $\beta >> 1$

Mini-groupwork 2

c)



$$\rho = n_e m_p = 6.1\cdot10^{6}\cdot1.67\cdot10^{-27} = 1.02\cdot10^{-20}$$

Then the kinetic energy density is (v = 313 km/s):

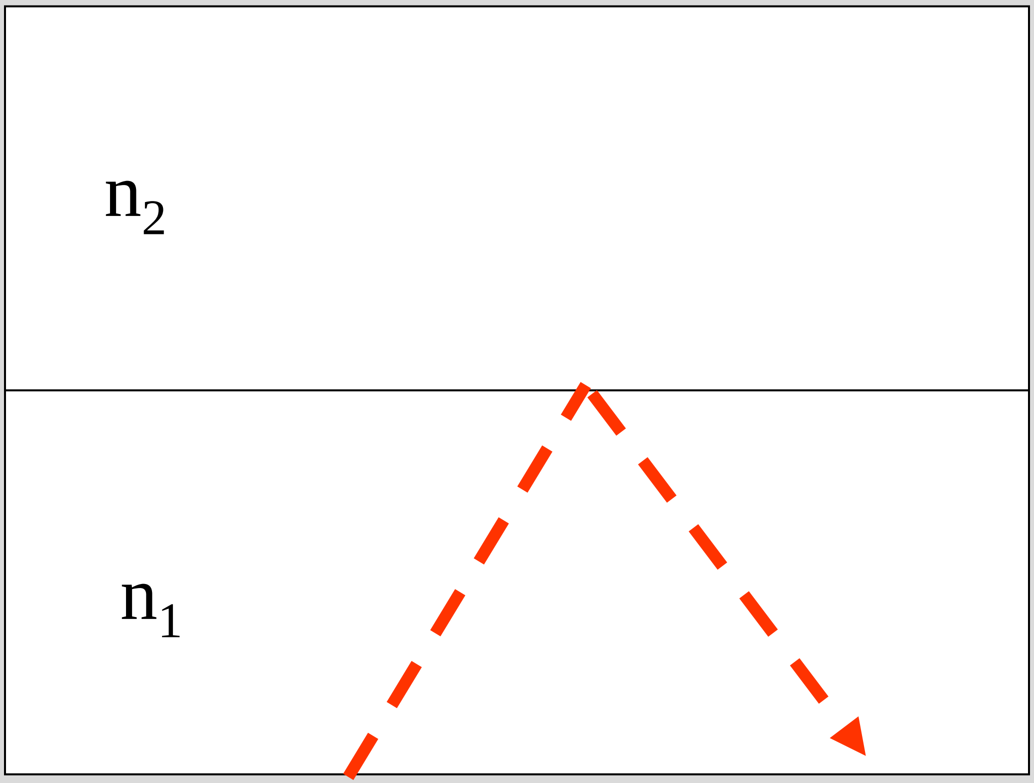
$$\rho v^2/2 = 5.0\cdot10^{-10}\ \mathrm{Jm}^{-3}$$

The magnetic energy density is (using values of figure)

$$\frac{B^2}{2\mu_0} = \frac{\left(B_x^2 + B_y^2 + B_z^2\right)^2}{2\mu_0} = (2.6^2 + 3.6^2 + 1.7^2)\cdot(10^{-9})^2/2\mu_0 = 9\cdot10^{-12}\ \mathrm{Jm}^{-3}$$

The ratio between the kinetic and magnetic energy densities is approximately **50,** thus the plasma motion determines the magnetic field configuration, and not the other way around.

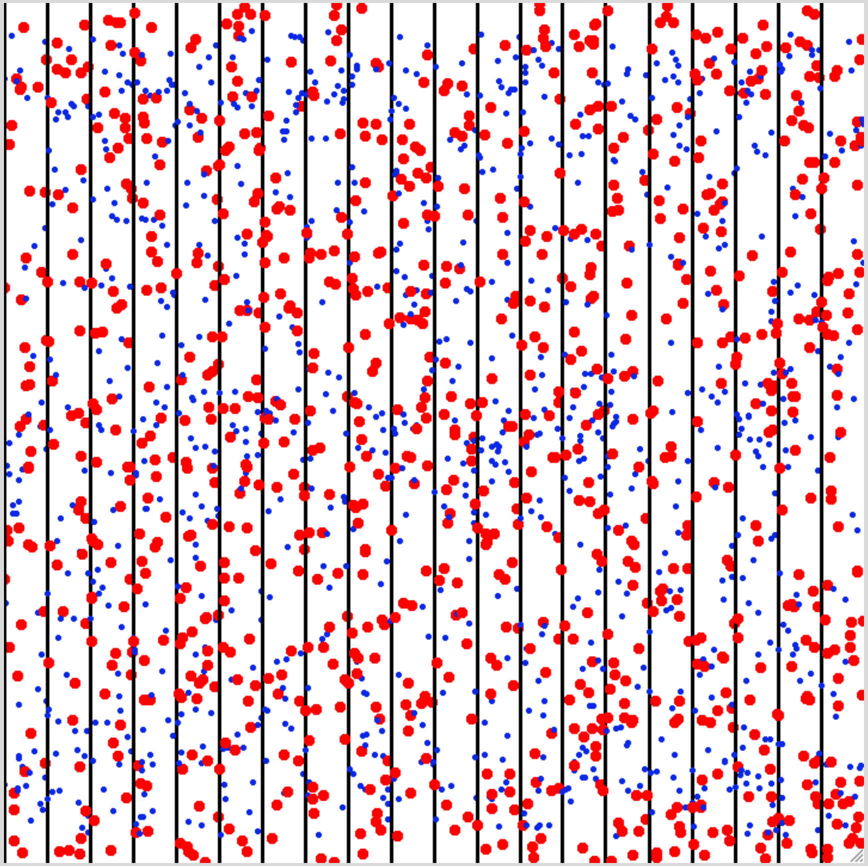
Reflection of radio waves

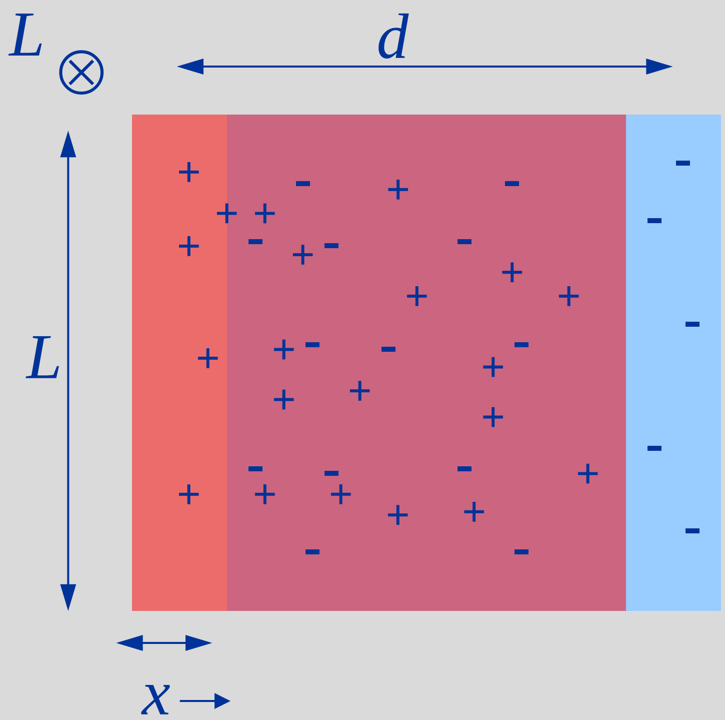


Total reflection at a sharp boundary (or large gradient) if

$$n_2 < n_1$$

Plasma oscillations parallel to B





$$F = m_e a$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$F = -eE$$

$$\sigma = e n_e x$$

$$-\frac{n_e e^2 x}{\varepsilon_0 m_e} = \frac{d^2 x}{dt^2}$$

$$x = \sin(\omega_{pe} t)$$

$$\boxed{\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}}$$

Index of refraction for electromagnetic waves in a plasma

(1) $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

(2) $$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

(3) $$\mathbf{j} = -e n_e \mathbf{v}_e$$

(4) $$m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\mathbf{E}$$

Assume all quantities vary sinusoidally, with frequency ω, *e.g.*:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

(1) $\Rightarrow$ $$\nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \frac{\partial \mathbf{B}}{\partial t}$$

(2) $\Rightarrow$ $$\nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\therefore \quad \nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial \mathbf{j}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \mu_0 e n_e \frac{\partial \mathbf{v}_e}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

Index of refraction for electromagnetic waves in a plasma

$$-\cancel{\mathbf{k}(\mathbf{k}\cdot\mathbf{E})}+k^2\mathbf{E}=\mu_0 e n_e(-i\omega\mathbf{v}_e)-\frac{1}{c^2}(-\omega^2)\mathbf{E}$$

Does not represent E.M. wave

(4) $\Rightarrow$

$$k^2\mathbf{E}=-i\mu_0 e n_e\omega\left(-\frac{ie\mathbf{E}}{\omega m_e}\right)-\frac{1}{c^2}(-\omega^2)\mathbf{E}$$

$\Rightarrow$

$$c^2k^2=-c^2\frac{\mu_0 n_e e^2}{m_e}+\omega^2=\frac{-1}{\cancel{\mu_0}\varepsilon_0}\frac{\cancel{\mu_0}n_e e^2}{m_e}+\omega^2$$

$$\therefore\quad \omega^2=c^2k^2+\omega_p^2$$

$$n^2=\frac{c^2}{v_{ph}^2}=\frac{c^2k^2}{\omega^2}=\frac{\omega^2-\omega_p^2}{\omega^2}=1-\frac{\omega_p^2}{\omega^2}$$

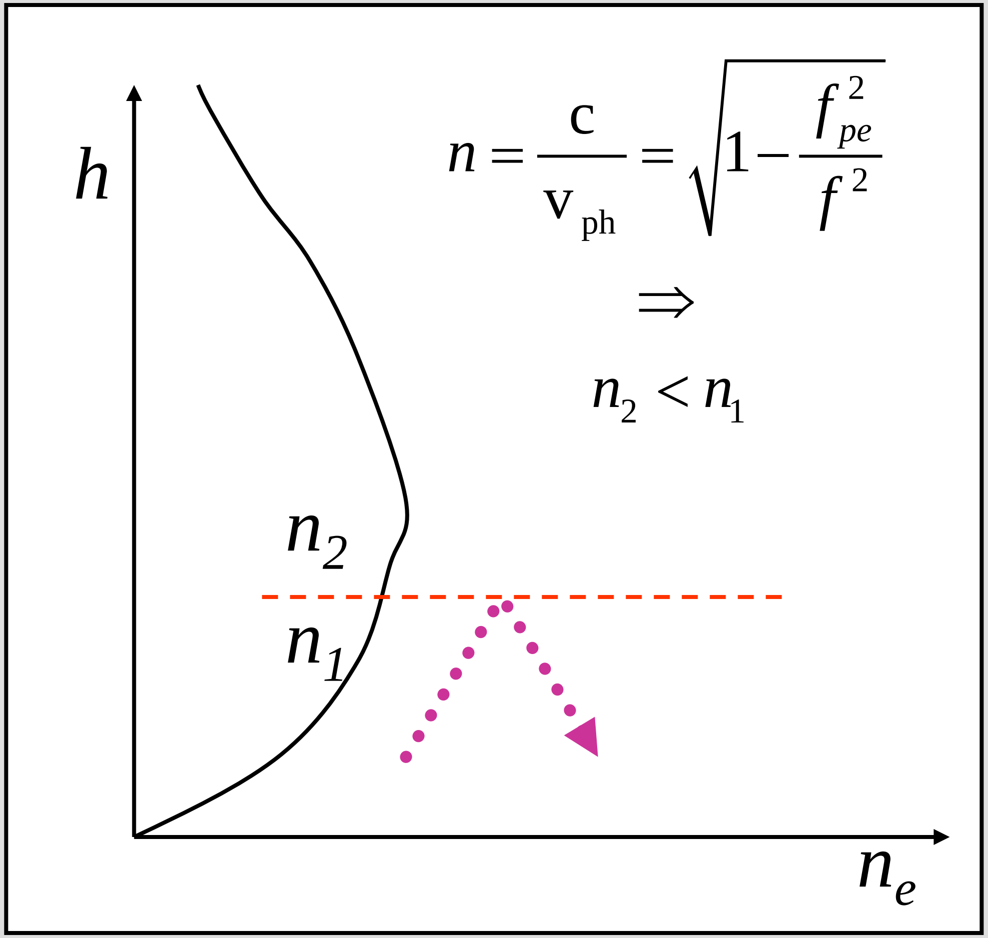
$\therefore$

$$\boxed{n=\sqrt{1-\frac{\omega_p^2}{\omega^2}}=\sqrt{1-\frac{f_p^2}{f^2}}}$$

Where does the total reflection take place?

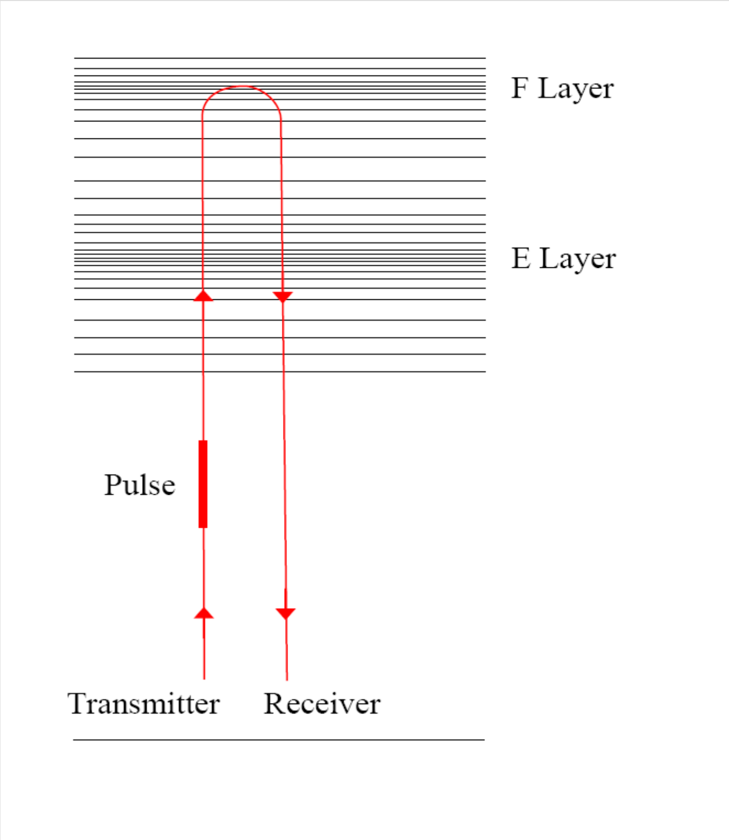
Large gradient when

$$f \approx f_{pe}$$



Higher frequencies $\rightarrow$ higher f_{pe} (n_e)

Ionosonde



The pulse will be reflected where

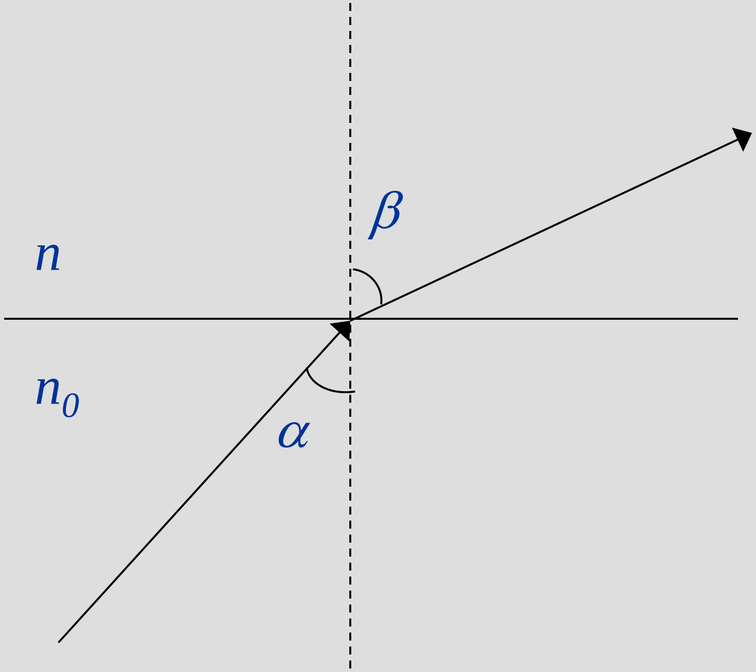
$$f = f_{pe}$$

The altitude will be determined by

$$2h = ct$$

Where t is the time between when the pulse is sent out and the registered again.

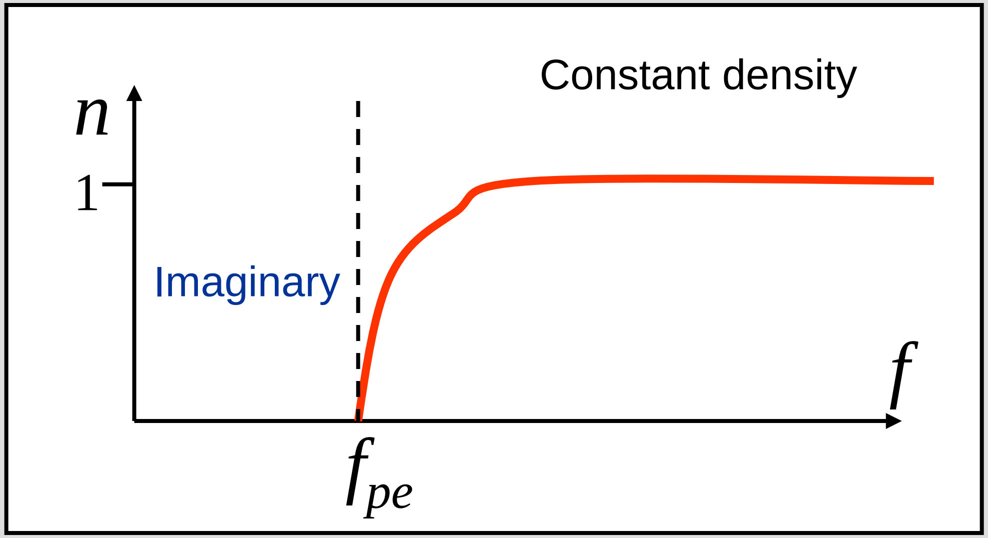
Reflection of radio waves, oblique incidence

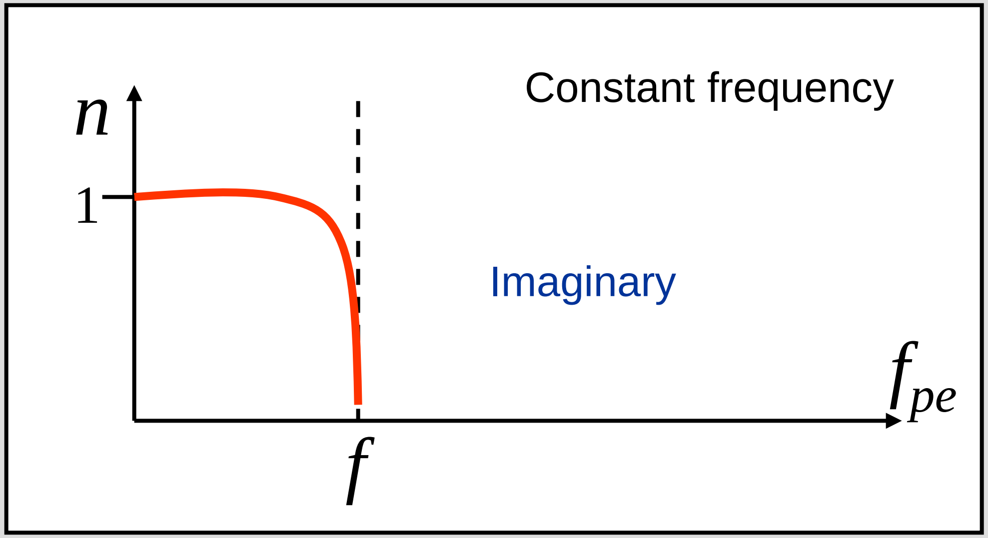


Snell's law:

$$n_0 \sin \alpha = n \sin \beta$$

Refraction index for plasma

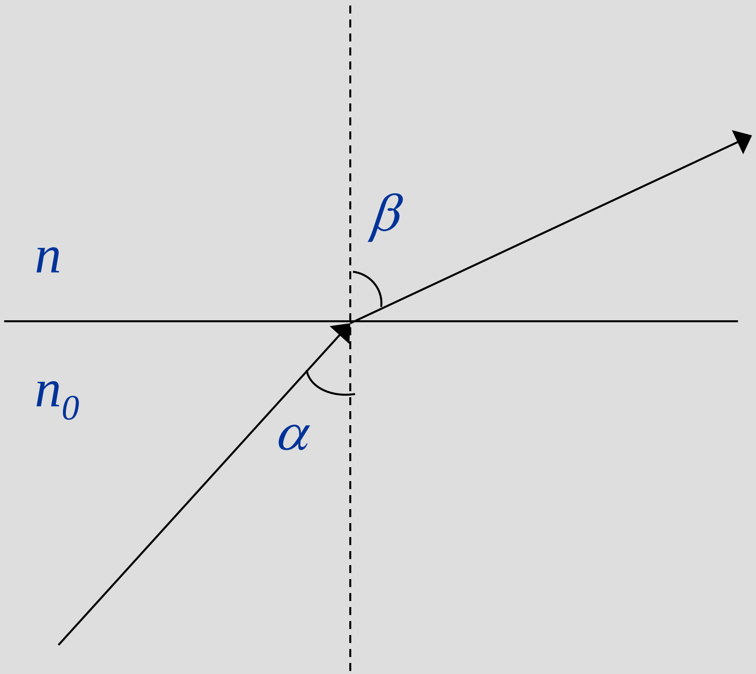




$$n = \frac{c}{v_{ph}} = \sqrt{1 - \frac{f_{pe}^2}{f^2}}$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

Reflection of radio waves, oblique incidence



Snell's law + condition for reflection:

$$n_0 \sin\alpha = n \sin\beta$$

$$\begin{cases} n_0 = 1, \\ \sin\beta > 1 \end{cases} \Rightarrow$$

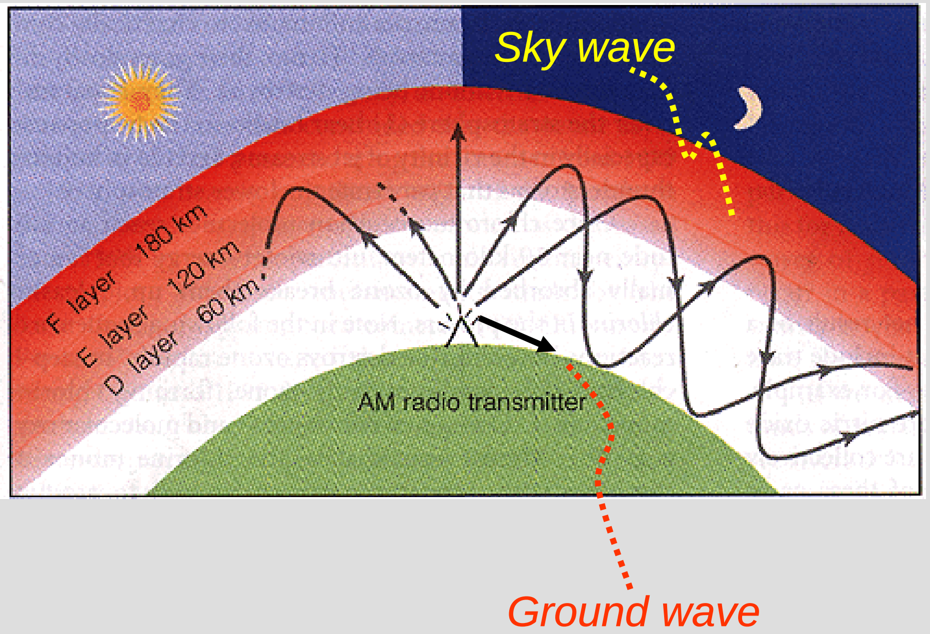
$$\sin\alpha > \frac{n}{n_0} = n \Rightarrow$$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} > n = \sqrt{1-\frac{f_p^2}{f}} \Rightarrow$$

$$-\cos^2\alpha > -\frac{f_p^2}{f} \Rightarrow$$

$$\boxed{f < \frac{f_p}{\cos\alpha}}$$

Reflection of radio waves

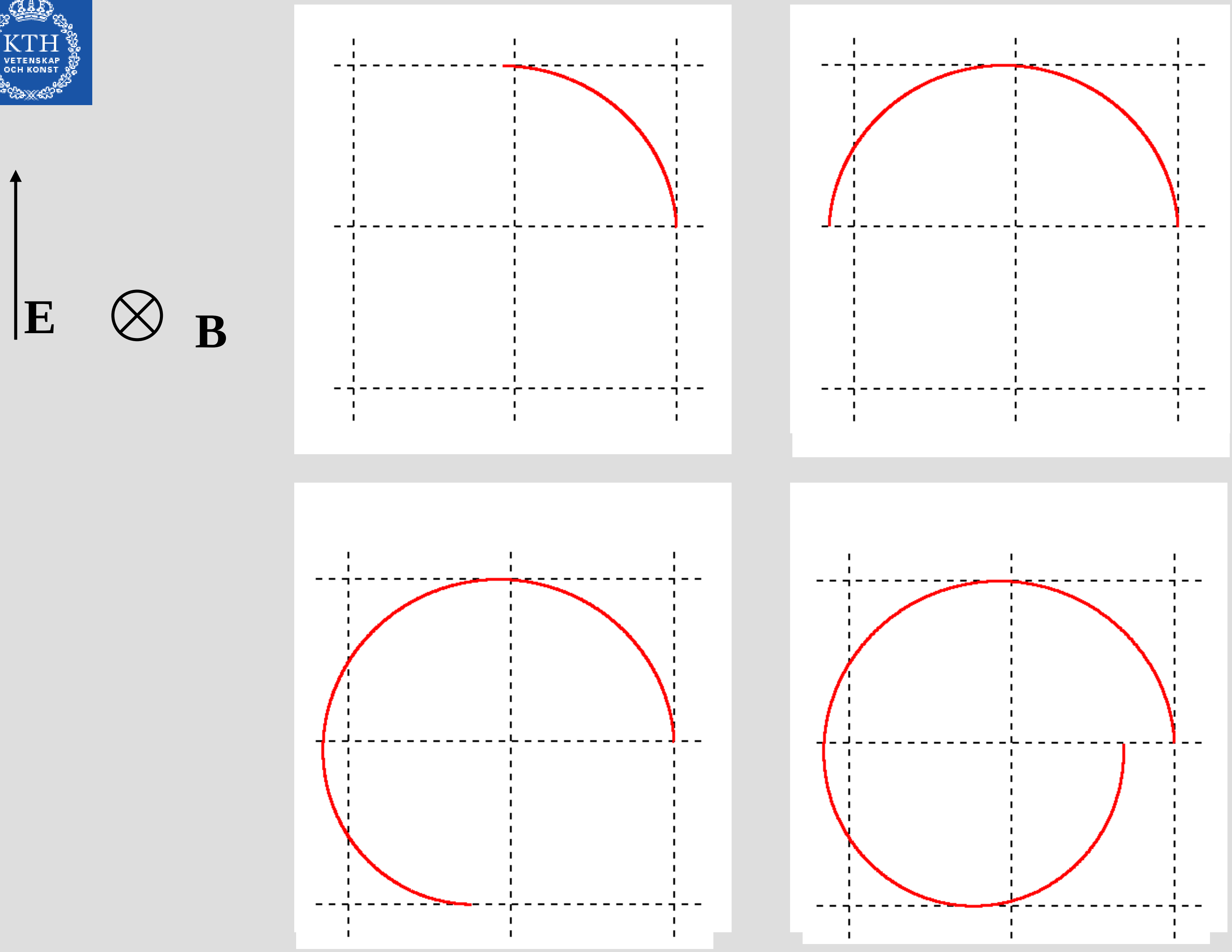


F2-layer during night:

$$n_e = 5 \cdot 10^{11} \text{ m}^{-3} \quad \Rightarrow$$

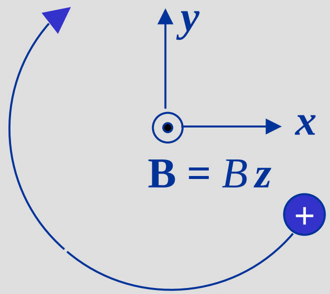
$$f_{pe} = 10^7 \text{ Hz} = 10 \text{ MHz}$$

= HF/short wave


E
B

Drift motion

Consider a charged particle in a magnetic field.



Assume an electric field in the x-z plane:

$$\mathbf{E} = \left(E_x, 0, E_z\right)$$

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times\mathbf{B}+\mathbf{E}\right) \Rightarrow$$

$$\begin{cases} m\dfrac{dv_x}{dt} = qv_yB + qE_x \\ m\dfrac{dv_y}{dt} = -qv_xB \\ m\dfrac{dv_z}{dt} = qE_z \end{cases}$$

Constant acceleration along z

$$\Rightarrow$$

$$\begin{cases} \dfrac{d^2v_x}{dt^2} = \dfrac{qB}{m}\dfrac{dv_y}{dt} = \omega_g\dfrac{dv_y}{dt} = -\omega_g^2 v_x \\ \dfrac{d^2v_y}{dt^2} = -\dfrac{qB}{m}\dfrac{dv_x}{dt} = -\omega_g\dfrac{dv_x}{dt} = -\omega_g^2 v_y - \dfrac{q^2B}{m^2}E_x \end{cases}$$

$$\Rightarrow$$

Drift motion

$$\begin{cases} \dfrac{d^2 v_x}{dt^2} = \dfrac{qB}{m}\dfrac{dv_y}{dt} = \omega_g \dfrac{dv_y}{dt} = -\omega_g^2 v_x \\ \dfrac{d^2 v_y}{dt^2} = -\dfrac{qB}{m}\dfrac{dv_x}{dt} = -\omega_g \dfrac{dv_x}{dt} = -\omega_g^2 v_y - \dfrac{q^2 B}{m^2} E_x \end{cases}$$

$$\therefore$$

$$\begin{cases} \dfrac{d^2 v_x}{dt^2} - \omega_g^2 v_x \\ \dfrac{d^2\left(v_y + \dfrac{E_x}{B}\right)}{dt^2} = -\omega_g^2 \left(v_y + \dfrac{E_x}{B}\right) \end{cases}$$

$\Rightarrow$

$$\begin{cases} v_x = v_\perp e^{i\omega_g t + \delta_x} \\ v_y = -\dfrac{E_x}{B} + v_\perp e^{i\omega_g t + \delta_y} \end{cases}$$

Average over a gyro period:

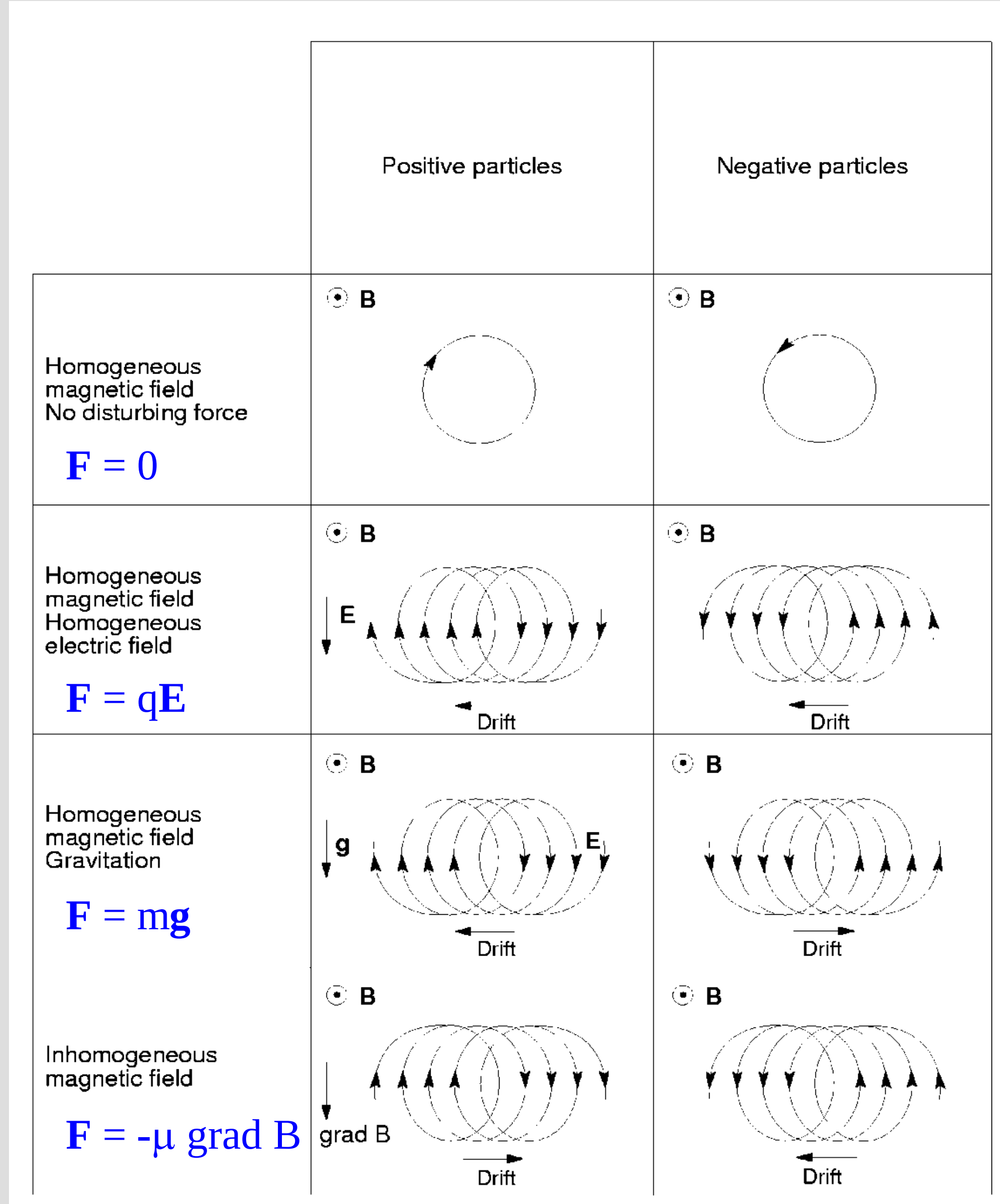
$$v_{drift,y} = -\frac{E_x}{B} = -\frac{E_x B_z}{B^2} = \frac{(\mathbf{E}\times\mathbf{B})_y}{B^2}$$

In general:

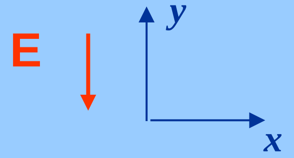
$$\mathbf{v}_{drift} = \frac{\mathbf{E}\times\mathbf{B}}{B^2} = \frac{q\mathbf{E}\times\mathbf{B}}{qB^2} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Drift motion

$$\mathbf{u}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$



Suppose you apply an electric field **E** in the direction showed in the figure, and that one electron and one ion (charge –*e* and *e*) is present. What will the resulting current be?



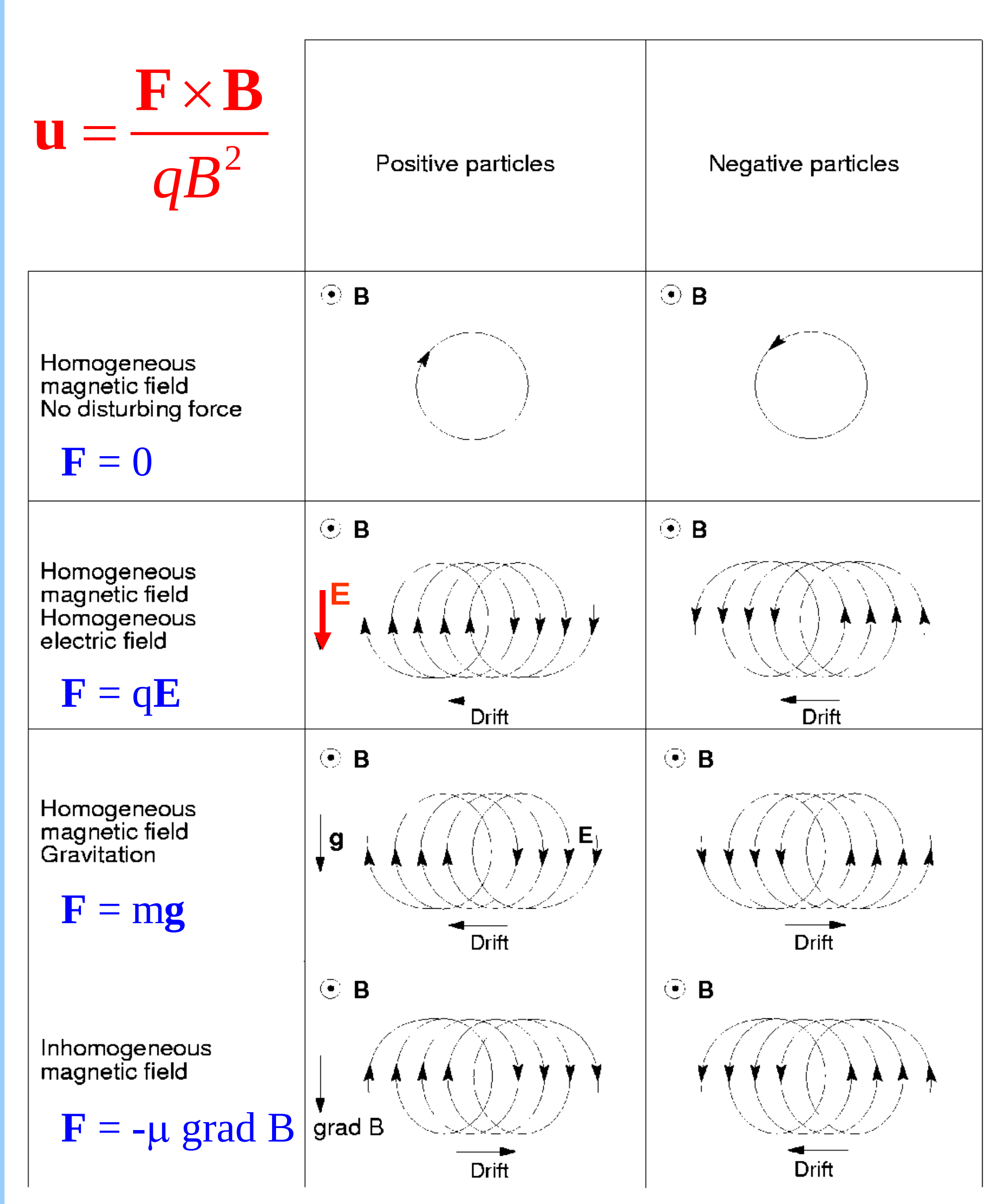
$$\mathbf{I} \equiv e\mathbf{u}_\mathrm{i} - e\mathbf{u}_\mathrm{e}$$

Yellow $\quad \mathbf{I} = -e\frac{E}{B}\hat{\mathbf{x}}$

Blue $\quad \mathbf{I} = 0$

Red $\quad \mathbf{I} = \frac{1}{2}e\frac{E}{B}\hat{\mathbf{x}} - \frac{1}{2}e\frac{E}{B}\hat{\mathbf{y}}$

Green $\quad \mathbf{I} = e\frac{E}{B}\hat{\mathbf{y}}$



$$\mathbf{I} \equiv e\mathbf{u_i} - e\mathbf{u_e}$$

$$\mathbf{u_i} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} = \frac{e\mathbf{E} \times \mathbf{B}}{eB^2} = -\hat{\mathbf{x}}\frac{EB}{B^2} = -\hat{\mathbf{x}}\frac{E}{B}$$

$$\mathbf{u_e} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} = \frac{-e\mathbf{E} \times \mathbf{B}}{-eB^2} = -\hat{\mathbf{x}}\frac{EB}{B^2} = -\hat{\mathbf{x}}\frac{E}{B}$$

$$\mathbf{I} \equiv e\mathbf{u_i} - e\mathbf{u_e} = e\left(\mathbf{u_i} - \mathbf{u_e}\right) = 0$$

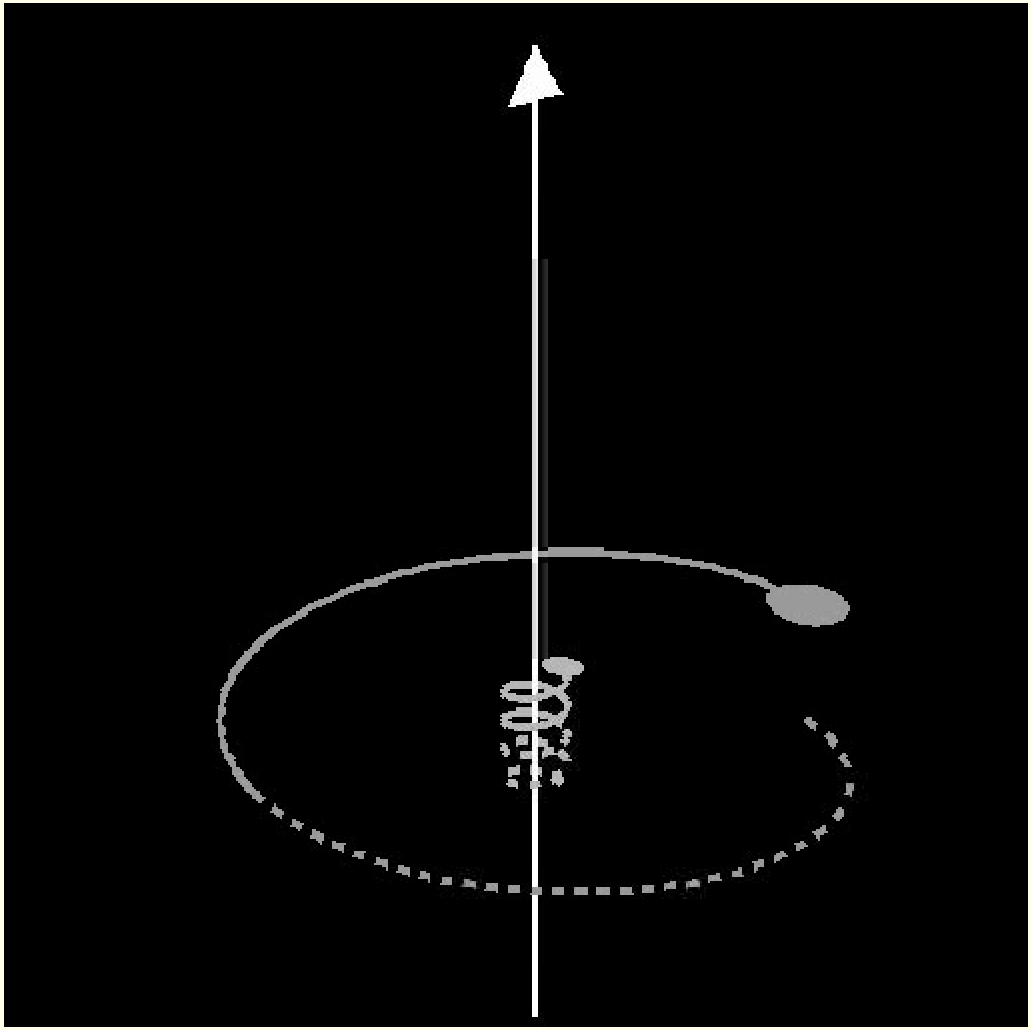
Blue

So, if there is no current when you apply an electric field, is the conductivity of the ionospheric plasma zero ?

What is the electron density at 100 km?

What is the neutral density at 100 km?

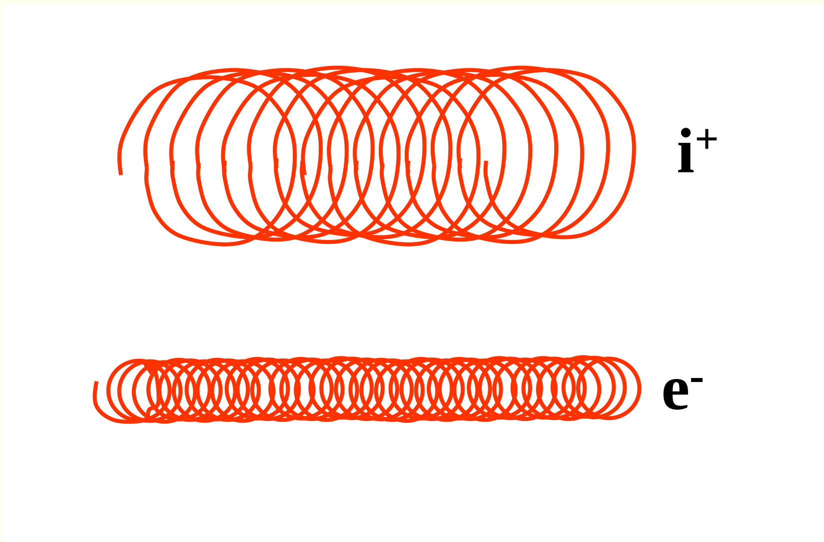
Gyro motion



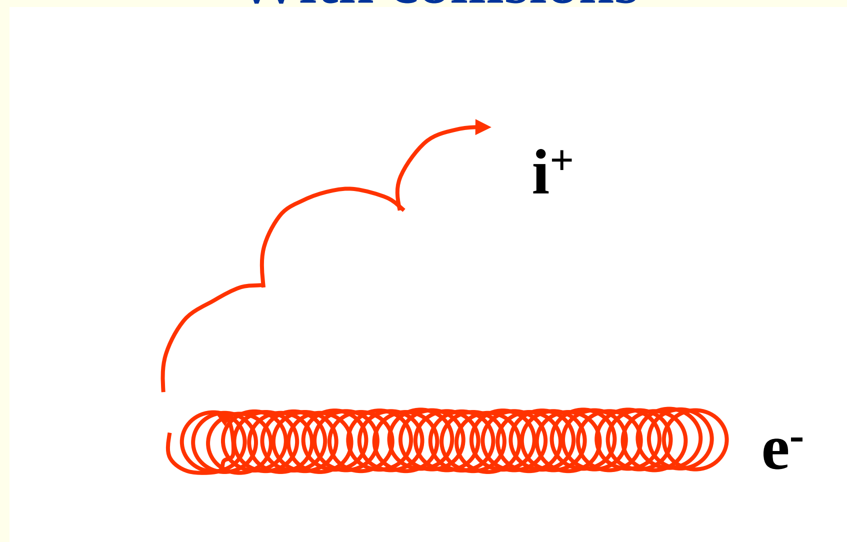
ExB-drift

E ⊙ B

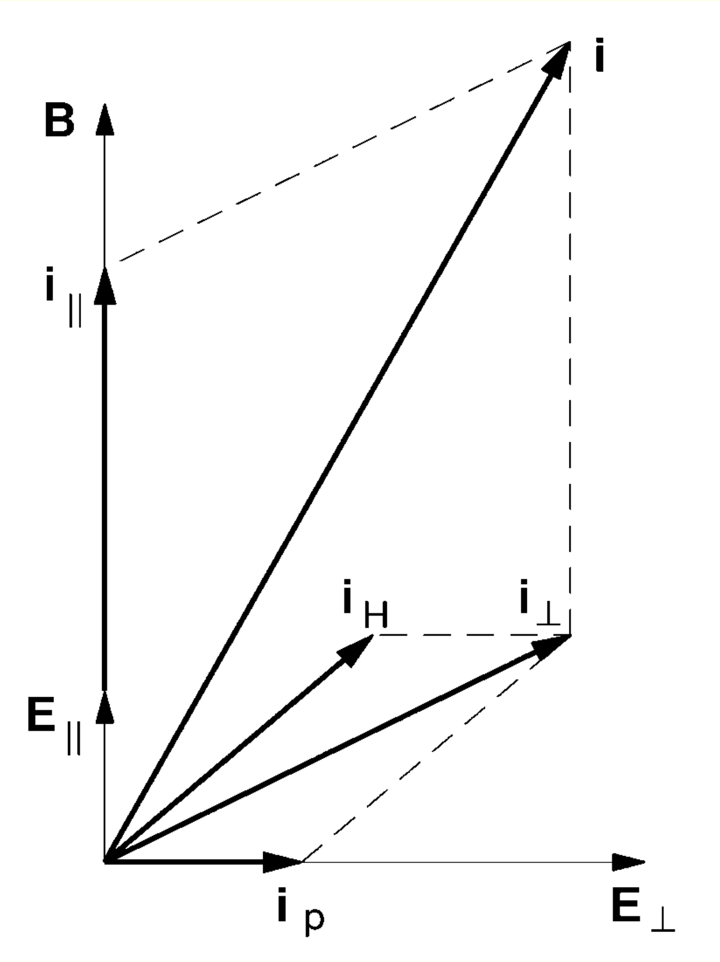
Without collisions



With collisions

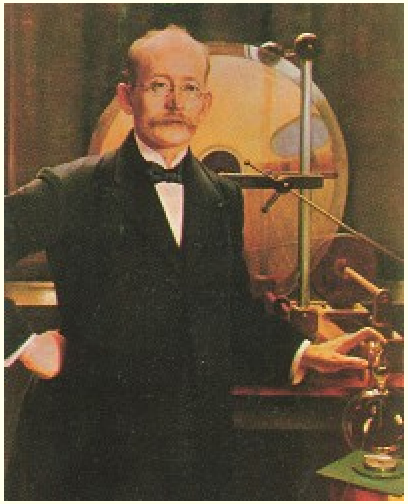


Electric conductivity in a magnetized plasma



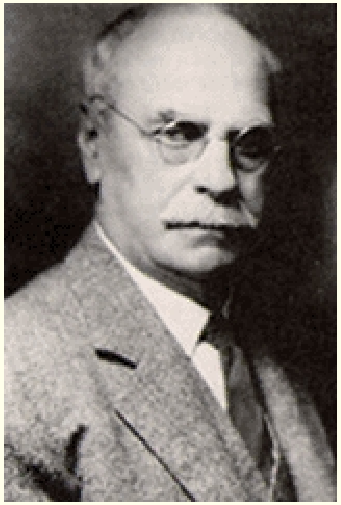
- $i_{//}$ = parallel current
- i_P = Pedersen current
- i_H = Hall current

Birkeland, Hall, Pedersen



Kristian Birkeland
1867-1917

Norwegian scientist



Edwin Hall
1855-1938

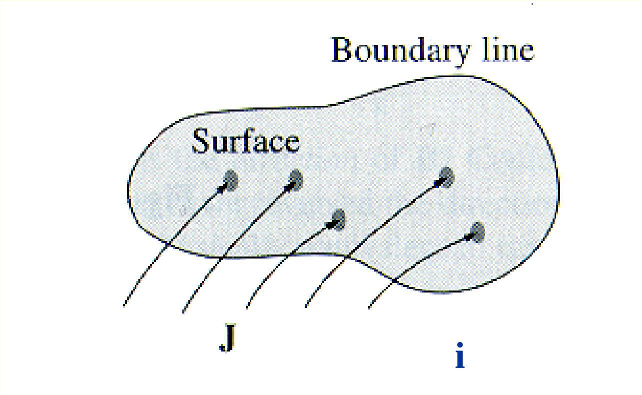
American physicist

Peder Oluf Pedersen
1874-1941

Danish engineer and physicist

Current density

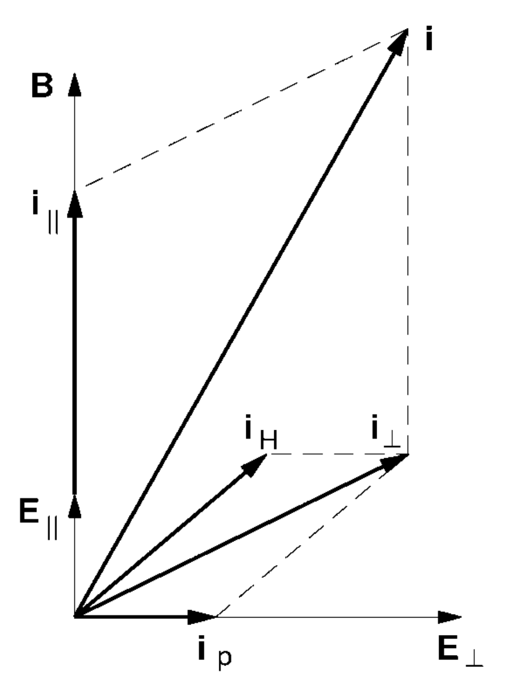
The current density **j** is a vector field with dimension [**i**] = Am^{-2}.



The total current I through the surface S is

$$I = \int_S \mathbf{j} \cdot d\mathbf{S}$$

Electric conductivity in a magnetized plasma II



$$\sigma_P = \sigma_e \frac{1}{1+\omega_{ge}^2\tau_e^2} + \sigma_i \frac{1}{1+\omega_{gi}^2\tau_i^2}$$

$$\sigma_H = \sigma_e \frac{\omega_{ge}\tau_e}{1+\omega_{ge}^2\tau_e^2} - \sigma_i \frac{\omega_{gi}\tau_i}{1+\omega_{gi}^2\tau_i^2}$$

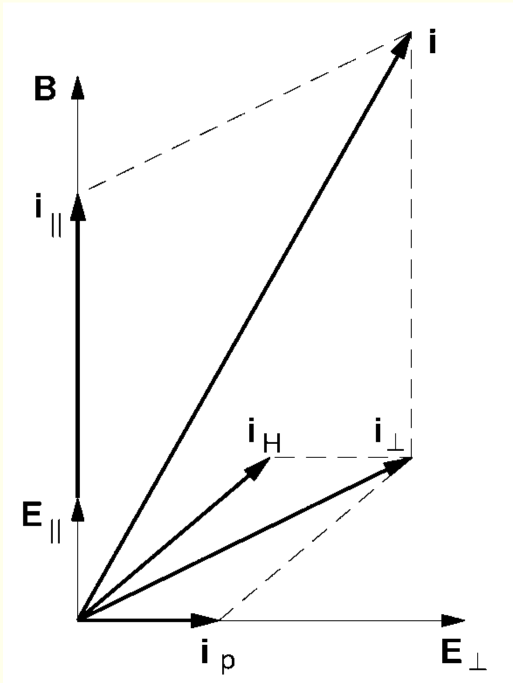
$$\sigma_{//} = \sigma_e + \sigma_i$$

$$\sigma_e = e^2 n \tau_e / m_e \qquad \sigma_i = e^2 n \tau_i / m_i$$

$$i_{//} = \sigma_{//} E_{//}$$

$$\left.\begin{aligned} i_P &= \sigma_P E_\perp \\ i_H &= \sigma_H E_\perp \end{aligned}\right\} \quad \text{or} \quad \mathbf{i}_\perp = \sigma_P \mathbf{E}_\perp + \sigma_H \frac{\mathbf{B}\times\mathbf{E}_\perp}{B}$$

Electric conductivity in a magnetized plasma II

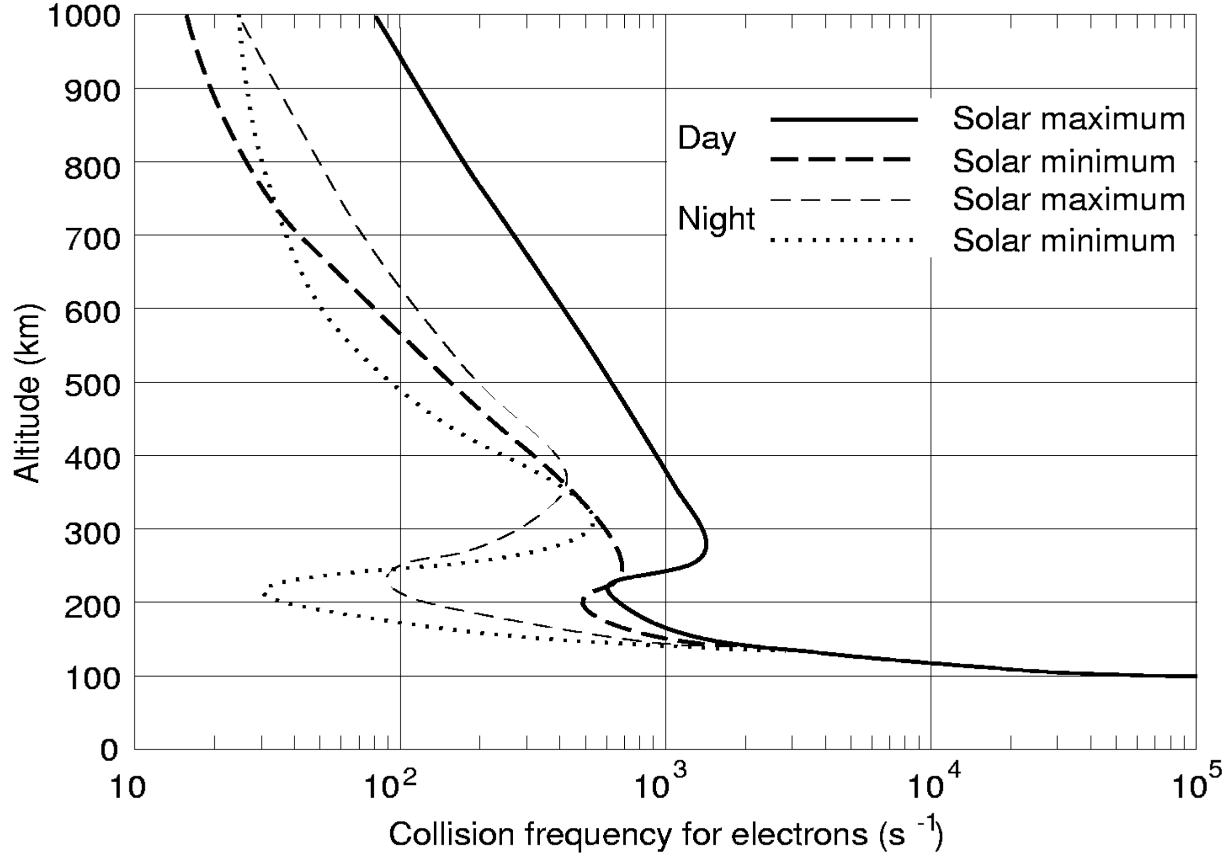


$$\mathbf{i} = \boldsymbol{\sigma} \cdot \mathbf{E}$$

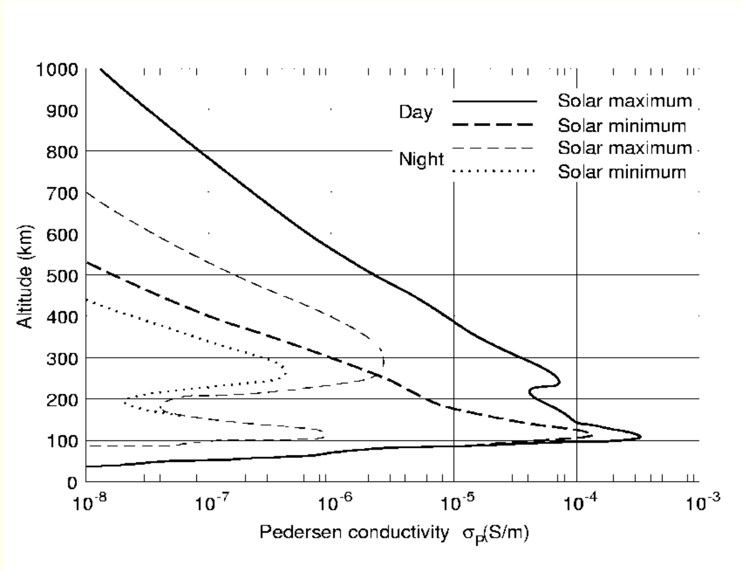
$$\boldsymbol{\sigma} = \underbrace{\begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{//} \end{pmatrix}}_{\textit{conductivity tensor}}$$

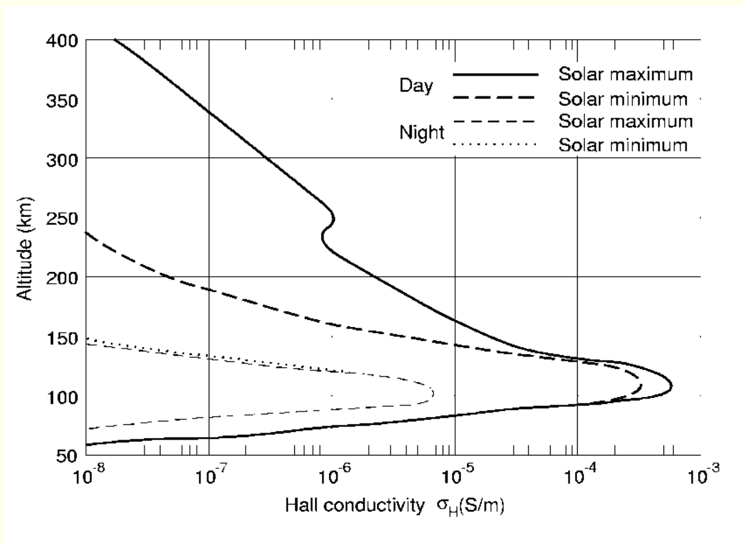
May be formulated as a tensor equation

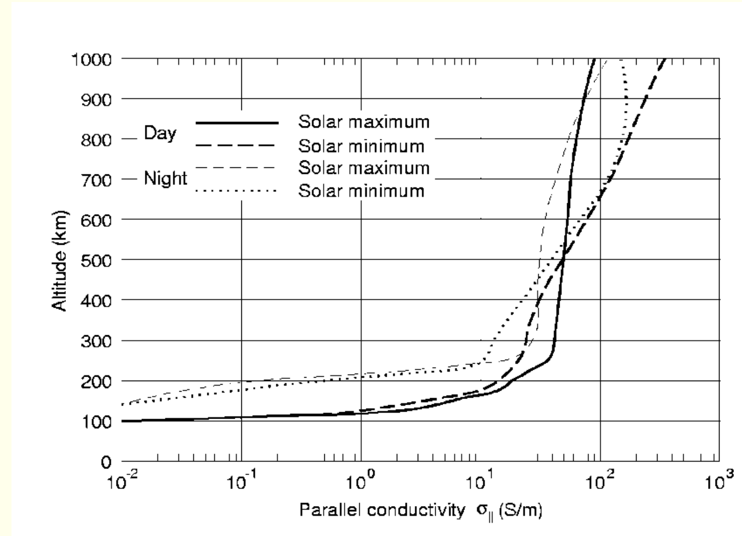
Collisional frequency



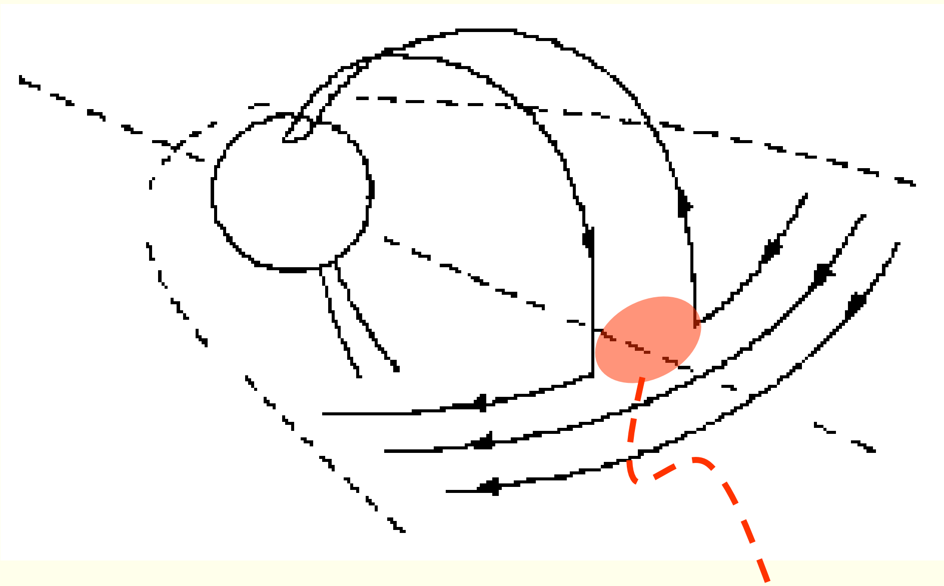
Ionospheric conductivities







Consequence: Birkeland currents



When the conductivity out in the magnetosphere is low, it is easier for the current to close through the ionosphere via currents parallel to the geomagnetic field. Such currents are called *Birkeland* currents.

Exemple: Electric field 700 km above the aurora.

$$\mathbf{E} = E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}}$$

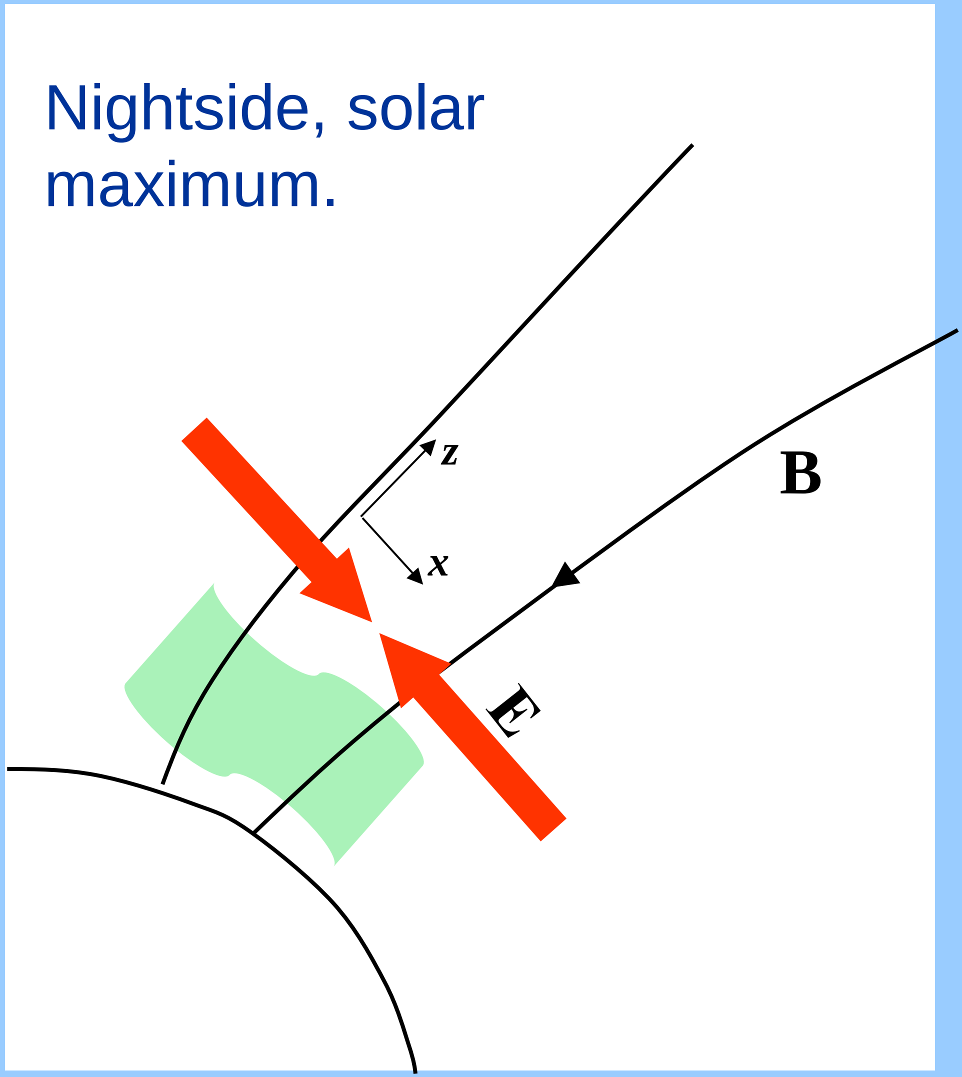
$$E_x = 1\,\mathrm{Vm}^{-1}$$

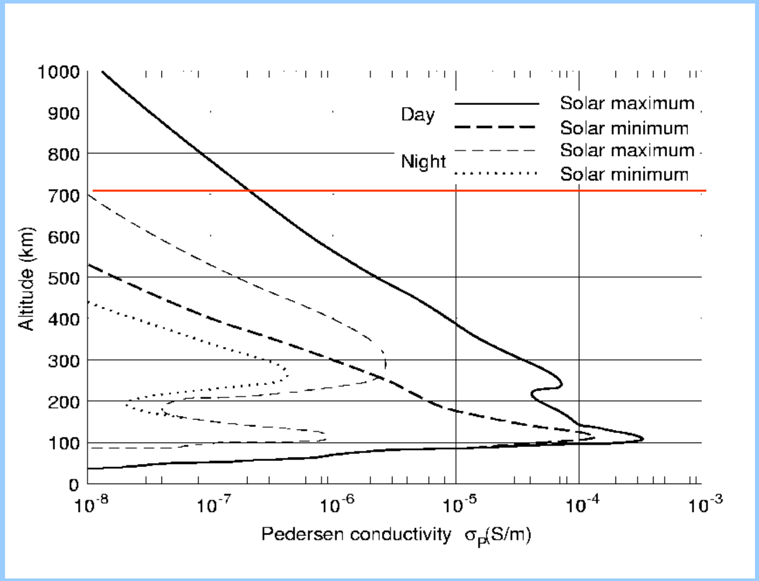
$$E_z = 1\,\mu\mathrm{Vm}^{-1}$$

$$\left.\begin{aligned} j_P = j_x &= 0.01\ \mu\mathrm{Am}^{-2} \\ j_{//} = j_z &= 40\ \mu\mathrm{Am}^{-2} \end{aligned}\right\}$$ Yellow

$$\left.\begin{aligned} j_P = j_x &= 10.0\ \mu\mathrm{Am}^{-2} \\ j_{//} = j_z &= 4.0\ \mu\mathrm{Am}^{-2} \end{aligned}\right\}$$ Red

$$\left.\begin{aligned} j_P = j_x &= 1.0\ \mu\mathrm{Am}^{-2} \\ j_{//} = j_z &= 40\ \mathrm{mAm}^{-2} \end{aligned}\right\}$$ Blue



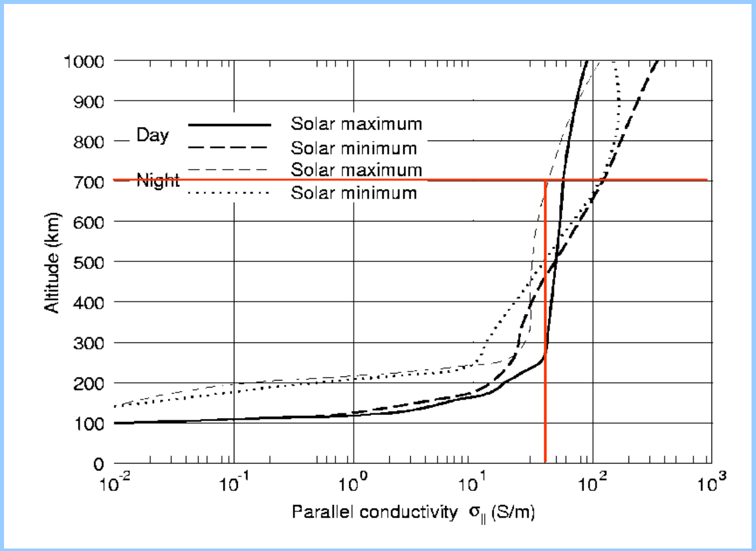


$$\sigma_P \approx 1\cdot 10^{-8}\,\mathrm{Sm}^{-1}$$

$$\sigma_{//} \approx 40\,\mathrm{Sm}^{-1}$$

$$j_P = j_x = \sigma_P E_x = 1\cdot 10^{-8}\,\mathrm{Am}^{-2} = 0.01\,\mu\mathrm{Am}^{-2}$$

$$j_{//} = j_z = \sigma_{//} E_z = 40\cdot 10^{-6}\,\mathrm{Am}^{-2} = 40\,\mu\mathrm{Am}^{-2}$$

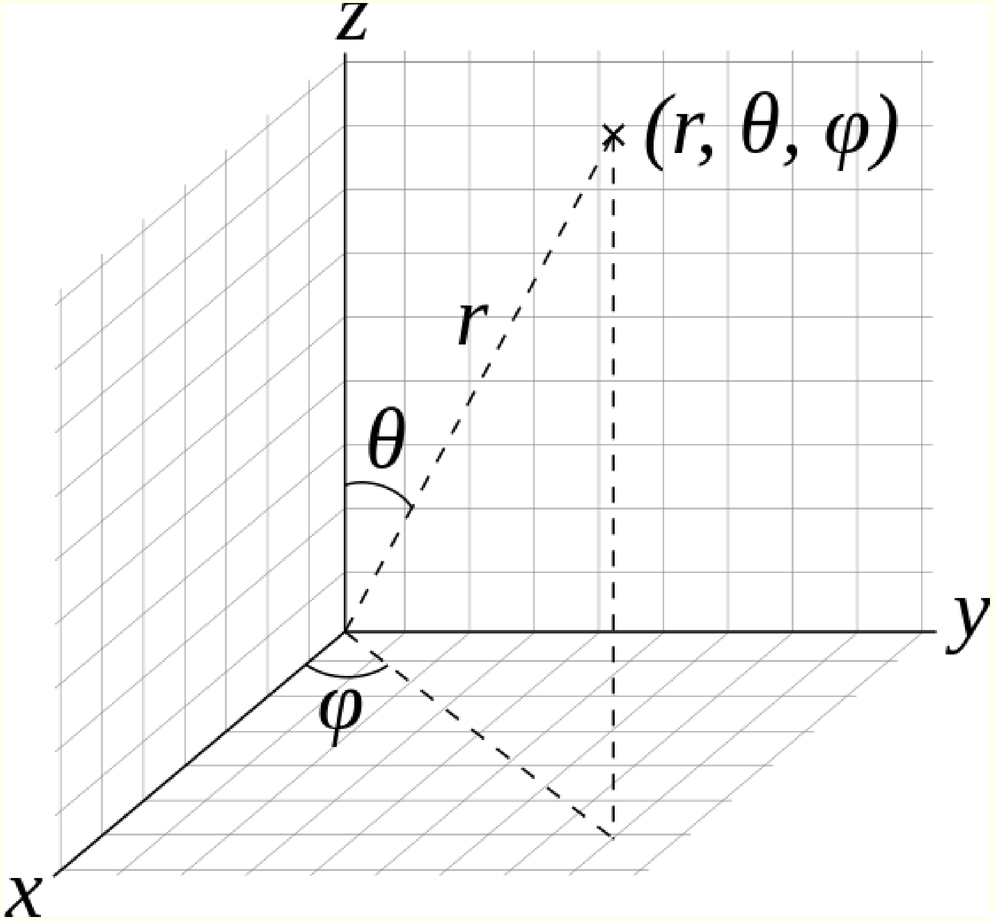


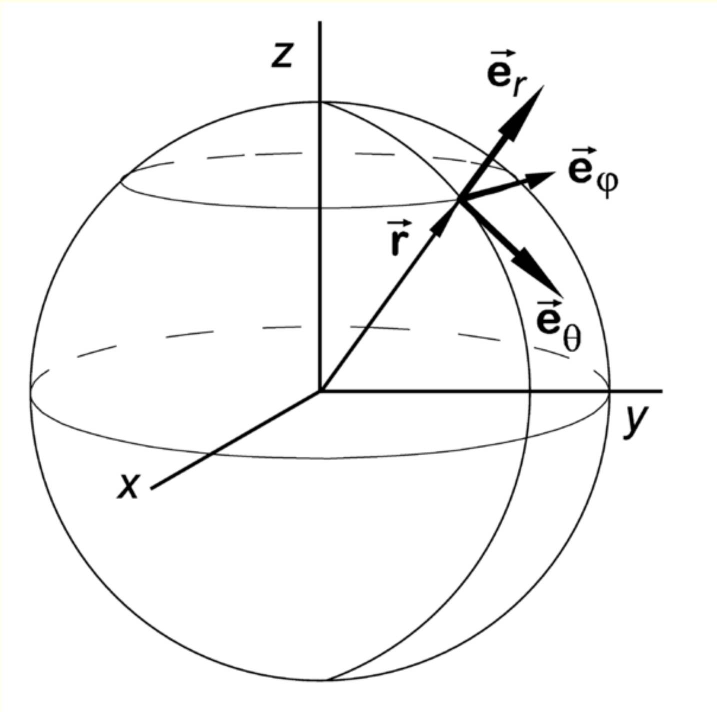
Yellow

How do we define ”the magnetosphere”?

The region in space where the magnetic field is dominated by the geomagnetic field.

Polar (spherical) coordinates





$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arccos\left(\frac{z}{r}\right)$$

$$\varphi = \arctan\left(\frac{y}{x}\right)$$

$$x = r\,\sin\theta\,\cos\varphi$$

$$y = r\,\sin\theta\,\sin\varphi$$

$$z = r\,\cos\theta$$

Geomagnetic field

Approximated by a dipole close to Earth.

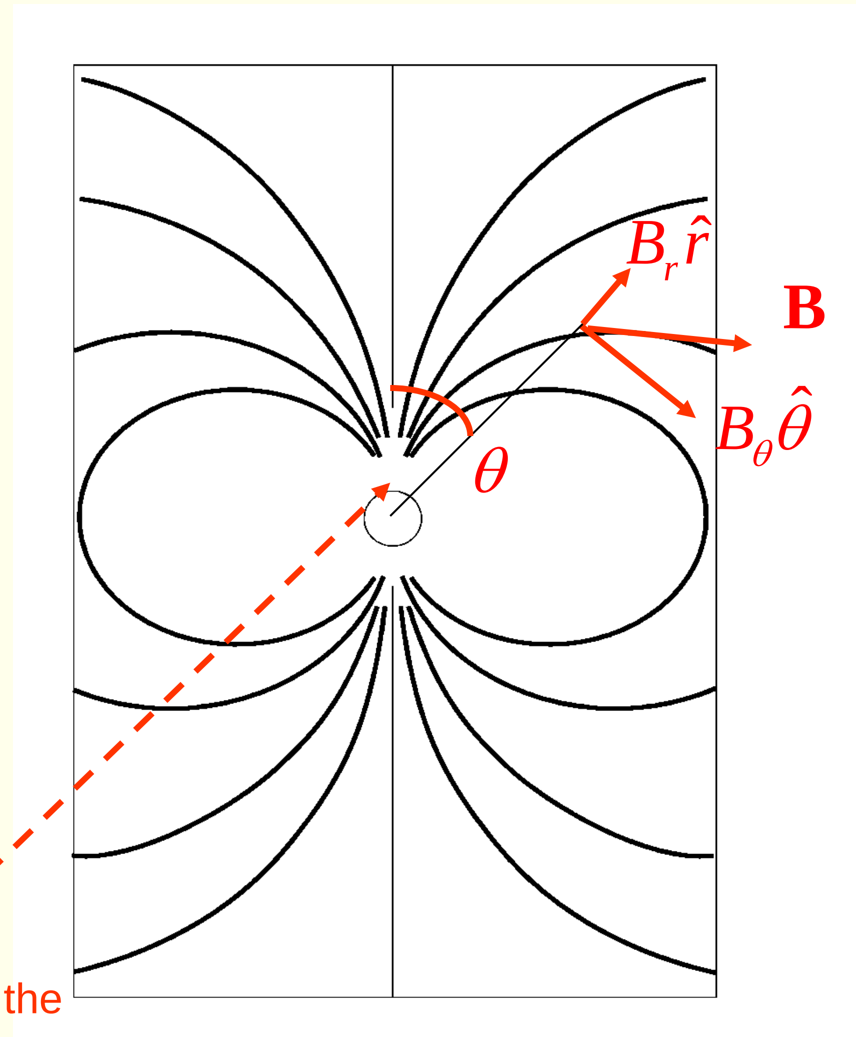
$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos\theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin\theta$$

$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$

magnetic dipole moment

Magnetic field at the "north pole"



Geomagnetic field

Alternative formulation of dipole field

$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos\theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin\theta$$

$$B_r = \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos\theta$$

$$B_\theta = \frac{\mu_0 a}{2\pi} \cdot \frac{1}{2} \cdot \frac{1}{r^3} \sin\theta$$

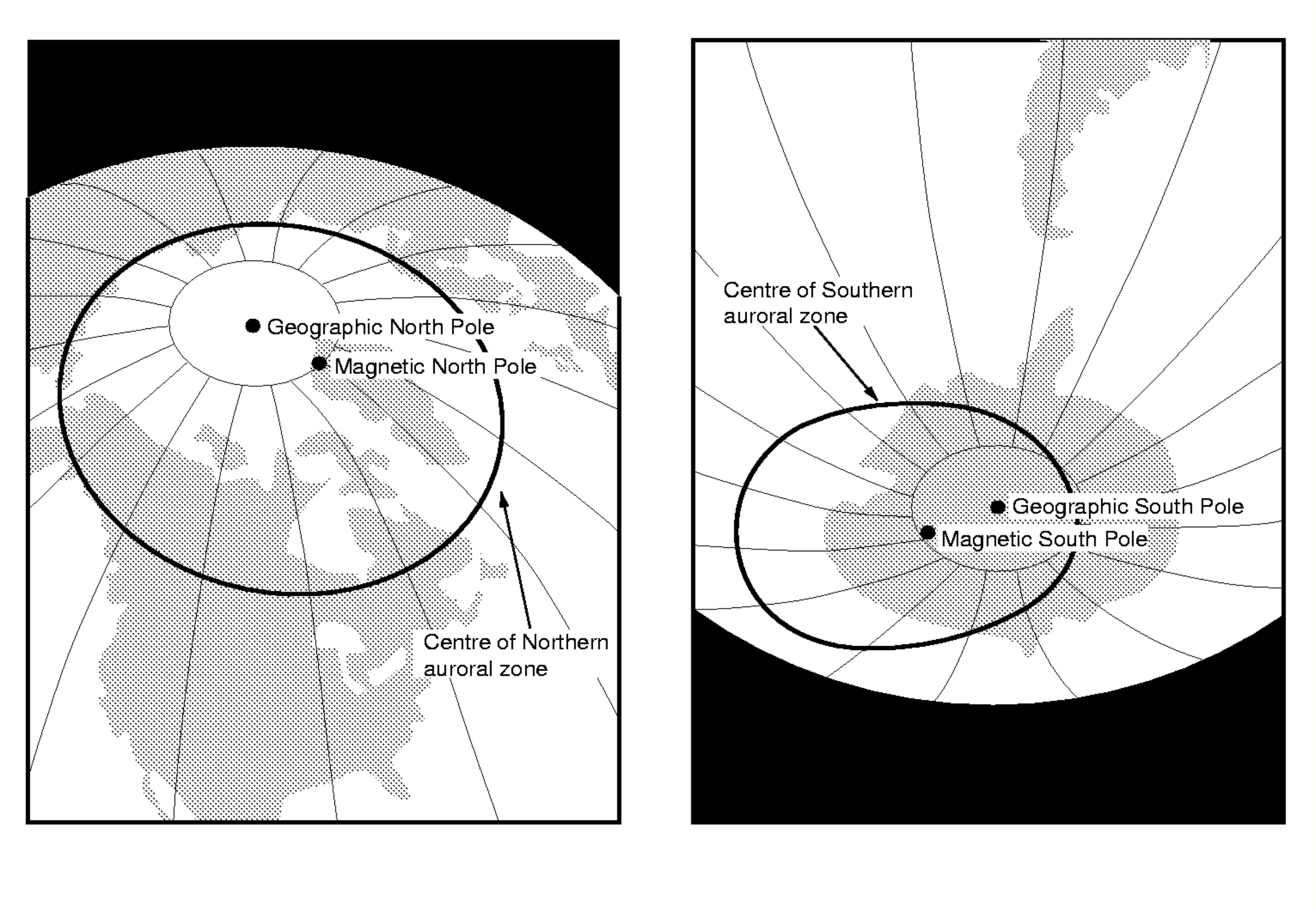
$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$

magnetic dipole moment

Geomagnetic field

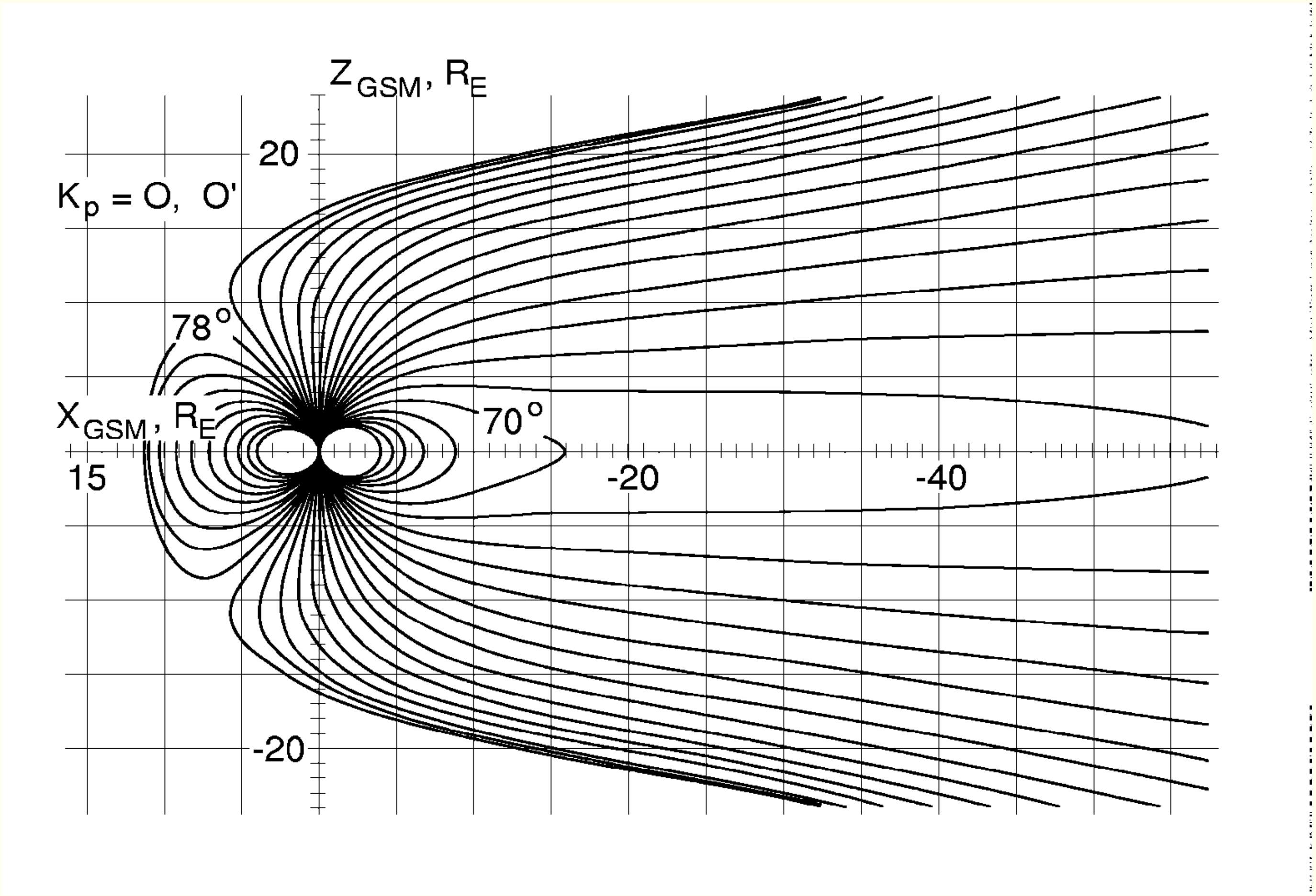
- Angle between dipole axis and spin axis: $\approx 11°$
- The geographic north pole is a magnetic south pole, and vice versa.
- $B_{equator} = 31\ \mu T$,

  $B_{pole} = 62\ \mu T$



Geomagnetic field

Modified by solar wind into tail-like configuration

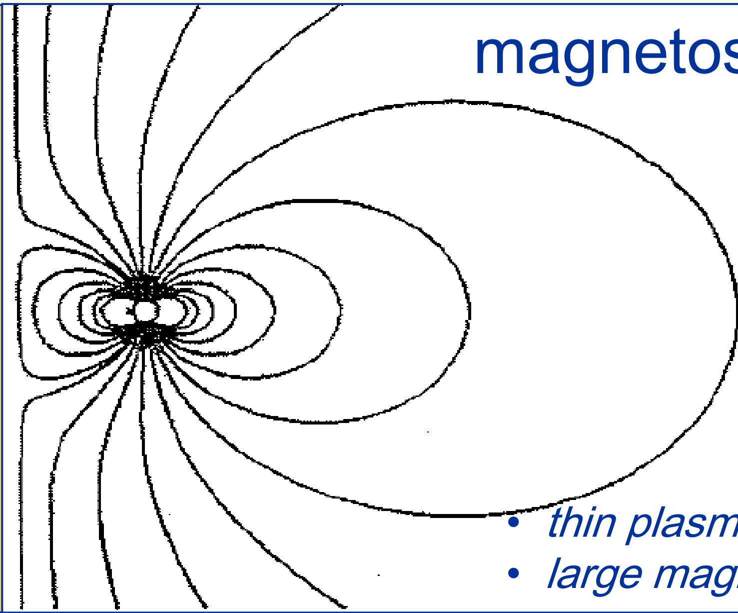


Stand-off distance from pressure balance

solar wind

- *dense plasma with* ***high conductivity***
- *weak magnetic field*

magnetosphere



- *thin plasma*
- *large magnetic field*

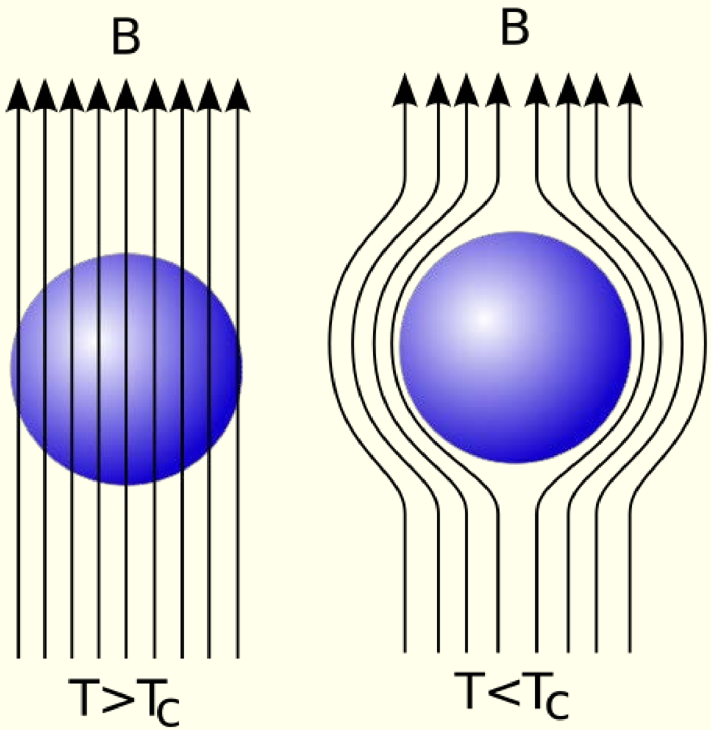
Dynamic pressure:

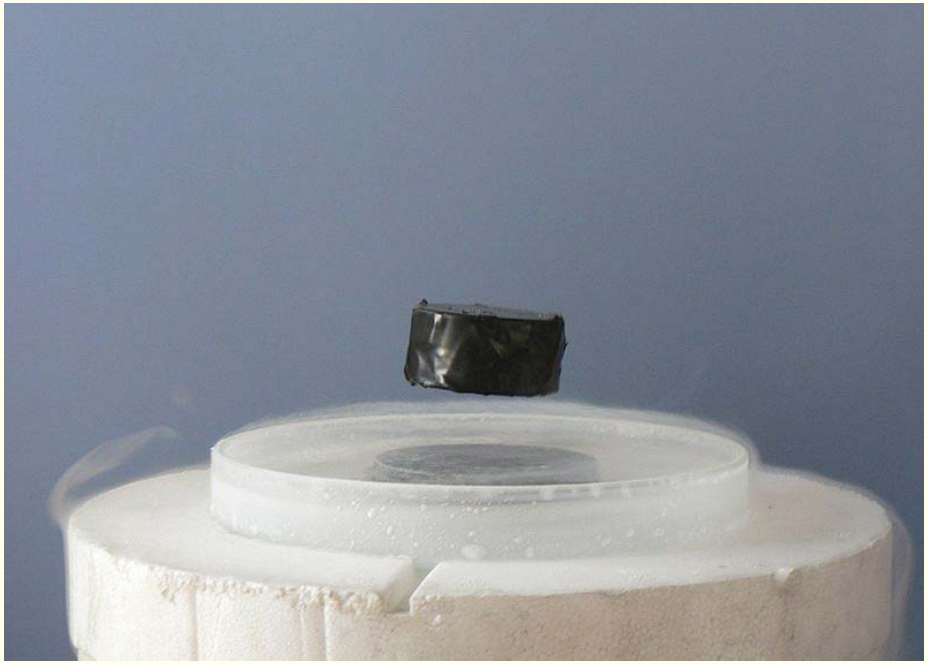
$$p_d = \rho_{SW} v_{SW}^2$$

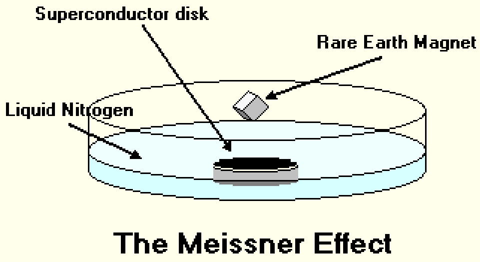
Magnetic pressure:

$$p_B = \frac{B^2}{2\mu_0}$$

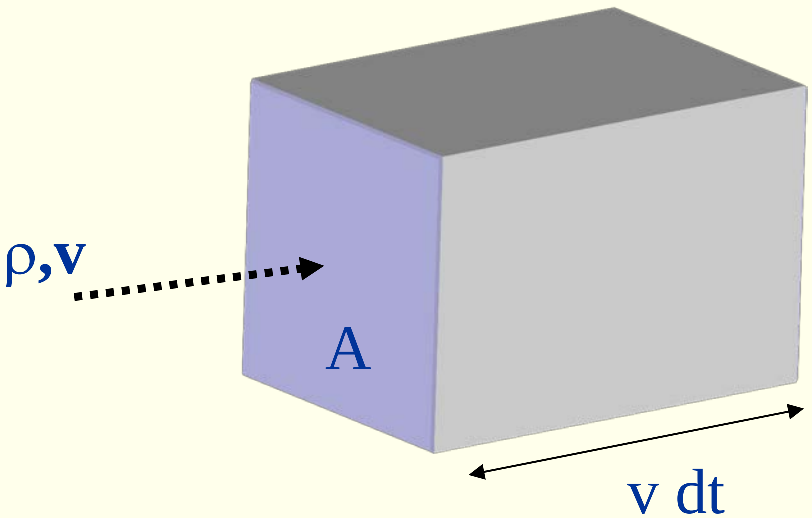
Meissner effect in super-conductors





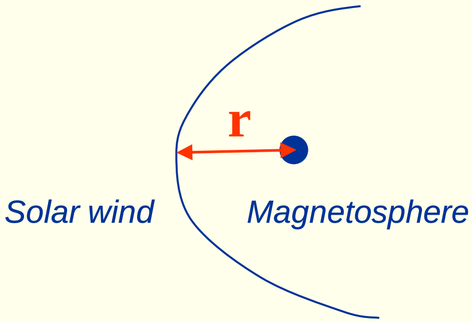


Dynamic (kinetic) pressure



$$p_d = \frac{F}{A} = \frac{d\left(mv\right)}{dt}\frac{1}{A} \approx \frac{\Delta\left(mv\right)}{\Delta t}\frac{1}{A} = \frac{\rho \cdot Av\Delta t \cdot v}{\Delta t A} = \rho v^2$$

Magnetopause "stand-off distance"



Dynamic pressure: $p_d = \rho_{SW} v_{SW}^2$

Magnetic pressure: $p_B = \dfrac{1}{2\mu_0} B^2$

Dipole field strength (in equatorial plane): $B = \dfrac{\mu_0 a}{4\pi} \dfrac{1}{r^3}$

$$p_d = p_B \quad \Rightarrow \qquad \rho_{SW} v_{SW}^2 = \left[\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 \quad \Rightarrow$$

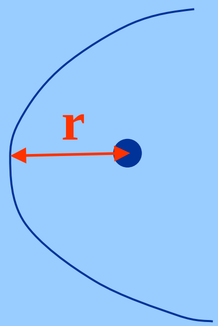
$$r = \left(\frac{\mu_0 a}{4\pi} \right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

$a = 8 \times 10^{22}$ Am^2, $v = 500$ km/s, $\rho_{SW} = 10^7 \times 1.7 \times 10^{-27}$ kg/m^3:

r = 7 R_e (1 R_e = 6378 km)

Standoff distance

v=500 km/s, $\rho_{SW}=10^7 \times 1.7 \times 10^{-27}$ kg/m^3: **r = 7 R$_e$**



$$r = \left(\frac{\mu_0 a}{4\pi}\right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2\right)^{-1/6}$$

How will the standoff distance change if the magnetosphere is hit by a coronal mass ejection (CME)? ($\rho = 10\rho_{SW}$, $v = 1000$ km/s)

Blue	r = 1.8 R$_e$	Yellow	r = 5.8 R$_e$
Green	r = 3.8 R$_e$	Red	r = 9.8 R$_e$

Standoff distance

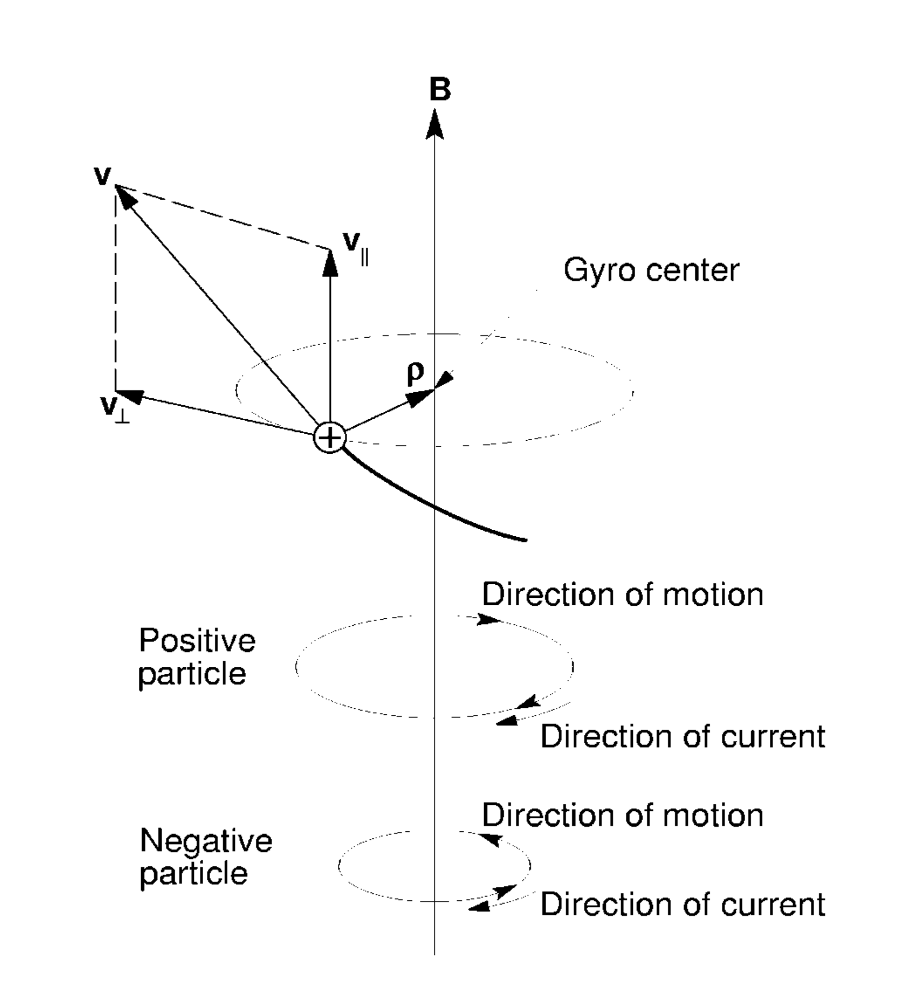
$$r = \left(\frac{\mu_0 a}{4\pi}\right)^{1/3} \left(2\mu_0 10 \rho_{SW} (2v)_{SW}^2\right)^{-1/6} = \left(\frac{\mu_0 a}{4\pi}\right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2\right)^{-1/6} 40^{-1/6}$$

$40^{-1/6} \cdot 7 = 0.54 \cdot 7 = 3.8$

Green

$r = 3.8\ R_e$

Particle motion in magnetic field



gyro radius

$$\rho = \frac{mv_\perp}{qB}$$

gyro frequency

$$\omega_g = \frac{qB}{m}$$

magnetic moment

$$\mu = IA = q\, f_g \pi\, \rho^2 = mv_\perp{}^2/2B$$

Adiabatic invariant

DEFINITION:

An **adiabatic invariant** is a property of a physical system which stays constant when changes are made slowly.

By ’slowly’ in the context of charged particle motion in magnetic fields, we mean much slower than the gyroperiod.

’First adiabatic invariant’ of particle drift:

$$\mu = \frac{mv_\perp^2}{2B}$$