Study Guide SF1677
Foundations of Analysis.

This is not really a study guide. It is an attempt to give a list of the most important definitions and Theorems of the course. To try to write such a list is, though, a fools errand. Everything is interconnected and mentioning one concept, say compactness, would immediately require an understanding of other concepts such as closed sets and completeness, but who could understand a closed set without also knowing what an open set is (?) and understanding completeness requires to understand Cauchy sequences which is incomprehensible without an understanding of... you get the point. The entire book, or at least the sections covered during the lectures, is required and may be covered by the exam. The list is however a guide to the more important theorems and definitions.

After the list of important results there will be a short description of the exam.

The most important Theorems and Definitions:

1. **Dedekind Cuts**, Definition on page 12. Understanding the completeness property of the real numbers and how the real numbers are constructed is the bedrock of all analysis.
2. **Cauchy sequences/Completeness**, page 18 (and p.77). These are absolutely vital since they allow us to discuss convergence without knowing the limit.
3. **Cardinality**. That $\mathbb{R}$ is uncountable whereas $\mathbb{Q}$ is countable has many deep consequences in mathematics.
4. **Max/Min and intermediate values.** Theorem 25 on page 41.
5. **Metric spaces**. Know the definition. Metric spaces are the right spaces to do analysis in.
6. **Homeomorphisms**. These are the most fundamental objects in topology. Know what they are.
7. **Compactness.** One of the fundamental concepts of analysis. Know the Heine-Borel Theorem p. 81. But also look at the definition of Covering compactness, Theorem 63 on p. 100.
8. **Compactness and uniform continuity**. Corollary 37 p. 84.
9. **Connectedness and the intermediate value property.** Theorem 43 p. 89 and its corollaries.
10. **Differentiation**. In one variable it is simple but in $\mathbb{R}^n$ it can be a little bit tricky to get the notation. Check for instance Theorem 8 p. 284 and also the mean value theorems on p 288-289.
11. **Taylor approximation**. You should know it from your first year calculus - make sure that you learn it now. Theorem 12 p. 160.
12. **Definition of the Riemann and Darboux Integrals**. But you probably figured this one out yourselves...
13. **The Riemann-Lebsegue Theorem.** p. 175 is the most important theorem in one dimensional integration (except maybe the fundamental thm of calc) - look at the list of corollaries.
14. **The Fundamental Theorem of Calculus.**, Theorem 34 on p. 183 You know it already but it is never wrong to repeat the fundamentals. Also look at the Antiderivative Theorem p 185 and the old calculus classics: Integration by substitution and by parts (Theorems 38 and 39 on p. 189).
15. **Series**, Look at the comparison, integral, root and ratio tests pp. 192-195
16. **Uniform convergence**. This is the right concept of convergence for continuous functions:
    (a) It preserves continuity and makes $C^0(M)$ into a complete space: Theorem 1 p213 and Theorem 2 p. 216
    (b) It preserves integrability. Theorem 6 p. 218
    (c) It preserves differentiability. Theorem 9 p. 219
17. **Equicontinuity.** This is what makes sets of continuous functions compact. Look in particular at the Arzela-Ascoli Theorem p. 224
18. **Approximation theorems**. In particular Weierstrass and Stone-Weierstrass Theorems p. 228 and p 234.
19. **Banach contraction principle**, Theorem 24 p. 240 and also Picard's Theorem p. 244 for an important application.
20. **Linear transformations and operator norms**.
21. **Second derivatives of multivariable functions**. Theorem 15-16 on pp. 291-292. Higher derivatives in $\mathbb{R}^n$ are more conceptually difficult since they are by their nature mappings of mappings.

22. **Implicit and inverse function theorems**. Very important even though the statements and the proofs are quite abstract. Theorems 22 and 23 on p. 298 and on p. 301.

23. **Multiple integrals**. Check the definition and the Riemann-Lebesgue Theorem. pp. 313-316

24. **Fubini's Theorem**. p. 316

25. **Change of variables formula**. Theorem 32 p. 319. Difficult but very important with many applications in analysis, geometry and topology.

26. **Stokes Theorem**, Theorem 45 p.345. This theorem is important and I want you to read it. But it is also quite demanding and the book does not contain enough details and examples to make it really understandable. Stokes theorem will therefore not be covered in detail at the exam. Though, simple questions to test familiarity with the theorem might be stated on the exam.

Some information about the exam:

The exam will contain 7 questions. Each question will be worth 5 marks for a total of 35 marks. The required marks for a certain grade will be

$$\mathbf{A}:\mathbf{31}\qquad \mathbf{B}:\mathbf{27}\qquad \mathbf{C}:\mathbf{23}\qquad \mathbf{D}:\mathbf{19}\qquad \mathbf{E}:\mathbf{15}\qquad \mathbf{FX}:\mathbf{14}.$$

A G^+ on either of the homework assignments will correspond to 5 marks of the corresponding question.

The exam questions will have a varying character. Some might be calculations, some might be slight extensions of the theory and some might be direct applications of a theorem. At least one question on the exam will be taken from the section *Prelim Questions* or *More Prelim Questions* in Pugh (on pp. 147-148 and pp. 270-276). So study those, they will give an idea of how the exam questions will look.

I might publish an example exam on the web-page at some point. It depends on the general work load. I have not been able to track down any old exams from SF2713 (they are not archived at the department and I couldn't find any online). If anyone have access to old exams through friends or older students I would be very happy if you could share them with me so that I can post some past exams online.