

Homework 3

Read Chapters 1 and 2 in Walliman: Research Methods.

Induction in Mathematics

Mathematics is similar to science when it comes to research activities (observations, experiments, models, hypotheses, theories). A simple example:

- OBSERVATIONS: $1 + 3 = 4, 1 + 3 + 5 = 9, 1 + 3 + 5 + 7 = 16$
- EXPERIMENT: `for n in range(10): sum(range(1,2*n+1,2))`
 $0, 1, 4, 9, 16, 25, 36, 49, 64, 81$
- MODEL: $1 + 3 + 5 + \dots + (2n - 1) = n^2$ for $0 < n < 10$
- HYPOTHESIS: $1 + 3 + 5 + \dots + (2n - 1) = n^2$ for $0 < n < \infty$

In science, the hypothesis, strengthened by the observations and the experiment (and also by the simple and plausible model) would then be temporarily accepted until falsified by a new observation. In mathematics, it would be called a *conjecture* until a proof has been presented.

One proof method is mathematical induction, which takes scientific induction a step further. The format is the following.

- HYPOTHESIS: $P(n)$ is true for all $n > 0$
- BASE CASE: $P(1)$ is true
- INDUCTION STEP: $P(k) \Rightarrow P(k + 1)$ for any k

Task: Try to write down a correct induction proof for our hypothesis.

Please turn over, homework continues on next page...

If the observations do not immediately suggest a model, you may consult the On-Line Encyclopedia of Integer Sequences. Let us say that you want a similar formula for the sum of squares $1 + 4 + 9 + \dots + n^2$. Make a small number of observations like

$$1 = 1$$

$$1 + 4 = 5$$

$$1 + 4 + 9 = 14$$

etc and search for the sequence $1, 5, 14, \dots$ in the database.

Task: What is the model you find? Did you get more interesting information?

Benford's law

"Thirty percent of all numbers start with the digit 1." This statement is supposed to be true for collections of numbers with widely varying orders of magnitude, like populations of cities or byte sizes of files.

Task: Test the law by generating $2, 4, 8, 16, \dots$ and printing the first digit statistics.

Task: To demonstrate your shell script skills you may also want to check the law on all byte sizes in your computer. (Not compulsory!)

Task: Do you consider the law a hypothesis, a conjecture or a mathematical theorem?

Handing in your solution

Please save your solution as a pdf file and hand it in on the peergrade page. Do not write your name in the pdf file.

Peer grading

You will be asked to review the homework of three other students. Your own solution will also be checked in this way.

Your teacher will read your submission and report the result in rapp.