

Minigroupwork 4, solutions

a)

$$\rho_{SW} v_{SW}^2 = \left[\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 \Rightarrow$$

$$r = \left(\frac{\mu_0 a}{4\pi} \right)^{1/3} \left(2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

Assuming the solar wind consists of protons

$$\rho_{SW} = n_{e,SW} m_p = 1.7 \cdot 10^{-22} \text{ kg m}^{-3}$$

Thus

$$r = 2.7 \cdot 10^9 \text{ m} \approx 38 R_J$$

b)

$$\rho_{SW} v_{SW}^2 = \left[\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 + 2n_e k_B T \Rightarrow$$

$$\rho_{SW} v_{SW}^2 = \left[\frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 + 2n_{e0} \left(\frac{R_J}{r} \right)^3 k_B T$$

Substitute $x = 1/r^3$. This gives you an equation on the form

$$ax^2 + bx + c = 0$$

with

$$a = \left[\frac{\mu_0 a}{4\pi} \right]^2 / 2\mu_0 = \frac{\mu_0 a^2}{32\pi^2} = 1.0 \times 10^{46}$$

$$b = 2n_{e0} R_J^3 k_B T = 3.6 \times 10^{18}$$

$$c = -\rho_{SW} v_{SW}^2 = -2.7 \times 10^{11}$$

The solution is

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}} = -1.8 \cdot 10^{-28} + \sqrt{3.24 \cdot 10^{-56} + 2.635 \cdot 10^{-57}} = \\ = -1.8 \cdot 10^{-28} + 1.87 \cdot 10^{-28} = 7.18 \cdot 10^{-30}$$

Then

$$r = x^{-\frac{1}{3}} = (7.18 \cdot 10^{-30})^{-\frac{1}{3}} = 5.2 \cdot 10^9 \text{ m} \approx 73 \text{ R}_J$$