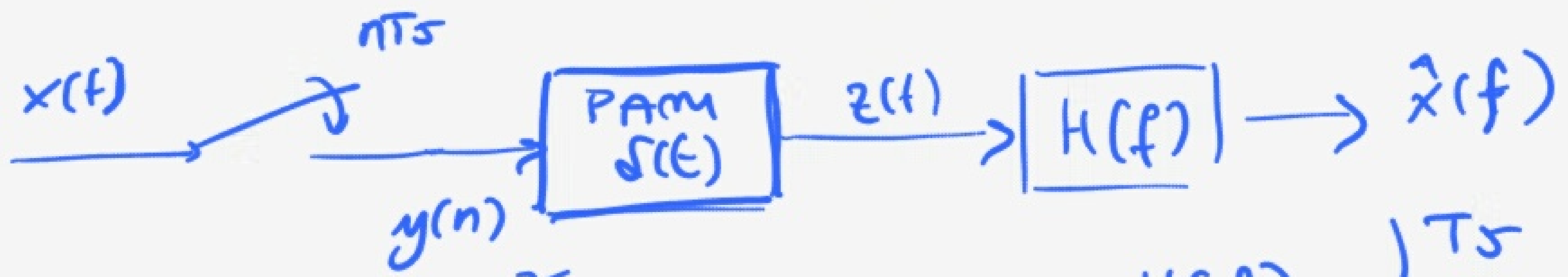
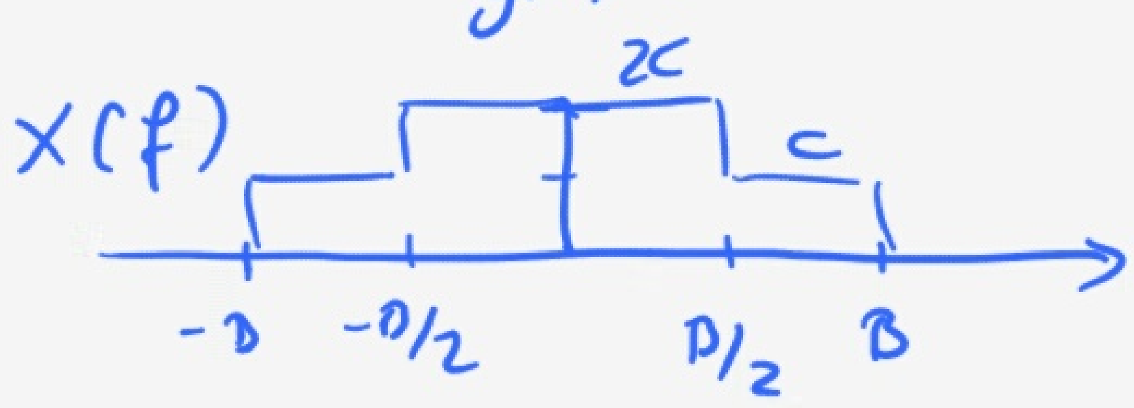
(12.1) $x(t)$ sampled $f_s = \frac{1}{T_s}$ Reconstructed using:



$$H(f) = \begin{cases} T_s & |f| \le B \\ 0 & \text{otherwise} \end{cases}$$

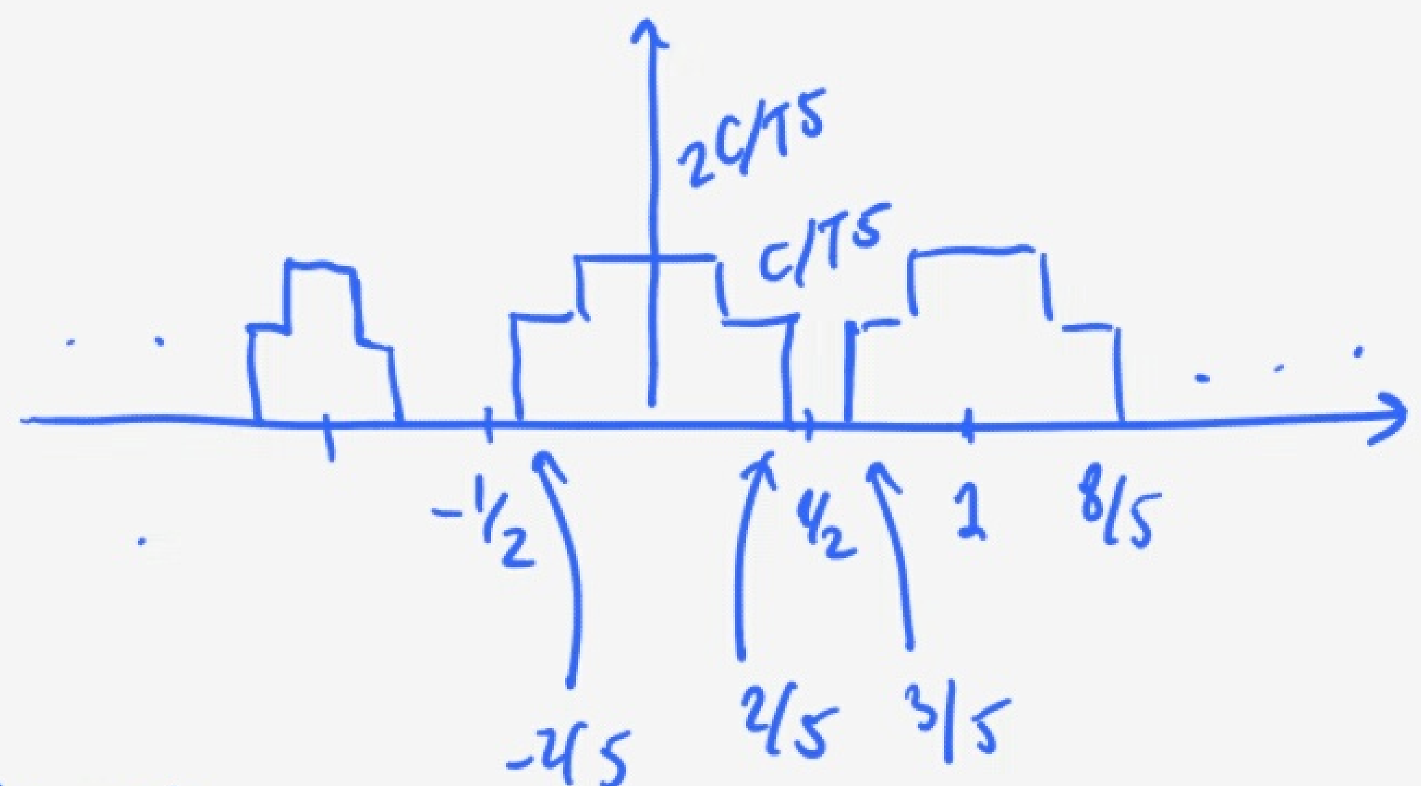


$$X(f) = C(\mathrm{Rect}_B(f) + \mathrm{Rect}_{2B}(f))$$

(a) $\hat{x}(f)$ if $T_s = 2/5B$ $\quad f_s = \frac{5B}{2} > 2B$ hence there is no aliasing.

$$Y(\nu) = \frac{1}{T_s} \sum_m X\left(\frac{\nu - m}{T_s}\right)$$

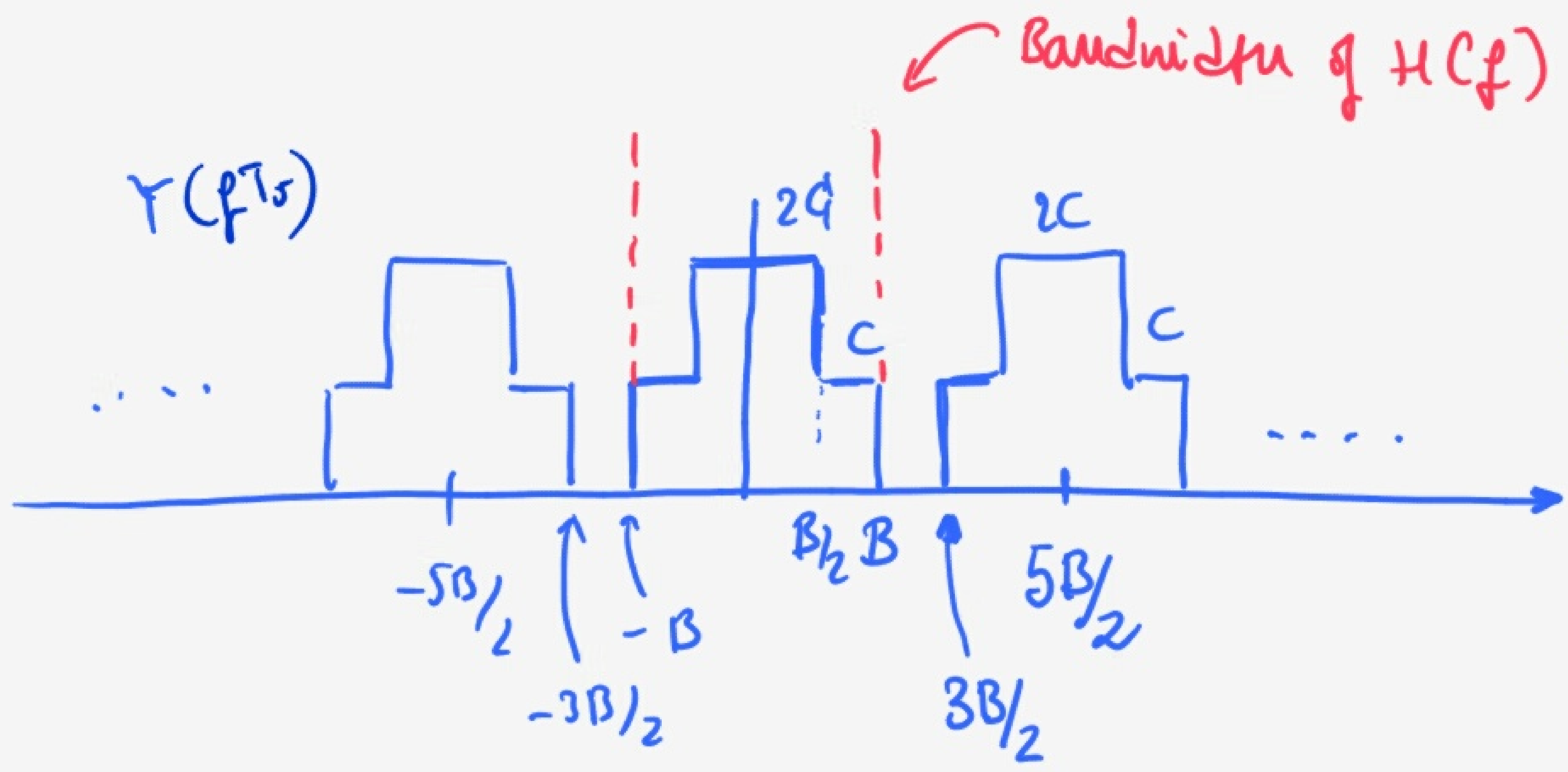
$$Y(\nu) = \frac{C}{T_s} \sum_m \mathrm{Rect}_B\left(\frac{\nu - m}{T_s}\right) + \mathrm{Rect}_{2B}\left(\frac{\nu - m}{T_s}\right)$$



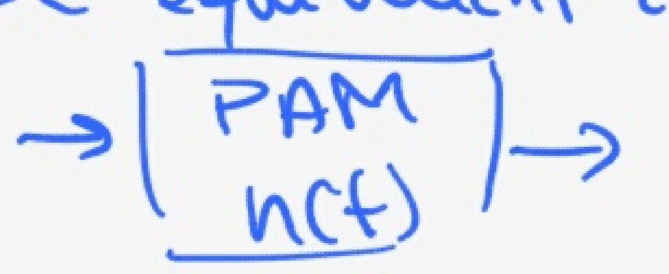
$$\hat{x}(f) = H(f)\, Y(f T_s)^*$$

$$Y(fT_s) = \frac{C}{T_s} \sum_m \mathrm{Rect}_B\left(\frac{fT_s - m}{T_s}\right) + \mathrm{Rect}_{2B}\left(\frac{fT_s - m}{T_s}\right) \quad \text{for } |f| \le B$$

Hence, $\hat{x}(f) = C(\mathrm{Rect}_B(f) + \mathrm{Rect}_{2B}(f))$



* the two blocks together are equivalent to



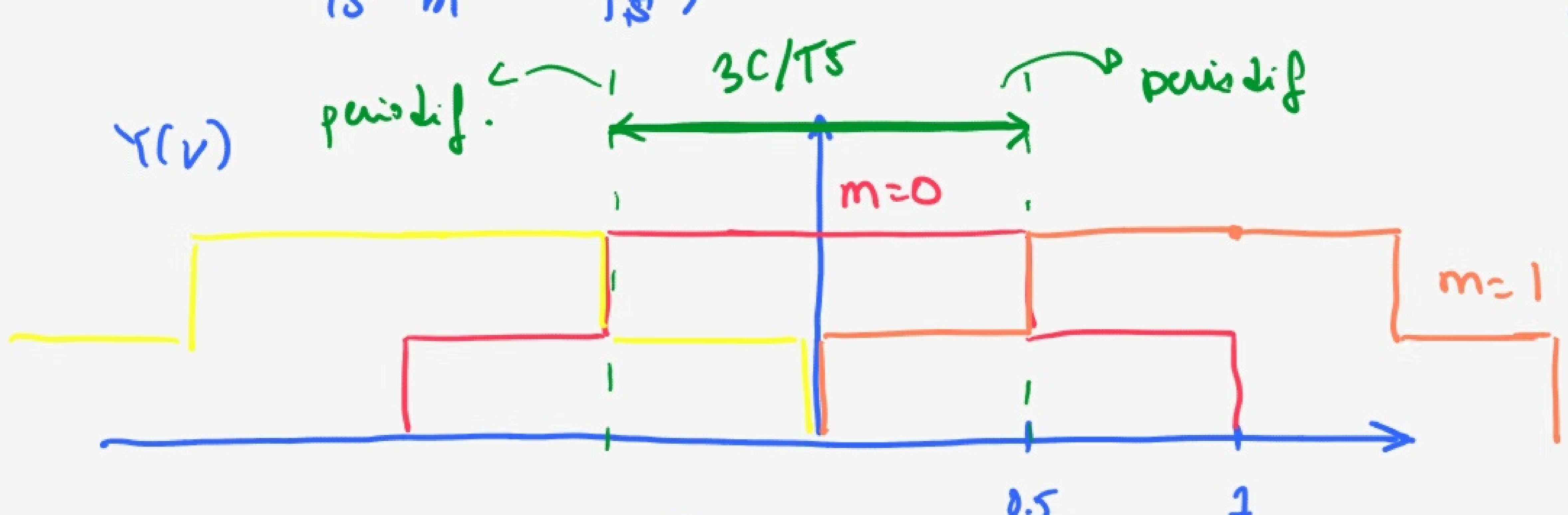
$$\hat{x}(t) = \sum_n y(n) h(t - nT_s)$$

$$\hat{x}(f) = \sum_n y(n) e^{-j2\pi f n T_s} H(f)$$

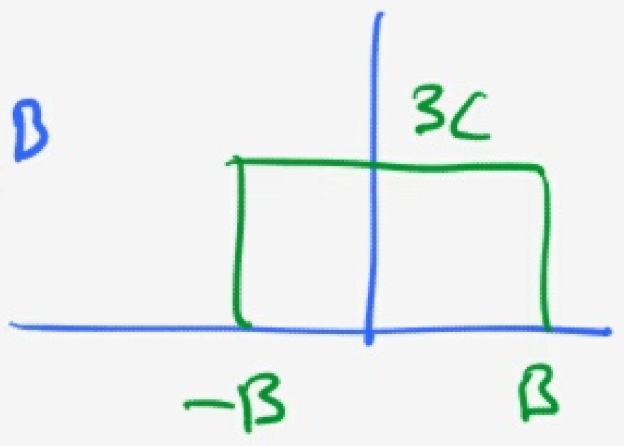
$$\hat{x}(f) = H(f)\, Y(fT_s)$$

(b) Find $\hat{x}(f)$ if $T_s = 1/B \Rightarrow$ will have aliasing!

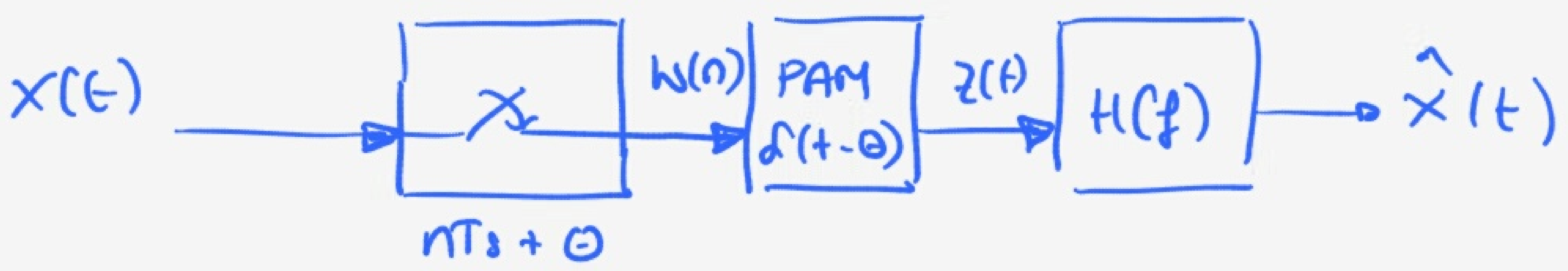
$$Y(\nu) = \frac{1}{T_s} \sum_m X\left(\frac{\nu - m}{T_s}\right) \qquad X(f) = C(\mathrm{Rect}_B(f) + \mathrm{Rect}_{2B}(f))$$



$Y(\nu) = \frac{3C}{T_s}$ Then $\hat{x}(f) = H(f)\, Y(fT_s) = \begin{cases} 3C & \text{for } |f| \le B \\ 0 & \text{otherwise} \end{cases}$



12.4



$\Theta \sim \mathcal{U}[0, T_s)$

$r_x(\tau) = \sigma_x^2 e^{-|\tau|/T_s}$ $\qquad H(f) = \begin{cases} T_s & |f| \le 1/2T_s \\ 0 & \text{otherwise} \end{cases}$

$E[w(n+k)w(n)] = E[X((n+k)T_s + \Theta)X(nT_s + \Theta)] =$ same value

$= r_x((n+k)T_s + \Theta - nT_s - \Theta) =$

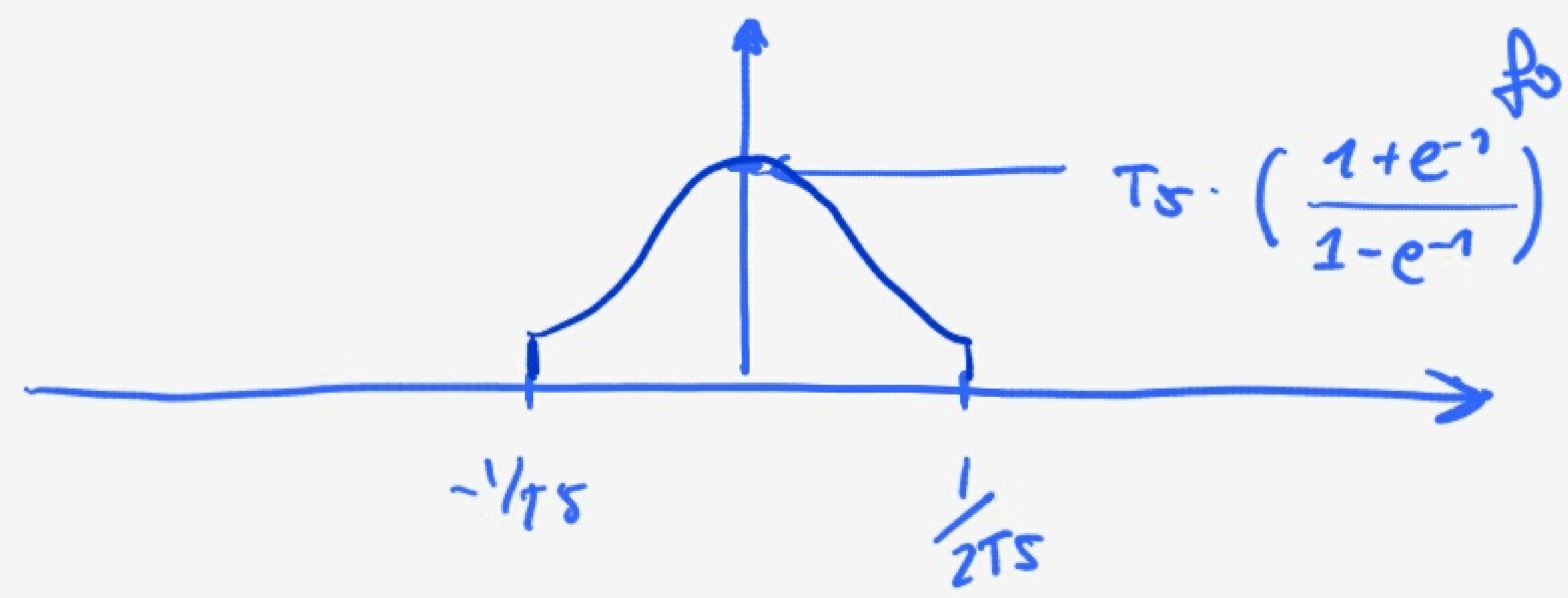
$= r_x(kT_s) = \sigma_x^2 e^{-|k|}$ $\qquad R_w(\nu) = \dfrac{1 - e^{-2}}{1 + e^{-2} - 2e^{-1}\cos(2\pi\nu)} \cdot \sigma_x^2$

$R_z(f) = \dfrac{|P(f)|^2}{T_s} R_w(fT_s) = \dfrac{1}{T_s} R_w(fT_s)$

then $R_{\hat{x}}(f) = |H(f)|^2 \dfrac{1}{T_s} R_w(fT_s)$ $\qquad |H(f)|^2 = \begin{cases} T_s^2 & |f| \le 1/2T_s \\ 0 & \text{otherwise} \end{cases}$

$= T_s R_w(fT_s)$

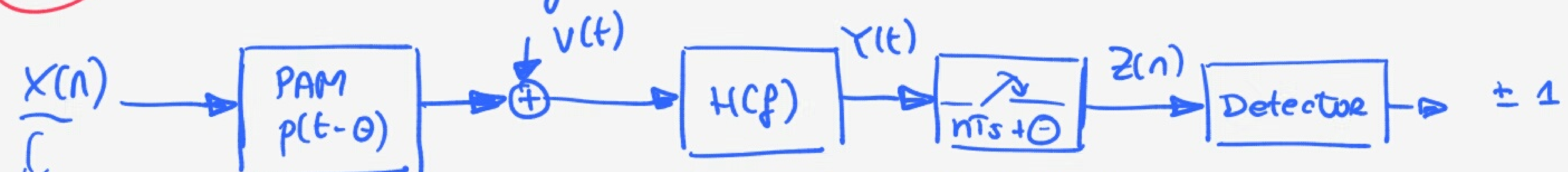
for $|f| \le 1/2T_s$.



Compare to $R_x(f) = \sigma_x^2 \dfrac{2/T_s}{\left(\frac{1}{T_s}\right)^2 + (2\pi f)^2} = \sigma_x^2 \dfrac{2T_s}{1 + (2\pi f T_s)^2}$ not band-limited $\Rightarrow$ aliasing effect.

write first in board

12.17 $X(n)$ reconstructed using PAM.



± 1 equiprob. indep. variables

$p(t) = C\left[\dfrac{\sin(\pi t/T_s)}{\pi t/T_s}\right]^2 = C\phi^2(t)$

$R_v(f) = R_0$ $\qquad H(f) = \begin{cases} \dfrac{1}{1 - |f|T_s} & |f| \le \frac{1}{2T_s} \\ 0 & \text{otherwise} \end{cases}$

SNR at $Z(n)$? $\qquad \boxed{r_x(k) = \delta(k)}$

$\dfrac{1}{T_s}|P(f)|^2 R_x(fT_s) = \dfrac{1}{T_s}|P(f)|^2$ $\qquad$ spectrum before anti-aliasing filter: $\dfrac{1}{T_s}|P(f)|^2 + R_0$

$R_y(f) = |H(f)|^2\left(\dfrac{1}{T_s}|P(f)|^2 + R_0\right)$ for $|f| \le \frac{1}{2T_s}$

0 otherwise

$C \cdot \mathcal{F}\left\{\dfrac{\sin(\pi t/T_s)}{\pi t/T_s}\right\} = C T_s \cdot \mathrm{Rect}_{1/T_s}(f)$

$\mathrm{Rect}_{1/T_s} * \mathrm{Rect}_{1/T_s}(f) =$

$\Rightarrow P(f) = \begin{cases} C T_s(1 - fT_s) & 0 \le f \le 1/T_s \\ C T_s(1 + fT_s) & -1/T_s \le f \le 0 \end{cases}$

$$P(f) = \begin{cases} c_1 T_s(1-|f|T_s) & |f| \le \frac{1}{T_s} \\ 0 & \text{elsewhere} \end{cases}$$

Then $R_y(f) = |H(f)|^2 \left(\frac{1}{T_s} |P(f)|^2 + R_0\right) = \left|\frac{1}{1-|f|T_s}\right|^2 \left(\frac{c^2 T_s^2}{T_s} \cancel{|1-|f|T_s|^2} + R_0\right)$

$$= \begin{cases} \underbrace{c^2 T_s}_{R_{yS}(f)} + \underbrace{\frac{R_0}{|1-|f|T_s|^2}}_{R_{yN}(f)} & \text{for } |f| \le \frac{1}{2T_s} \\ 0 & \text{elsewhere.} \end{cases} \Rightarrow \underline{\text{no aliasing}}$$

$R_{z_S}(\nu) = \frac{1}{T_s} \sum_m{}' R_{y_S}\left(\frac{\nu-m}{T_s}\right) = c^2$. Then $P_S = \int_{-1/2}^{1/2} c^2 d\nu = c^2(\nu)\Big]_{-1/2}^{1/2} = c^2$

$$P_N = \int_{-1/2}^{1/2} \frac{R_0}{T_s} \sum_m{}' \frac{1}{|1-|\nu-m||^2} d\nu = \int_{-1/2}^{1/2} \frac{R_0}{T_s} \frac{1}{(1-|\nu|)^2} d\nu =$$

$$= 2\int_0^{1/2} \frac{R_0}{T_s} \frac{1}{(1-\nu)^2} d\nu = \frac{2R_0}{T_s}\left[\frac{1}{1-\nu}\right]_0^{1/2} = \frac{2R_0}{T_s} \qquad SNR = \frac{c^2}{2R_0/T_s}$$