"FÖRSLAG TILL LÖSNING. TENTAMEN MATEMATIK BASKURS 1 (4)

SF1659. 2016-10-03

1. a) $x + \sqrt{5-x} = -1$ $\quad 5-x \geq 0 \Leftrightarrow 5 \geq x$

$\Leftrightarrow$

$\sqrt{5-x} = -1-x$

$\Leftrightarrow$ KVADRERA!

$5-x = (-1-x)^2$

$\Leftrightarrow$

$5-x = 1+2x+x^2$

$\Leftrightarrow$

$x^2+3x-4=0 \Leftrightarrow x = -\frac{3}{2} \pm \sqrt{\frac{9}{4}+\frac{16}{4}} \Leftrightarrow$

$x = -\frac{3}{2} \pm \frac{5}{2} \begin{cases} 1 \\ -4 \end{cases}$, $\quad x \leq 5$

PRÖVA RÖTTER! $\quad x=1$ GER $1+\sqrt{4} = 1+2 = 3 \neq -1$!! EJ ROT

$x=-4$ GER $-4+\sqrt{9} = -4+3 = -1$!! ROT

SVAR: $x = -4$

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b) $2^{3x} \cdot 5^x = 2\sqrt{10} \Leftrightarrow (2^3)^x \cdot 5^x = \sqrt{2^2 \cdot 10} \Leftrightarrow$

$\Leftrightarrow (2^3 \cdot 5)^x = \sqrt{2^2 \cdot 10} \Leftrightarrow (40)^x = (40)^{\frac{1}{2}}$!! $x = \frac{1}{2}$

SVAR: $x = \frac{1}{2}$

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2. $f(x) = \sqrt{5x-6-x^2}$ $\quad D_f$: $5x-6-x^2 \geq 0$

$5x-6-x^2 \geq 0 \Leftrightarrow x^2-5x+6 \leq 0 \Leftrightarrow$

$(x-3)(x-2) \leq 0$

x		2		3	
$x-2$	$-$	0	$+$		$+$
$x-3$	$-$		$-$	0	$+$
$(x-2)(x-3)$	$+$	0	$-$	0	$+$

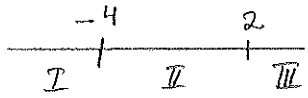
$2 \leq x \leq 3$.

SVAR: D_f: $2 \leq x \leq 3$

3.

$$|x-2| \le 3|x+4|$$

$$|x-2| = \begin{cases} x-2 & \text{DÅ } x \ge 2 \\ -(x-2) & x < 2 \end{cases} \qquad |x+4| = \begin{cases} x+4 & x \ge -4 \\ -(x+4) & x < -4 \end{cases}$$



I: $x < -4$

$$-(x-2) \le 3 \cdot -(x+4)$$
$$-x+2 \le -3x-12$$
$$2x \le -14$$
$$x \le -7 \in I$$

II: $-4 \le x < 2$

$$-(x-2) \le 3(x+4)$$
$$-x+2 \le 3x+12$$
$$-4x \le 10$$
$$x \ge -\frac{5}{2} \quad !! \quad -\frac{5}{2} \le x < 2$$

III: $x \ge 2$

$$x-2 \le 3(x+4)$$
$$x-2 \le 3x+12$$
$$-2x \le 14$$
$$x \ge -7 \Rightarrow x \ge 2$$

II och III GER TILLSAMMANS $x \ge -\frac{5}{2}$

SVAR: $x \le -7$ ELLER $x \ge -\frac{5}{2}$

4.

$$\left(2x - \frac{1}{4x^2}\right)^7 = \sum_{k=0}^{7} \binom{7}{k} (2x)^{7-k} \cdot \left(-\frac{1}{4x^2}\right)^k$$

$$(2x)^{7-k} \cdot \left(-\frac{1}{4x^2}\right)^k = 2^{7-k} \cdot x^{7-k} \cdot (-1)^k \cdot \left(\frac{1}{4x^2}\right)^k =$$

$$= 2^{7-k} \cdot (-1)^k \cdot x^{7-k} \cdot \left(\frac{1}{4}\right)^k \cdot \left(x^{-2}\right)^k =$$

$$= 2^{7-k} \cdot (-1)^k \cdot \frac{1}{4^k} \cdot x^{7-k} \cdot x^{-2k} =$$

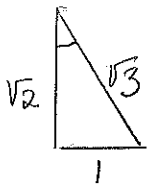
$$= (-1)^k \frac{2^7}{2^k} \cdot \frac{1}{4^k} \cdot x^{7-k-2k}; \qquad 7-k-2k = 1 \Leftrightarrow k = 2$$

$$k=2 \quad \binom{7}{2} \cdot (-1)^2 \cdot \frac{2^7}{2^2} \cdot \frac{1}{4^2} = 21 \cdot \frac{2^5}{4^2} = 21 \cdot 2 = 42.$$

SVAR: 42

5. $\sin(2\arctan\frac{1}{\sqrt{2}}) = |\sin 2x = 2\sin x\cos x| =$

$= 2\sin(\arctan\frac{1}{\sqrt{2}})\cos(\arctan\frac{1}{\sqrt{2}}) =$



$= 2\sin(\arcsin\frac{1}{\sqrt{3}})\cos(\arccos\frac{\sqrt{2}}{\sqrt{3}}) =$

$= 2\cdot\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}}{3}$ SVAR: $\frac{2\sqrt{2}}{3}$

6. $\cos 2x = (\cos x + \sin x)^2$

$\Leftrightarrow$

$\cos 2x = \cos^2 x + 2\cos x\sin x + \sin^2 x$

$\Leftrightarrow$

$\cos^2 x - \sin^2 x = \cos^2 x + 2\cos x\sin x + \sin^2 x$

$\Leftrightarrow$

$2\cos x\sin x + 2\sin^2 x = 0$

$\Leftrightarrow$

$\sin x(\cos x + \sin x) = 0$

1) $\sin x = 0 \Leftrightarrow x = n\cdot\pi$

2) $\cos x + \sin x = 0$

$\sin x = -\cos x$

$\frac{\sin x}{\cos x} = -1$

$\tan x = -1$ $\rightarrow$ $x = -\frac{\pi}{4} + n\cdot\pi$

SVAR: $x = n\cdot\pi$, $x = -\frac{\pi}{4} + n\cdot\pi$

7. ${}^x\log(2x+12) = 2 + {}^x\log 2$ $\quad y = x^t \Leftrightarrow t = {}^x\log y$, $x > 0$

$\Leftrightarrow$

${}^x\log(2x+12) = 2\cdot{}^x\log x + {}^x\log 2$

$\Leftrightarrow$

${}^x\log(2x+12) = {}^x\log x^2 + {}^x\log 2$

$\Leftrightarrow$

${}^x\log(2x+12) = {}^x\log(x^2\cdot 2)$

$\Leftrightarrow$

$2x+12 = 2x^2 \Leftrightarrow x^2 - x - 6 = 0 \Leftrightarrow$

$x = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{24}{4}}$

$x = \frac{1}{2} \pm \frac{5}{2} = \begin{cases} 3 \\ -2 \end{cases}$

$x = 3$

$(x = -2)$ EJ DEF.

SVAR: $x = 3$

8. $x^2 + ax - 2a^2 = 0$ HAR ROT $x = 1$

$\Rightarrow 1^2 + a \cdot 1 - 2a^2 = 0 \Leftrightarrow a^2 - \frac{a}{2} - \frac{1}{2} = 0$

$\Leftrightarrow a = \frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{8}{16}} \Leftrightarrow a = \frac{1}{4} \pm \frac{3}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$

$a = 1$ GER $x^2 + x - 2 = 0 \Leftrightarrow$

$x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{8}{4}} \Leftrightarrow x = -\frac{1}{2} \pm \frac{3}{2} = \begin{cases} 1 \\ -2 \end{cases}$

$a = -\frac{1}{2}$ GER $x^2 - \frac{x}{2} - \frac{1}{2} = 0 \Leftrightarrow$

$x = \frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{8}{16}} \Leftrightarrow x = \frac{1}{4} \pm \frac{3}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$

SVAR! $a = 1$ OCH $x = -2$ (OCH $x = 1$)

$a = -\frac{1}{2}$ OCH $x = -\frac{1}{2}$ (OCH $x = 1$)

9. $\ln(x^2 - 4) \leq \ln(x + 8)$

D_f: $x^2 - 4 > 0 \Leftrightarrow x > 2$ ELLER $x < -2$

$x + 8 > 0 \Leftrightarrow x > -8$

$0 \leq \ln(x+8) - \ln(x^2-4)$

$0 \leq \ln \frac{x+8}{x^2-4}$, $\ln x \nearrow$ DÅ $x \nearrow 1$

$\frac{x+8}{x^2-4} \geq 1 \Leftrightarrow \frac{x+8}{x^2-4} - 1 \geq 0 \Leftrightarrow \frac{x+8-(x^2-4)}{x^2-4} \geq 0$

$\Leftrightarrow \frac{-x^2+x+12}{x^2-4} \geq 0 \Leftrightarrow \frac{-(x^2-x-12)}{x^2-4} \geq 0 \quad \frac{-(x+3)(x-4)}{(x+2)(x-2)} \geq 0$

$\Leftrightarrow \frac{(x+3)(4-x)}{(x+2)(x-2)} \geq 0$

x	-3		-2		2		4
$x+3$	$-$	0	$+$	$+$	$+$	$+$	
$4-x$	$+$		$+$	$+$	$+$	0	$-$
$x+2$	$-$		$-$	0	$+$	$+$	$+$
$x-2$	$-$		$-$		$-$	0	$+$
$\frac{(x+3)(4-x)}{(x+2)(x-2)}$	$-$	0	$+$	$\nexists$	$-$	$\nexists$	$+$ 0 $-$

SVAR! $-3 \leq x < -2$ ELLER $2 < x \leq 4$