Rigid Body Dynamics (SG2150)
Exam, 2016-10-27, 8.00-12.00

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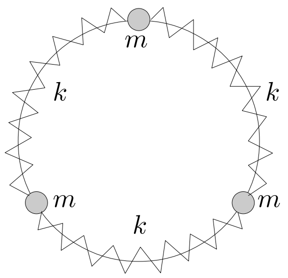
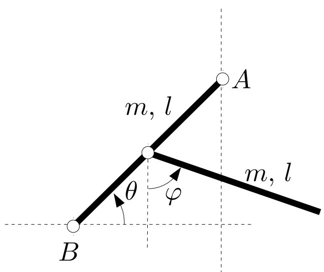
Each problem gives a maximum of 3 points, so that the total maximum is 18. Grading: 1–3 F; 4–5 FX; 6: E; 7–9 D; 10–12 C; 13–15 B; 16–18 A.

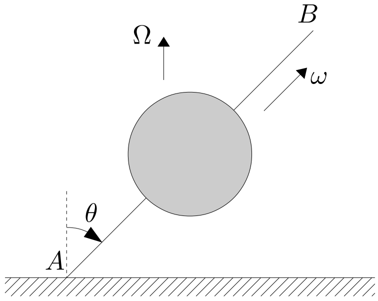
Allowed equipment: Handbook of mathematics and physics. One A4 page with your own compilation of formulae.



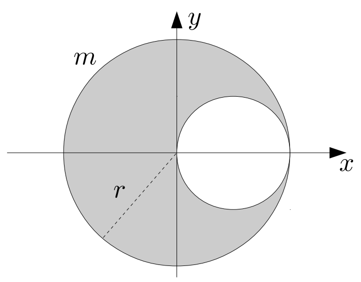
Problem 1. Three equal particles of mass m are sliding on a smooth, horizontal, circular ring. Three equal linear thin springs with spring constant k are also threaded on the ring, connecting the particles. Formulate the normal mode equations as an eigenvalue problem, and in particular find the angular frequencies of the oscillating modes, and the mode shape for the non-oscillating mode.



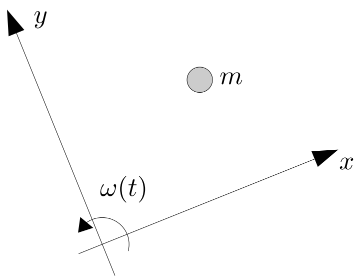
Problem 2. A thin, homogeneous rod of mass m and length l has one end point A sliding on a smooth, vertical, fixed track. The other end point B is sliding on a similar horizontal track. Both tracks lie in the same vertical plane. An identical second rod has one end point attached to the mid point of the first rod through a smooth hinge. Using the angles θ and φ as generalised coordinates, find Lagrange's equations for the system. Use the equations to determine the initial values of $\ddot{\theta}$ and $\ddot{\varphi}$, if the system is started from rest with initial angles $\theta_0 = \pi/3$, $\varphi_0 = -\pi/6$.



Problem 3. A light, thin, rod AB of length $2l$ passes through a homogeneous spherical ball of mass m and radius $r < l$ in such a way that the rod and ball centres coincide. The resulting top is spun up and placed on a smooth, horizontal surface, so that one end point A of the rod is in slipping contact with the surface. A motion is observed with constant tilt angle θ, and with the angular velocity of the top consisting of one part (precession) with constant component Ω upwards and another part (spin) with constant component ω along the top axis from A to B. Show that this motion is possible, and find a relation between θ, Ω, and ω.



Problem 4. From a thin, homogeneous, circular plate of mass m and radius r, a circular hole of radius $r/2$ is cut. The hole is tangent to the circumference of the plate. Find the new centre of mass G, and compute the 3×3 inertia matrix $\boldsymbol{J}_G$ for the axes indicated in the figure.



Problem 5. A particle of mass m is moving on a smooth horizontal plane. The x–y coordinate system has a fixed origin but is rotating with a given counter-clockwise angular velocity component $\omega(t)$ with respect to an inertial frame of reference. Using x and y as generalised coordinates, find Lagrange's equations of motion for the particle. Interpret the equations in terms of the inertial forces $\boldsymbol{F}_{\text{sp}} = -m\,[\boldsymbol{a}_{O'} + \dot{\boldsymbol{\omega}} \times \boldsymbol{r}_{\text{rel}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_{\text{rel}})]$ and $\boldsymbol{F}_{\text{cor}} = -2m\boldsymbol{\omega} \times \boldsymbol{v}_{\text{rel}}$ in an accelerating reference frame.

Problem 6. Write down Lagrange's equations for a system with non-conservative generalised forces. Suppose the generalised forces partly consists of impulsive forces. Starting from Lagrange's equations, use the *impact approximation* that the impulsive forces are much larger that non-impulse forces, and also of very short duration, and derive *Lagrange's equations for impact*, relating the change in velocities during the short duration to the generalised impulses.