SF 2720, Homework assignment 2 – due Tuesday 11. 10. 2016.

(1) (Exercise 3.9, [BV]). Determine whether the maps f and $g : \mathbb{R} \to \mathbb{R}$ are topologically conjugate for
  a). $f(x) = x$, $g(x) = x^2$;
  b). $f(x) = x/3$, $g(x) = 2x$;
  c). $f(x) = 2x$, $g(x) = x^3$.

(2) (Exercise 3.16, [BV]). Compute the topological entropy of the endomorphism of the torus $\mathbb{T}^2$ induced by the matrix

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}.$$

(3) (Exercise 3.18, [BV]). Let $f : X \to X$ be a continuous map of a compact metric space, and take $k \in \mathbb{N}$.

This exercise shows that $h(f^k) = kh(f)$.

  a). Writing $d_{n,f} = d_n$ and $N_f(n, \varepsilon) = N(n, \varepsilon)$, show that $d_{n,f^k}(x, y) = d_{nk,f}(x, y)$, and thus

$$N_{f^k}(n, \varepsilon) \leq N_f(nk, \varepsilon).$$

  b). Conclude that $h(f^k) \leq kh(f)$.
  c). Show that

$$\lim_{\varepsilon \to 0} \limsup_{n \to \infty} \frac{1}{n} \log N_f(nk, \varepsilon) \leq h(f^k).$$

  Hint: By the uniform continuity of f, given $\varepsilon > 0$, there exists a $\delta(\varepsilon) \in (0, \varepsilon)$ such that $d_k(x, y) < \varepsilon$ when $d(x, y) < \delta(\varepsilon)$. Hence it follows from

$$d_{nk,f}(x, y) = \max\{d_k(f^{ik}(x), f^{ik}(y)) : 0 \leq i \leq n - 1\}$$

  that

$$d_{n,f^k}(x, y) \geq \delta(\varepsilon) \ \text{ when } \ d_{nk,f}(x, y) \geq \varepsilon,$$

  which yelds the inequality

$$N_f(nk, \varepsilon) \leq N_{f^k}(n, \delta(\varepsilon))$$

  d). Use inequality (3.23) and Theorem 3.4 [BV] to conclude that $h(f^k) \geq kh(f)$.

(4) (Exercise 8.3.7, ["small" KH]). Consider the closed unit disk in $\mathbb{R}^2$ and the map f_λ on it defined in polar coordinates by $f_\lambda(re^{i\theta}) = \lambda re^{2i\theta}$, where $0 \leq \lambda \leq 1$.
  a). Show that $h_{top}(f_1) \geq \log 2$;
  b). Show that $h_{top}(f_\lambda) = 0$ for $\lambda < 1$.