



KTH Informations- och kommunikationsteknik

Written exam with solutions IE1204-5 Digital Design Friday 21/10 2016 09.00-13.00

General Information

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Exam text does not have to be returned when you hand in your writing.

Aids: No aids are allowed!

The exam consists of three parts with a total of 14 tasks, and a total of 30 points:

Part A1 (Analysis) contains ten short questions. Right answer will give you one point. Incorrect answer will give you zero points. The total number of points in Part A1 is **10 points**. To **pass the Part A1 requires at least 6p**, *if you get fewer points, we will not look at the rest of your exam.*

Part A2 (Methods) contains two methodology-related problems with a total of 10 points.

To **pass the exam** requires at least **11 points** from A1 + A2 , *if you get fewer points, we will not look at the rest of your exam.*

Part B (Design problems) contains two design problems with total of 10 points. Part B is corrected only if there are at **least 11p** from the exam A- Part.

NOTE ! At the end of the exam text there is a submission sheet for Part A1, which should be separated and submitted together with the solutions for A2 and B.

A passing grade (**E**) requires at **least 11 points on the exam**. If you get exactly 10p from A1(6p)+A2(4p), (FX), completion to (E) will be offered.

Grades are given as follows:

0 –	11 –	16 –	19 –	22 –	25
F	E	D	C	B	A

The result is expected to be announced before Friday 11/11 2016.

Part A1: Analysis

Only final answers are required in Part A1. Write these answers on the submission sheet for Part A1, which can be found at the end of the exam text.

1. 1p/0p

A function $f(x, y, z)$ is described by the expression:

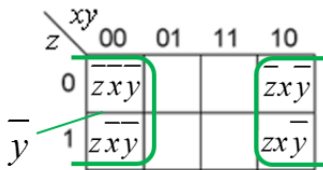
$$f(x, y, z) = (\overline{z \oplus x} + z \oplus x) \cdot (\overline{yx} + \overline{yx})$$

Write down a minimized two-level sum-of-products (SoP) form for this function..

$$f(x, y, z) = \{SoP\}_{\min} = ?$$

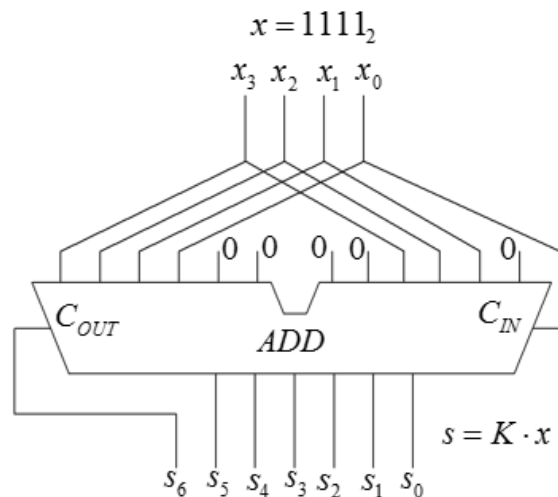
1. Proposed solution.

$$\begin{aligned} f(x, y, z) &= (\overline{z \oplus x} + z \oplus x) \cdot (\overline{yx} + \overline{yx}) = (\overline{z \oplus x} + z \oplus x) \cdot \overline{y(x + \overline{x})} = \\ &= zxy + \overline{z}xy + \overline{z}xy + zxy = \overline{y} \end{aligned}$$



2. 1p/0p

A four-bit unsigned integer $x = (x_3x_2x_1x_0)$ is connected to a 6-bit adder in the way that it is multiplied with a constant K , so that the result is $s = K \cdot x$ (see the diagram below to figure out the value of K). If the input $x = (1111)_2$ is applied to the diagram below, then what will the number $s = ?$ Answer with s as a binary number.



2. Proposed solution. $s = 1 \cdot x + 4 \cdot x = 5 \cdot x$ $5 \cdot 15 = 75$ 1001011_2

$$\begin{array}{r} \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \\ \quad \quad 0 \quad 0 \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{0} \\ \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{0} \quad \underline{0} \\ \hline 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \end{array}$$

3. 1p/0p

A four bit adder adds two 4 bit signed numbers represented as 2's complements. If $x = 0101$ and $y = 0111$, then what will be the four bit sum s computed by the adder? Answer with a signed decimal number. $\pm s_{10}$

3. Proposed solution.

$x = 0101$ $y = 0111 = 1100$ overflow! The absolute value is the corresponding positive number $0011 + 1 = 0100$ $s_{10} = -4$

4. 1p/0p

Consider a Karnaugh map for a function of four variables $y = f(a, b, c, d)$ given below. Write the function in a minimized y_{\min} product-of-sums (PoS) form. "-" in the map means "don't care".

		cd			
		00	01	11	10
ab	00	0	0	1	0
	01	1	0	0	-
	11	-	-	1	1
	10	0	0	1	0

4. Proposed solution.

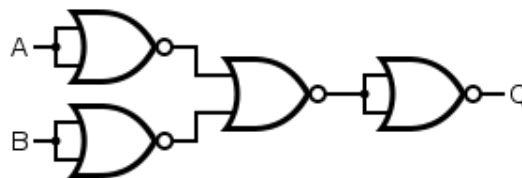
$$y = (c + \bar{d})(b + d)(a + \bar{b} + \bar{d})$$

		cd			
		00	01	11	10
ab	00	0	0	1	0
	01	1	0	0	-
	11	-	-	1	1
	10	0	0	1	0

$(a + \bar{b} + \bar{d})$
 $(b + d)$
 $(c + \bar{d})$

5. 1p/0p

The figure below shows a circuit with four NOR gates. Simplify the function $Q = f(A, B)$ as much as possible and write down the resulting expression for it.



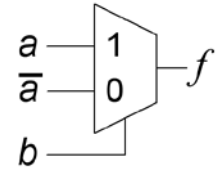
5. Proposed solution.

$$Q = \overline{\overline{A + A + B + B}} = \overline{\overline{A + B}} = \overline{dM} = \overline{A \cdot B} \quad \text{NAND}$$

6. 1p/0p

What logic function does this multiplexor circuit represent?

$f(b,a) = ?$

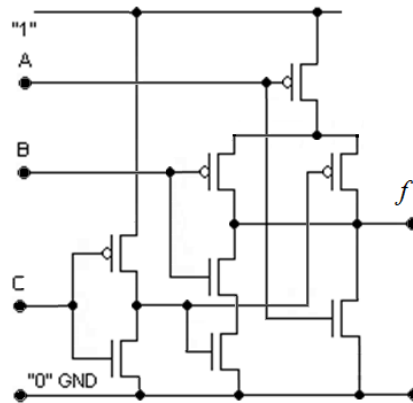


6. Proposed solution. $f(b,a) = b \cdot a + \bar{b} \cdot \bar{a}$ XNOR

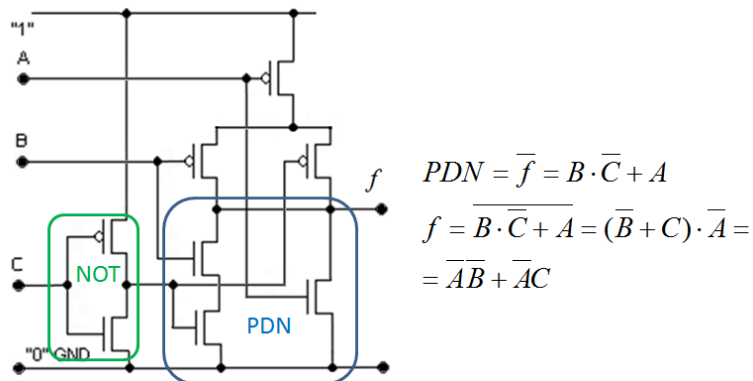
7. 1p/0p

Give an expression for the logic function realized by the CMOS circuit in the figure.

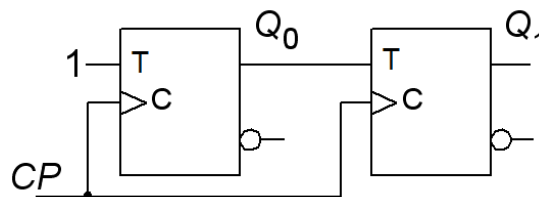
Answer with a sum of products (SoP) form. $f(A, B, C) = ?$



7. Proposed solution.



8. 1p/0p



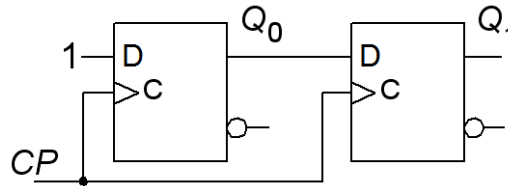
A synchronous counter with **T** flip-flops in the figure starts in the state $Q_1Q_0 = 00$. Give its sequence of states for the following four clock pulses.

8. Proposed solution.

Q_0 toggles each clock pulse, Q_1 toggles when $Q_0=1$.

$Q_1 Q_0$: $00 \rightarrow 01 \rightarrow 10 \rightarrow 11 \rightarrow 00$ so, it acts as an ordinary binary counter.

9. 1p/0p



For the shift register with D flip-flops in this figure setup time is $t_{su} = 4$ ns, delay time for the flip-flop outputs is $t_{pdQ} = 3$ ns and the hold time is $t_h = 2$ ns.

- How long needs to be the time between the clock pulses, $T_{CP} > ?$, for the counter function to be correct?
-

9. Proposed solution.

The clock period is determined by data path from Q_0 to Q_1 .

$$T_{CP} > t_{pdQ0} + t_{suD1} = 3+4 = 7 \text{ ns.}$$

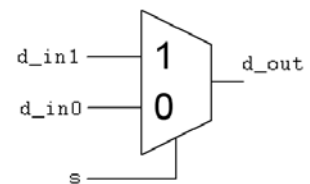
Hold time is the time the data input must be stable after the clock edge. This is no problem because Q_0 can change first after $t_{pdQ} = 3$ ns while the demand on stable signal after clock edge is $t_h > 2$ ns.

10. 1p/0p

Consider the following the VHDL code for a 2-input multiplexer given below. One line of the code in the circuit **architecture** is missing. Write the missing code line on the submission sheet.

Variabls are: **d_in0, d_in1, d_out, s**. Useful keywords are: **and, or, not**.

```
entity MUX2_1 is
  port( d_in0, d_in1, s : in std_logic;
        d_out           : out std_logic);
end entity MUX2_1;
```



```
architecture behave of MUX2_1 is
begin
  d_out <= Oooooops! What should be here?
end architecture behave;
```

10. Proposed solution.

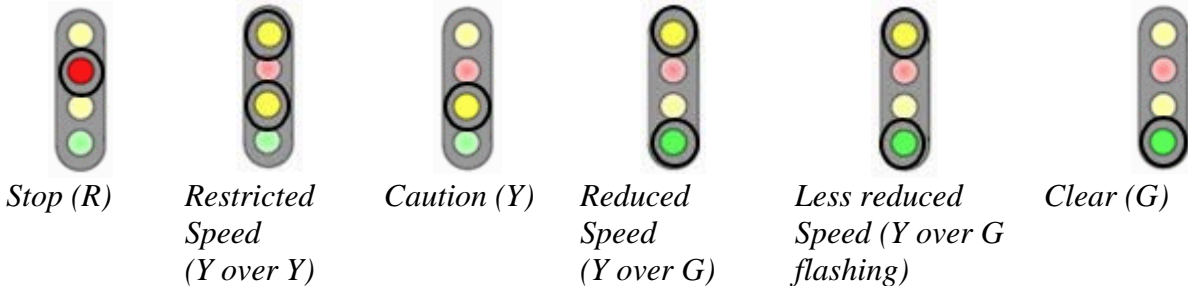
```
architecture behave of MUX2_1 is
begin
  d_out <= (not s and d_in0) or (s and d_in1);
end architecture behave;
```

Part A2: Methods

Note! Part A2 will only be corrected if you have passed part A1 ($\geq 6p$)

11. 4p

Japanese train signals are more complex than ours, but it is needed because Japan has some of the world's fastest train. The signals allow speed in six steps from stand still up to full speed.

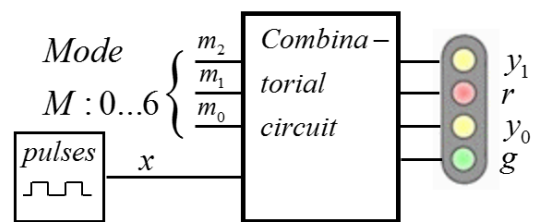


Design a *signal decoder*, a combinatorial logic network, which uses a three-bit mode signal M ($m_2 m_1 m_0$) to generate output signals controlling four colored lights ($y_1 r y_0 g$).

Mode 0, $m_2 m_1 m_0 = 000$, will mean that the signal off.

Mode 1...6, $m_2 m_1 m_0 = 001 \dots 110$ (in binary), select signals for incrementally increasing allowable speed according to the standard shown in the figure above.

Flashing signal means that y_1 and g should blink in an alternating manner, controlled by the pulse x from a pulse generator.



a) (1p) Set up the relationship between the output signals $y_1 r y_0 g$ and the input signals $x m_2 m_1 m_0$ as a table (as a **truth table**).

b) (2p) Set up Karnaugh maps for the four lamp signals and produce the minimized expressions $y_1 r y_0 g$ in the SoP form. Make use of "don't cares".

c) (1p) Draw the **schematic diagram** for one of the four signal decoder outputs (select the most complex network) using the optional gates of your choice.

11. Proposed solution.

a)

$xm_2m_1m_0$	y_1ry_0g	$xm_2m_1m_0$	y_1ry_0g
0 0000	0000	8 1000	0000
1 0001	0100	9 1001	0100
2 0010	1010	10 1010	1010
3 0011	0010	11 1011	0010
4 0100	1001	12 1100	1001
5 0101	$\frac{0}{1}00\frac{1}{0}$	13 1101	$\frac{1}{0}00\frac{0}{1}$
6 0110	0001	14 1110	0001
7 0111	—	15 1111	—

K-map setup:

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

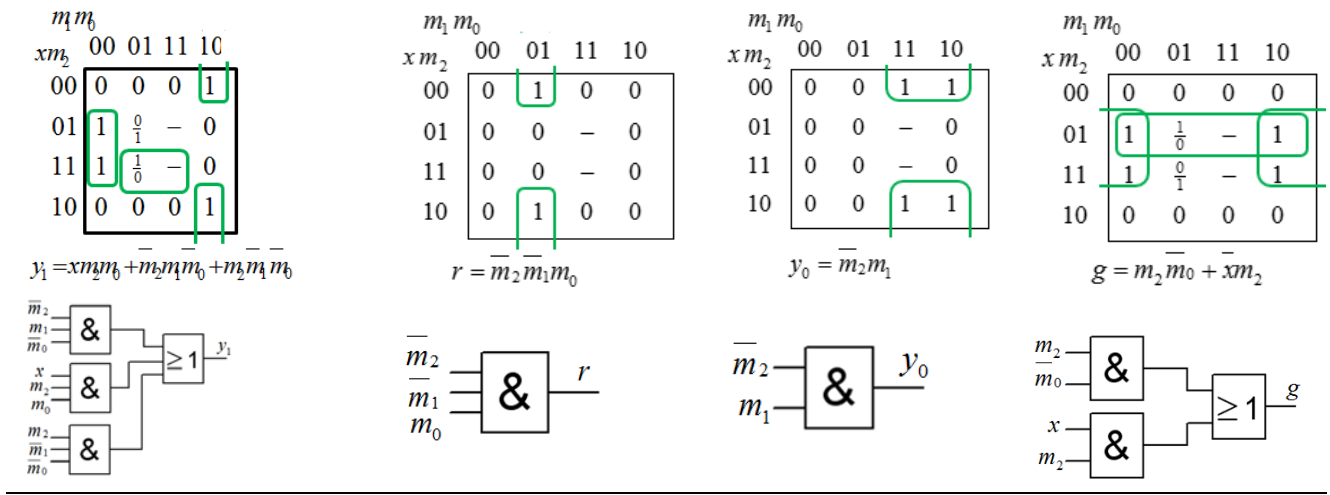
b)

$$y_1 = xm_2m_0 + \overline{m_2}m_1\overline{m_0} + m_2\overline{m_1}\overline{m_0}$$

$$r = \overline{m_2}m_1m_0$$

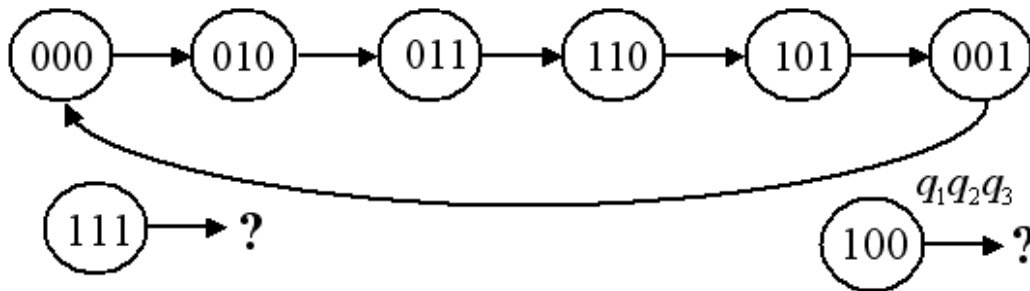
$$y_0 = \overline{m_2}m_1$$

$$g = m_2\overline{m_0} + xm_2$$



12. 6p

A modulo-6 synchronous counter is implementing the sequence $q_1q_2q_3$: 000, 010, 011, 110, 101, 001.



a) (1p) Derive the **State table**. The two states 111 and 100 which are not included in the sequence can be treated as don't cares, but they should lead to the modulo-6 sequence (in other words their next state should be one of the 6 states in the sequence above).

b) (1p) Derive **minimized** expressions for next state $q_1^+ = ?$ $q_2^+ = ?$ $q_3^+ = ?$

c) (3p) The Synchronous Counter is constructed with three D flip-flops as in the figure. Implement the functions:

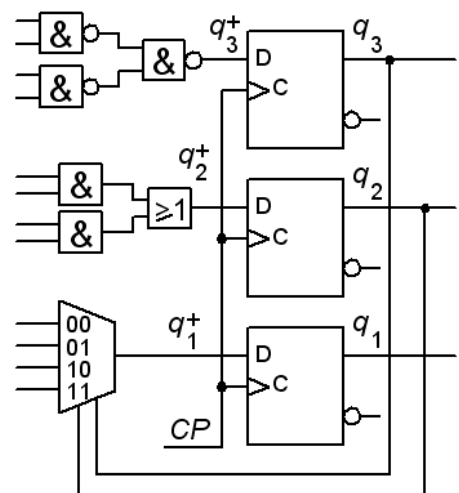
q_1^+ with a 4:1 multiplexer.

q_2^+ with AND-OR gates

q_3^+ with only NAND gates

By assigning the inputs in the figure on the right,

d) (1p) to which state in the modulo-6 sequence will **111** and **100** go to in your implementation?



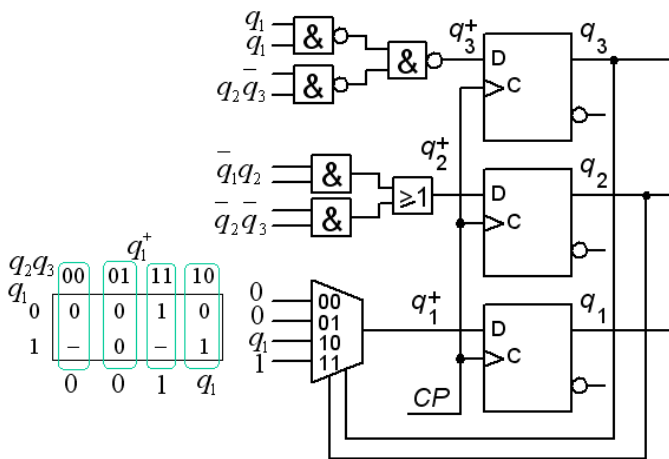
12. Proposed solution.

$q_1 q_2 q_3$	$q_1^+ q_2^+ q_3^+$	$q_2 q_3$	q_1^+
		00 01 11 10	
000	010	q_1 0	q_1^+ 0 0 1 0
001	000	1	- 0 - 1
010	011	$q_2 q_3$	q_2^+
011	110	00 01 11 10	
100	---	0	1 0 1 1
101	001	1	- 0 - 0
110	101	$q_2 q_3$	q_3^+
111	---	00 01 11 10	
		0	0 0 0 1
		1	- 1 - 1

$$q_1^+ = q_1 q_2 + q_2 q_3$$

$$q_2^+ = \bar{q}_1 q_2 + \bar{q}_2 \bar{q}_3$$

$$q_3^+ = q_1 + q_2 \bar{q}_3$$



$$q_1^+(111) = 1 \quad q_1^+(100) = 0$$

$$q_2^+(111) = 0 \quad q_2^+(100) = 1$$

$$q_3^+(111) = 1 \quad q_3^+(100) = 1$$

$$(111) \rightarrow (101) \quad (100) \rightarrow (011)$$

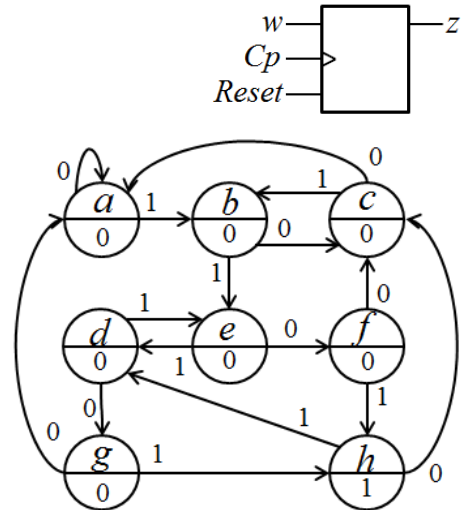
Part B. Design Problems

Note! Part B will only be corrected if you have passed part A1+A2 ($\geq 11p$).

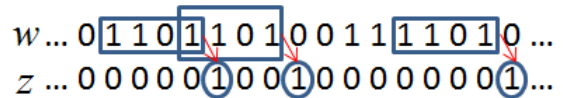
13. 6p Synchronous sequential circuit. Sequence detector.

(To solve this **independent task** may well prove to be a good use of time for solving the rest of the task!)

a) (2p) Minimize the states in the state diagram on the right (a,b,c,d,e,f,g,h). Then draw the minimal state diagram.



b) (4p) A sequence detector is expected to detect each occurrence of a sub-sequence ... 1101 ... within a sequence of bits received by the input w .



The sequence detector output signal z should be **1** in a clock pulse interval immediately after the sub-sequence 1101 has occurred (see the figure). Implement the sequence detector circuit as a Moore machine with positive edge-triggered D flip-flops with reset inputs. Derive the circuit **state table** and draw its **state diagram**.

Use binary code to encode the states and derive the **encoded state table**. Assumed that no 1 can be received directly after Reset. Derive the minimized expressions for **next state** and for the **output**.

$$q_2^+ q_1^+ q_0^+ = f(q_2 q_1 q_0, w) \Rightarrow q_2^+ = f(q_2 q_1 q_0, w) \quad q_1^+ = f(q_2 q_1 q_0, w) \quad q_0^+ = f(q_2 q_1 q_0, w)$$

$$z = f(q_2 q_1 q_0, w)$$

13. Proposed solution. a)

z	S	S^+	
		$w=0$	$w=1$
0	a	a	b
0	b	c	e
0	c	a	b
0	d	g	e
0	e	f	d
0	f	c	h
0	g	a	h
1	h	c	d

(abcdefg)(h)

$w=0$: a→(abcdefg) b→(abcdefg) c→(abcdefg) d→(abcdefg)

e→(abcdefg) f→(abcdefg) g→(abcdefg)

$w=1$: a→(abcdefg) b→(abcdefg) c→(abcdefg) d→(abcdefg)

e→(abcdefg) f→(h) g→(h)

(abcde)(fg)(h)

$w=0$: a→(abcde) b→(abcde) c→(abcde) d→(fg) e→(fg)

$w=1$: a→(abcde) b→(abcde) c→(abcde) d→(abcde) e→(abcde)

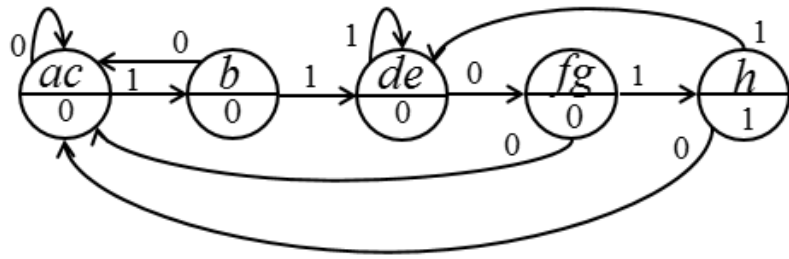
(abc)(de)(fg)(h)

$w=0$: a→(abc) b→(abc) c→(abc)

$w=1$: a→(abc) b→(de) c→(abc)

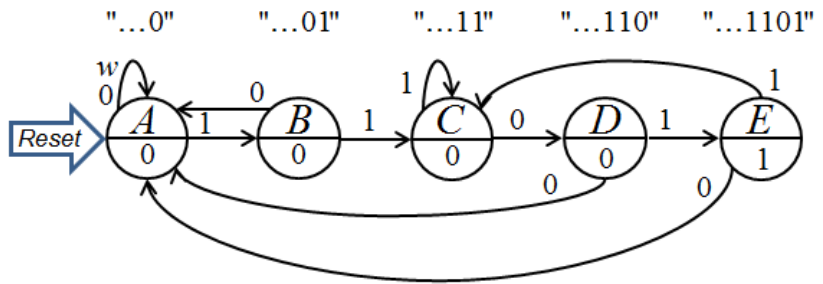
(ac)(b)(de)(fg)(h)

z	S	S ⁺
	w=0	w=1
0	ac	ac b
0	b	ac de
0	de	fg de
0	fg	ac h
1	h	ac de



b)

z	S	S ⁺
	w=0	w=1
0	A	A B
0	B	A C
0	C	D C
0	D	A E
1	E	A C



A:000 B:001 C:010 D:011 E:100

$$q_2^+ q_1^+ q_0^+ = f(q_2 q_1 q_0, w)$$

$$q_2^+ = f(q_2 q_1 q_0, w)$$

$q_2 q_1$	$q_0 w$	00	01	11	10
00		A → A:000	A → B:001	B → C:010	B → A:000
01		C → D:011	C → C:010	D → E:100	D → A:000
11		---	---	---	---
10		E → A:000	E → C:010	---	---

$q_2 q_1$	$q_0 w$	00	01	11	10
00		0	0	0	0
01		0	0	1	0
11		-	-	-	-
10		0	0	-	-

$$q_1^+ = f(q_2 q_1 q_0, w)$$

$q_2 q_1$	$q_0 w$	00	01	11	10
00		0	0	1	0
01		1	1	0	0
11		-	-	-	-
10		0	1	-	-

$$q_0^+ = f(q_2 q_1 q_0, w)$$

$q_2 q_1$	$q_0 w$	00	01	11	10
00		0	1	0	0
01		1	0	0	0
11		-	-	-	-
10		0	0	-	-

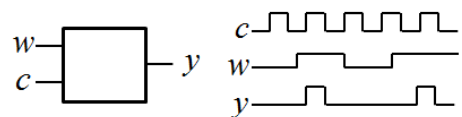
$$q_2^+ = q_1 q_0 w$$

$$q_1^+ = q_1 \bar{q}_0 + q_2 w + \bar{q}_1 q_0 w$$

$$q_0^+ = q_1 q_0 w + \bar{q}_2 \bar{q}_1 \bar{q}_0 w$$

14. 4p Single pulse generator.

Design an asynchronous sequential circuit which has an input w and an input c . A sequence of pulses are arriving at the input w . Each time the input w is 1, a complete pulse of the sequence c should appear at the output y , as soon as possible. A pulse w is always longer than a pulse c (see figure on the right).

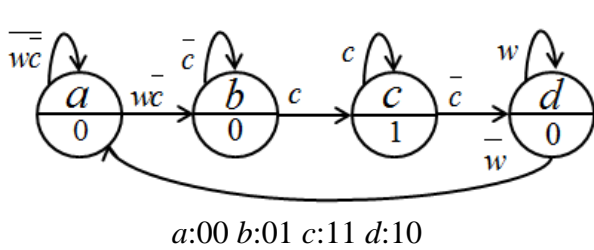


a) Set up a proper **flow table** for the circuit. Draw its **state diagram**.

b) Do a suitable **state assignment** with an excitation-table which gives a circuit which is **free of critical race**. You should also derive **hazard free expressions** for the next state and an **expression for the output**, and draw the **circuit diagrams** with the gates of your choice.

14. Proposed solution.

Pulse could be started at $c = 1$, after $w = 1$ and $c = 0$.



wc	$q_2^+q_1^+$			
	00	01	11	10
a:00	00	00	00	01
b:01	-	-	11	01
c:11	-	-	11	10
d:10	00	00	10	10

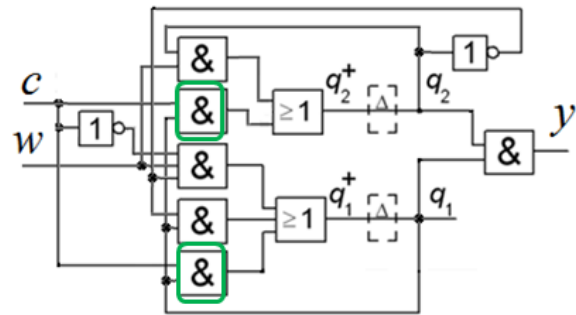
Gray code gives the states hamming distance 1

w pulse is long, so if $w = 0$ state b and c are don't care

wc	q_2^+				wc	q_1^+			
	00	01	11	10		00	01	11	10
00	0	0	0	0	00	0	0	0	1
01	-	-	1	0	01	-	-	1	1
11	-	-	1	1	11	-	-	1	0
10	0	0	1	1	10	0	0	0	0

$q_2^+ = wq_2 + cq_1$ $q_1^+ = cq_1 + \overline{q_2}q_1 + \overline{q_2}w\overline{c}$

(with hazard cover) output: $y = q_2q_1$



q_2^+ and q_1^+ could share one gate cq_1

Hope all went well!

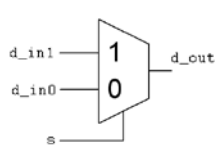
Submission sheet for Part A1 Sheet 1

(remove and hand in together with your answers for part A2 and part B)

Last name: _____ Given name: _____

Personal code: _____ Sheet: **1**

Write down your answers for the questions from Part A1 (1 to 10)

Question	Answer
1	$f(x, y, z) = \{SoP\}_{\min} = ?$
2	$s = K \cdot x \quad x = 1111_2 \quad s = ?$ [binary]
3	$x = 0101 \quad y = 0111 \quad s = x + y \quad \pm s_{10}$ [two complement number as signed decimal value]
4	$f(a, b, c, d) = \{PoS\}_{\min} = ?$
5	$Q = f(A, B) = ?$
6	$f(b, a) = ?$
7	$f(A, B, C) = ?$
8	$Q_1 Q_0 : 00 \rightarrow$
9	$T_{CP} > ?$ [ns]
10	<div style="display: flex; align-items: flex-start;"> <div style="margin-right: 20px;">  </div> <div> <p>architecture behave of MUX2_1 is begin $d_out <=$ end architecture behave;</p> </div> </div>

This table is completed by the examiner!!

Part A1 (10)	Part A2 (10)		Part B (10)		Total (30)	
Points	11	12	13	14	Sum	Grade