Lecture 3

Induction, The HD Method and Bayesianism

Introduction

- Induction - We study the simplest scientific principle
- HD-Method - We study a more general and advanced scientific principle
- Probabilistic variant - We see how the HD-Method can be modified with probabilistic reasoning. We look at Bayesian methods

Induction

- The basic idea: We make observations and try to see a pattern in them.
- If the observations are many and all agree with the pattern we conjecture that the pattern always applies.
- There are at least two different standardized forms of the method.

Induction: A basic form

- We make observations of objects which all have property A.
- Let us assume that in all observations the objects also have property B.
- We conclude that all objects with property A also have property B.

Induction: Form 2

- This is a more general form.
- Assume that we make observations of situations of a certain type P.
- Then assume all these situations are of type Q.
- We conclude that all situations of type P also are of type Q.

Induction: Logical formulas

If we use logical formulas we can write the first form as

$$\forall x \big(A(x) \rightarrow B(x)\big)$$

More generally we could have some statement F and want to prove that F is always true

$$\forall\, x\, F(x)$$

We then observe instances $F(x_1)$, $F(x_2)$, ...

If they are all true, can we conclude that $\forall x\ F(x)$ is true?

Does induction work?

- Yes, basically. There are however counter-examples.
- The set of observations must be chosen in a sufficiently general way.
- What is the logical basis for induction?
- One motivation for induction is the Principle of Uniformity of Nature (PUN).

PUN

- The idea is that there are *regularities* in nature
- If there are a lot of regularities to be found out there, then there is a big chance that an observed regular pattern can be an instance of a basic regularity
- If this is the case then it seems as if induction could be a logically meaningful tool for finding regularities

A critic

David Hume 1711-1776

There is no scientific ground for induction!

- Induction cannot be proved to be correct using logic.
- Induction cannot be proved using induction (circular reasoning).
- We believe in induction since it seems to work.
- But it cannot be used for scientific proofs.

A solution?

Karl Popper 1902-1994

- Popper claims that he has solved the riddle of induction.
- The solution is that we never really use induction!
- We can never verify hypotheses.
- We can only falsify them.

Can induction generate theories?

- The idea is that we can see patterns and we can generalize them into theories.
- By using the induction principle we can "prove" the theory.
- But can this be done? There are at least three objections.
- The fact (if it is a fact) that we must first have a theory before we can make observations.
- Goodman's paradox
- Underdetermination

Goodman's paradox

- It can be stated in several essentially equivalent forms. This is one:
- An object is *grue* if it has been observed and was green or has not been observed (yet) and is blue.
- All observed emeralds have been grue.
- Should we conclude that all emeralds are grue?
- Another way of defining grue is that x is grue if observed before September 26th 2016 and was green or x will be blue after the same date.
- So emeralds are grue?

Underdetermination

- To each set of observations there are always different theories that fit the data.
- Take the sequence 1,2,3,4,5 …. (five observations)
- One hypothesis is that the sequence is 1,2,3,4,5,6,7,8,…
- Another hypothesis is that the sequence is 1,2,3,4,5,5,5,5, …
- Observation (induction) confirms both hypotheses!
- Perhaps we should chose the simplest theory (Occam's razor). But will that always give the best result?
- Goodman's paradox is an example of this problem.

So what's the problem?

- An obvious conclusion is that induction should be used with a certain measure of *common sense.*
- The problem with common sense is that it is impossible (?) to *formalize* it.
- If that is so, it seems impossible to give an algorithmic description of scientific procedure (using induction).
- A simple way of viewing this problem is that we use induction since it is successful - A pragmatic view!
- Should we be satisfied with this?

In spite of this ...

- It seems as if it is impossible not to use induction, at least in everyday situations
- But what should we do in science?
- We will describe a method that is a sort of development of the induction method.

The two methods of science

- In science we work both with deductions and observations.
- In mathematics it is almost always deductions.
- In physics we work with both methods.
- In social sciences and humanities the situation is more uncertain. But in a way observations must be used.

Is there a general scientific method?

- Experimental science has at least four different components:
- To set up hypotheses.
- To verify the hypotheses with logic.
- To evaluate the hypotheses by making observations.
- To perform experiments that generate observations.

Is there a general scientific method?

- A suggestion: It could be the Hypothetico-Deductive Method.
- It is certainly used in physics and chemistry.
- In a specialized sense it is used in mathematics.
- It seems as if it used sometimes in Social Sciences.

Carl Hempel 1905-1997

The HD Method

- Let us assume that we have a hypothesis H. We want to know if it is true or not.
- H can be a single fact or a general law.
- We have different observations E1, E2, …, En.
- (The observations can be generated by an experiment. They can also exist before H.)
- Does the observation *confirm* or *disconfirm* the hypothesis H?
- The HD Method is a way to find an answer to that question.

The HD Method used for falsification

- We have a hypothesis and want to show that it is false.
- We have a set of observations E1, E2, …, En.
- Assume that there is an observation Ei such that H => not Ei.
- Then Ei falsifies H.

A CS-example

- Let H be ”There is an algorithmic way of solving all kinds of problems”
- A logical consequence would be E: ”There is an algorithmic way of solving the *Halting problem”.*
- But E is false! (as Turing showed)
- So H is falsified!

An example from chemistry

Scheele

Lavoisier

Chemistry

- Great steps are taken in the 18th century.
- At the beginning of the century almost nothing is known about atoms and chemical elements. There are only two known gases: Air and carbon dioxide.
- Oxygen is discovered. (Scheele/Priestley).
- Hydrogen is discovered (Cavendish). It is discovered that water is composed of hydrogen and oxygen.
- Lavoisier disproves the so called phlogiston theory of combustion.

Chemistry II

- John Dalton discovers the atom.
- Berzelius describes the composition of elements.
- He creates the modern chemical notation for substances.
- Mendeleyev creates the periodic table.

The Phlogiston Theory

Antoine Lavoisier

The Phlogiston Theory: When an object is burning it is phlogiston leaving the object.

The Phlogiston Theory was falsified by Lavoisier.

The falsification of The Phlogiston Theory

- Let H be The Phlogiston Theory.
- A consequence of The Phlogiston Theory must be that burning objects get lighter.
- But we can find certain metals that become heavier after burning. Let us call this observation E.
- Since H => not E, we have falsified H.

Supporting hypotheses

- It might not be possible to prove H => not E *directly.* We might need a supporting hypothesis A such that H&A => not E.
- A could be all our *background knowledge.*
- Eg: H = "The illness is caused by bacteria".
- A = “Penicillin kills bacteria".
- E = “The illness is not cured by penicillin".

Ad hoc hypotheses

- Supporting hypotheses should be well established and secure. Sometimes they are not:
- If H => not E and E has been observed, someone might want to *save* H.
- This can maybe be done by assuming that the implication has the form (H&A=> not E). Then we substitute A1 for A and get (H&A1 => E).
- If A1 seems very unlikely, if considered by itself, we call A1 an ad hoc hypothesis.

Example: The Phlogiston Theory

- Let H = The Phlogiston Theory.
- E was the observation of a metal getting heavier after burning.
- We can argue that the implication is H&A => not E, where A is "The phlogiston has positive weight".
- We can replace A with A1 = "The phlogiston in the metal has negative weight". Then H&A1 => E!
- But how probable is A1?

A more critical example: Uranus and Neptune

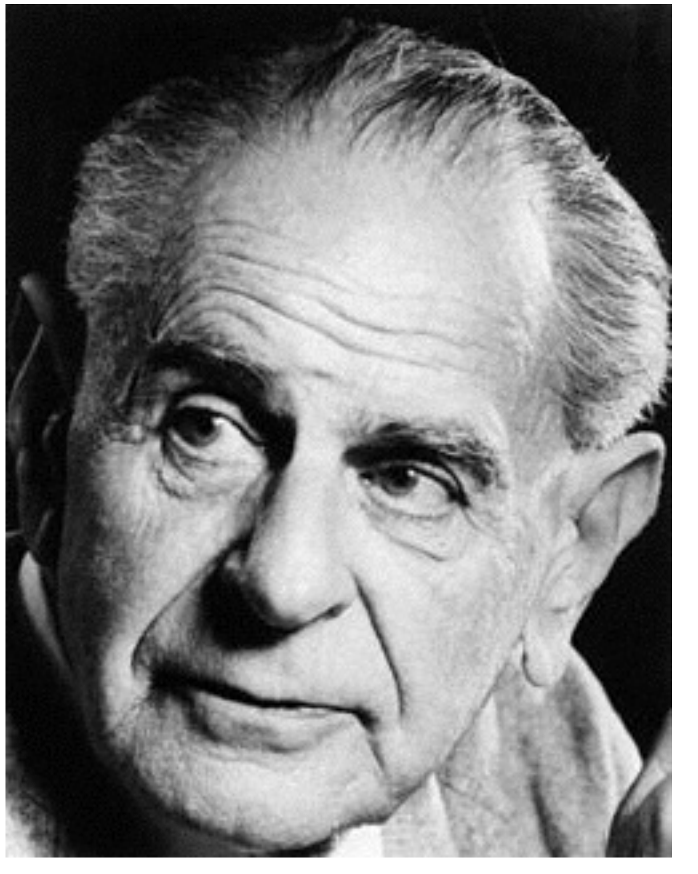
- The planet Uranus was discovered with telescope in 1781.
- In the beginning of the 19th century it was observed that Uranus didn't move in the way Newton's laws predicted.
- Call this observation E and Newton's laws H. Then we have H => not E.
- So Newton's laws were falsified!?
- But wait! The implication is really H&A => not E where A, amongst other things, contained the statement that there are seven planets.
- But if we replace A with A* where A* says that there are unknown planets we don't get a falsification.
- and in 1846 Neptune (the eight planet) was observed!
- So A* wasn't really an ad hoc hypothesis (or was it?).

The HD Method for falsification. Summary.

- We have a hypothesis and want to test if it is false.
- We use a supporting hypothesis A and deduce H&A => not E.
- We then observe E.
- We have then falsified H.

This is what Popper believed in

- The HD-Method can be used for falsification
- But in some cases we feel that a theory can be *confirmed* by positive experiments
- Popper denied this but the logical positivists thought so
- A simple example is induction
- Now let's look at a more advanced form of induction



The HD Method used for verification

- Assume that we have a hypothesis H and observations E1, E2, ... , En.
- When can we say that the observations confirm H?
- One possibility is that E1&E2&…&En => H. In that case H is verified.
- But let us assume that this is not the case.

Observations that confirm

- We have H and E1, E2, ..., En.
- Assume that they are all rather improbable.
- Assume that we have a hypothesis A that we already believe is true and that H&A => E1&E2&…&En.
- Then the observations confirm H.

Arguments for and against a hypothesis

- Assume that we have observations E1, E2, ... ,En and a hypothesis H.
- Some of the observations confirm H if they together with a supporting hypothesis Ai give H&Ai => Ei.
- Other observations disconfirm H if they together with a supporting hypothesis Bk H&Bk => not Ek. Observe that we don't know if Bk is true. We have not falsified H with absolute certainty.

Making a decision

- We form a type of weighted average. If the supporting hypotheses Ai are more natural than the Bk we say that H is strengthened, otherwise it is weakened.
- This works best if we can use probability theory.

A third form of the HD-Method. To choose between hypotheses.

- If we have a set of observations E1, E2, … ,En and a hypothesis H we can try to find supporting hypotheses Ai such that H&Ai => Ei for all i.
- If another hypothesis H* can do the same thing with more natural supporting hypotheses Bi (that is H*&Bi => Ei), then we say that H* is a better hypothesis.

We use probability

- The previous methods were qualitative.
- We now try to do a probabilistic analysis of when observations confirm a hypothesis.
- So we have this problem: Given a hypothesis H and an observation E, when can we say that the observation confirms H?

An important formula

Thomas Bayes 1702-1761

He found an important formula connecting different types of conditional probabilities.

This formula is the basis for so called Bayesian Statistics.

Bayes' formula

Let H be an hypothesis and e an observation.
Remember that $P(A|B) = \frac{P(A\&B)}{P(B)}$.
We assume that $P(e|H)$ and $P(e|\neg H)$ can be estimated.
We know that $P(e) = P(e|H)P(H)+p(e|\neg H)P(\neg H)$.
Then $P(H|e) = \frac{P(e|H)P(H)}{P(e)}$

$= \frac{P(e|H)P(H)}{P(e|H)P(H)+p(e|\neg H)P(\neg H)}$

Example: Test of medicine

- Let us assume that we have a certain medicine that is supposed to cure a disease. Call the hypothesis that the medicine works H.
- We make an observation. It is that a sick Patient gets well after been given the medicine. Call this observation E.
- Can we decide to what degree E confirms H?

Test of medicine II

- We want to find $P(H|E)$
- We need to estimate some probabilities in Bayes' formula.
- $P(E|H) = 1$ seems reasonable.
- $P(E|\neg H)$ is more complicated. Let us assume that we have the probability 0.25
- $P(H)$ is even more complicated. Let us start with the guess $P(H) \; = 1$
- That gives us $P(H|E)$= 0.8

Test of medicine III

- Let us now assume that we have the guess $P(H) = 0.1$
- That gives us $P(H|E) = 0.36$
- In both cases we find that $P(H|E) > P(H)$
- We can use this this relation to define *strengthening*.

A CS-example

- Assume that we have a computer running a program. We observe errors in the output. Could it be a hardware error?
- Let us call this hypothesis (hardware error) H.
- Let us assume that we have an observed error e.
- We estimate that the probability for this type of error is $P(e) = 0.1$
- We estimate $P(e|H) = 0.4$ and $P(H) = 0.05$ Then Bayes gives us a new estimate

$$P(H) = P(H|e) = 0.2$$

- This observation *strengthens* H.

A CS-example

- Let us look at the hypothesis $\neg H$
- We estimate $P(\neg H) = 0.95$
- We must have $P(e|\neg H) = 0.08$ (Why?)
- Then Bayes gives us a new estimate
- $P(\neg H) = P(\neg H|e) = 0.76$
- This observation has weakened $\neg H$

Definition of strengthening

- We have a hypothesis H and an observation E.
- We say that E strengthens H if $P(H|E) > P(H)$.
- and we say that it weakens H if $P(H|E) < P(H)$.

Other ways of putting it

- We assume that $0 < P(E) < 1$.
- E strengthens H if $P(E|H)/P(E) > 1$, i.e. $P(E|H) > P(E)$.
- E weakens H if $P(E|H)/P(E) < 1$, i.e. $P(E|H) < P(E)$
- Or we can say it like this:
- E strengthens H if $P(E|H) > P(E|\neg H)$
- E weakens H if $P(E|H) < P(E|\neg H)$

Different views of probability

There are three different ways in which probability can be interpreted.

- Axiomatic: We postulate a set of equally probable *elementary events.* Every other event is expressed as a combination of these events.
- Frequency: The probability for an event is roughly the frequency with which the event will occur in repeated experiments.
- Subjective: We give a measure for the ”probability” of events without giving a formal basis for this measure.

It seems as if the Bayesian view of verification relies on an extensive use of subjective probability. This is a problem since subjective probability is not universally accepted as a stringent scientific concept.