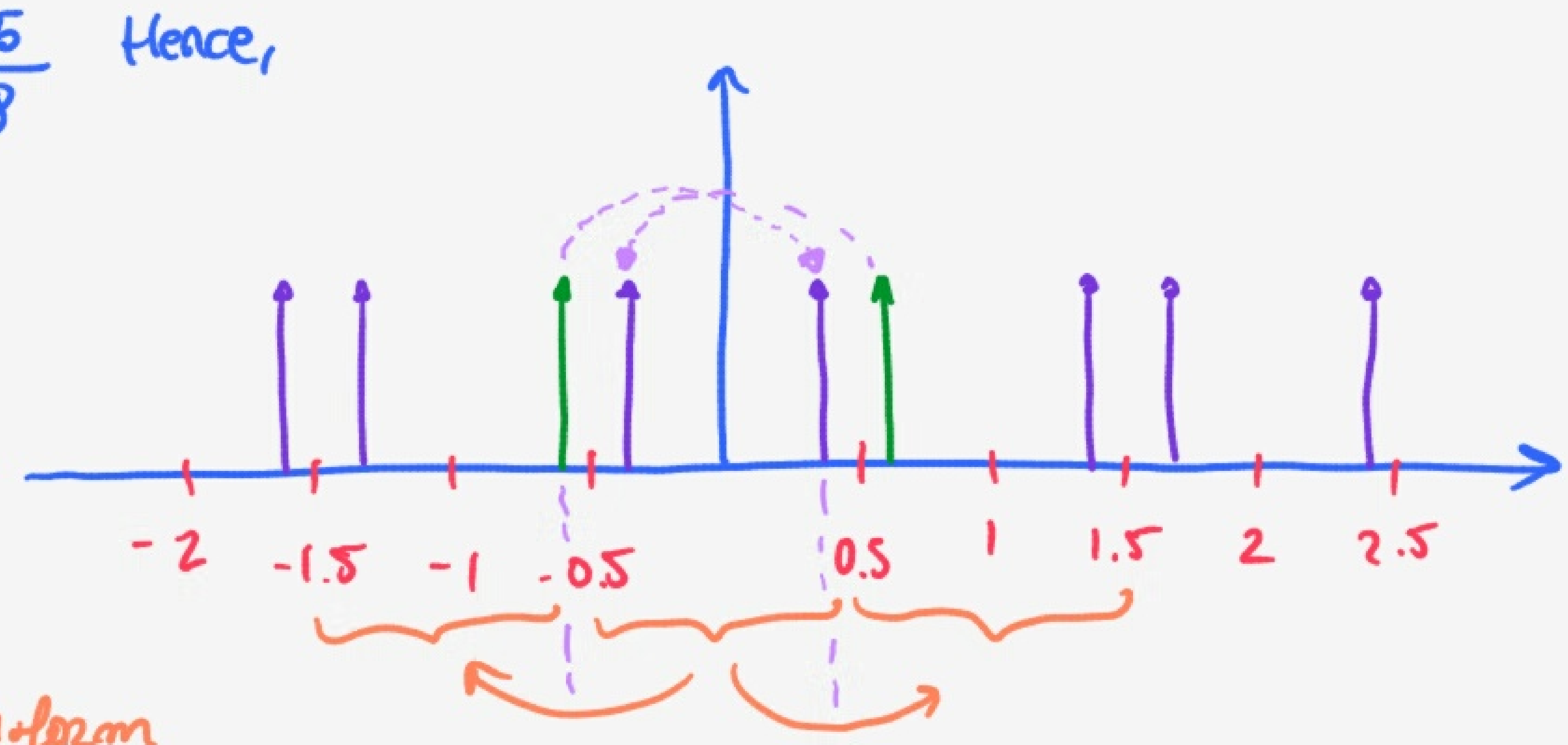
10.1 $f_0 = 625\,Hz$ $\quad (625 = 25^2 = 5^4)$

sampled at $f_s = 1000\,Hz$ $\quad (1000 = 5^3 2^3)$

Determine the normalized frequency between $[0, 1/2]$

$$\frac{625}{100} = \frac{5^4}{5^3 2^3} = \frac{5}{8}$$ Hence,



The fourier transform of any discrete time signal is periodic, where the interval $[-0.5, 0.5)$ repeats itself.

1

since it's periodic with period 1

$$1 - \frac{5}{8} = \boxed{3/8}$$

10.3 $X(t)$ $\quad r_x(k) = R_0 \delta(k)$

(a) Calculate $r_{YZ}(k+n, n)$

$$r_{YZ} = E[\,Y(n+k) Z(n)] = E\left[\int_{-\infty}^{\infty} h_1(\tau_1)\, x((n+k)T_s - \tau_1)\, d\tau_1 \int_{-\infty}^{\infty} h(\tau_2)\, x(nT_s - \tau_2)\, d\tau_2 =\right.$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[x((n+k)T_s - \tau_1)\, x(nT_s - \tau_2)]\, h_1(\tau_1)\, h_2(\tau_2)\, d\tau_1 d\tau_2 =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r_x(kT + \tau_2 - \tau_1)\, h_1(\tau_1)\, h_2(\tau_2)\, d\tau_1 d\tau_2 =$$

$$= R_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(\underbrace{kT + \tau_2 - \tau_1}_{d(\tau_2 - (\tau_1 - kT))})\, h_1(\tau_1)\, h_2(\tau_2)\, d\tau_1 d\tau_2 = R_0 \int_{-\infty}^{\infty} h_1(\tau_1)\, h_2(\tau_1 - kT)\, d\tau_1 =$$

$\rightarrow$ Parseval's $\Rightarrow$ $$R_0 \int_{-\infty}^{\infty} H_1(f) \left(H_2(f)\, e^{-j2\pi f kT}\right)^* df = R_0 \int_{-\infty}^{\infty} H_1(f)\, H_2^*(f)\, e^{j2\pi f kT}\, df$$

(b) f_1, f_2, f_3 so that they become uncorrelated

<u>Note that</u>: $$m_Y = E\left[\int_{-\infty}^{\infty} x(nT - \tau)\, h_1(\tau)\, d\tau\right] = \int_{-\infty}^{\infty} E[x(nT - \tau)]\, h_1(\tau) = 0$$

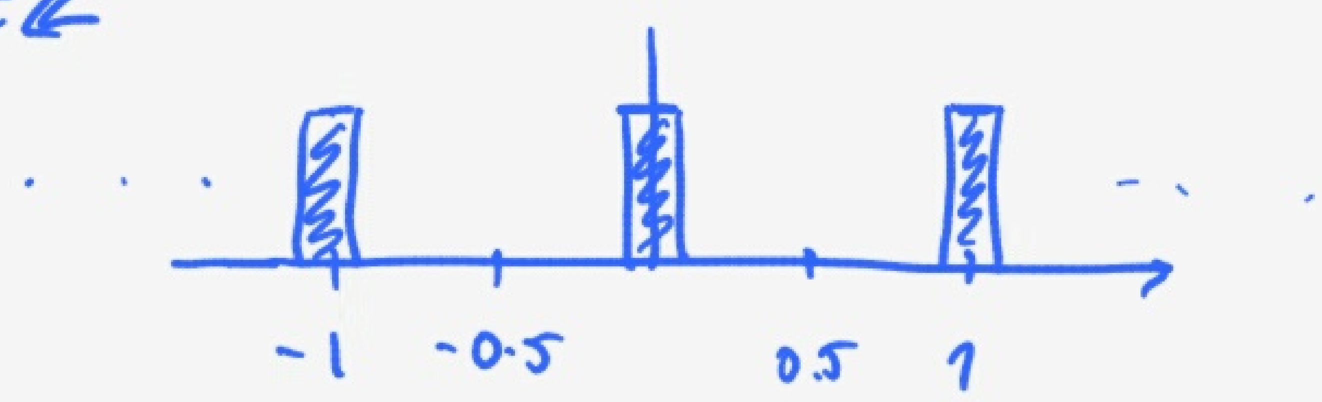
$$m_Z = 0$$

Hence for them to be uncorrelated it suffices that $r_{ZY}(k) = 0$

$$r_{YZ}(k) = R_0 \int_{-\infty}^{\infty} H_1(f) H_2^*(f)\, e^{j2\pi f kT}\, df = 0 \;\; \forall k \iff H_1(f) H_2^*(f) \Rightarrow \underline{f_1 < f_2 < f_3}$$

11.1 $x(n)$ deterministic and real $X(\nu) = \begin{cases} 1 & |\nu|<0.1 \\ 0 & 0.1 \le |\nu| < 0.5 \end{cases}$

$X(\nu+m) = X(\nu) \quad m \in \mathbb{Z}$



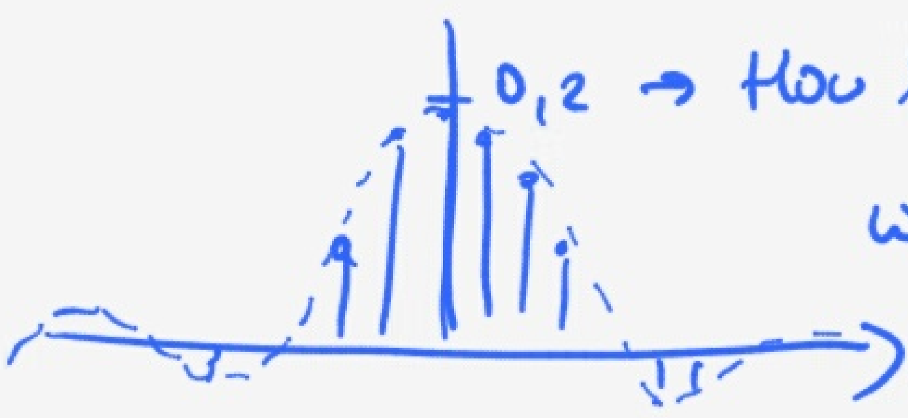
Subject to PAM

$$x(n) \rightarrow \boxed{\begin{matrix}\text{PAM} \\ \delta(t)\end{matrix}} \rightarrow y(t) = \sum_n x(n)\delta(t-nT)$$

$$x(n) = \int_{-0.1}^{0.1} X(\nu) e^{j2\pi\nu n} = \int_{-0.1}^{0.1} e^{j2\pi\nu n} d\nu = \frac{1}{j2\pi n}\left(e^{j2\pi 0.1 n} - e^{-j2\pi 0.1 n}\right) =$$

$$= \frac{\sin(2\pi 0.1 n)}{\pi n} \quad \text{"sampled sinc"} \qquad \frac{e^{j2\pi 0.1n} - 0e^{-j2\pi 0.1 n}}{2j} = \sin(2\pi 0.1 n)$$

$$y(t) = \sum_n \frac{\sin(2\pi 0.1 n)}{\pi n} \delta(t-nT)$$



$\rightarrow$ How to know the heigh at $n=0$.

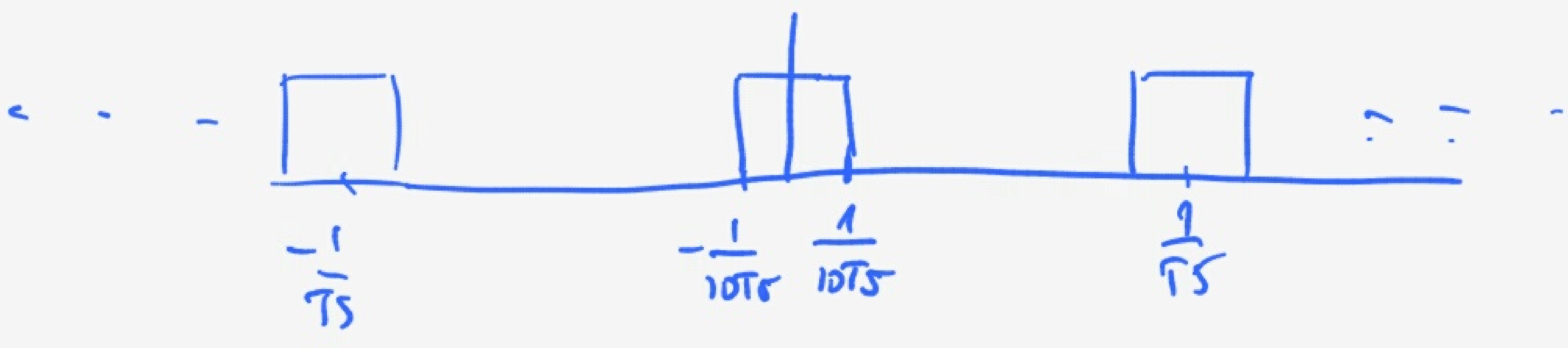
when $n=0$ $x(n) = \int_{-0.1}^{0.1} X(\nu)\cdot 1 d\nu = \underline{\underline{0.2}}$

$$y(t) = \sum_n x(n)\delta(t-nT) \qquad Y(f) = \mathcal{F}\left\{\sum_n x(n)\delta(t-nT)\right\} =$$

$$= Y(f) = \sum_n x(n)\,\mathcal{F}\{\delta(t-nT)\} = \sum_n x(n) e^{-j2\pi f n T} = X(\underbrace{fTs}_{fTs=\nu})$$

Therefore $\frac{-1}{10Ts} \le f \le \frac{1}{10Ts}$ $Y(f) = 1$

also when $\frac{-1}{10Ts} \le f + \frac{m}{Ts} \le \frac{1}{10Ts}$ $m \in \mathbb{Z}$ and $\emptyset$ elsewhere



11.4 $z(t) = \sum_n Y(n)p(t-nTs-\Theta)$ with $r_Y(k) = \sigma^2\delta(k)$ $p(t) = \frac{\sin(\pi t/Ts)}{\pi t/Ts}$

$$R_Z(f) = \frac{1}{Ts}|P(f)|^2 R_Y(fTs) \qquad R_Y(f) = \sigma^2 \Rightarrow R_Y(fTs) = \sigma^2$$

$$p(t) = Ts \cdot \frac{\sin(\pi t/Ts)}{\pi t} \xrightarrow{\mathcal{F}} P(f) = Ts\,\mathrm{Rect}_{\frac{1}{Ts}}(f)$$

$$R_Z(f) = \begin{cases} Ts\sigma^2 & \text{for } |f| \le \frac{1}{2Ts} \\ 0 & \text{for the rest} \end{cases}$$