EXAM IN FOUNDATION OF ANALYSIS SF2713 MARCH 14 2016

The solutions shall be clearly written and every step carefully motivated.
Each problem gives at most 3 points. The grades are:
A: 22-24 p; B: 19-21p; C: 16-18p; D:14-15p; E:12-13p; Fx: 11p

1 Define the following concepts:
a) a real number
b) an equivalence relation
c) uniform continuity

2 Formulate the following theorems:
a) the chain rule
b) the contraction principle (Banach`s fixed point theorem
c) the Stone-Weierstrass theorem

3 Let f be the limit of a uniformly convergent sequence of uniformly continuous functions between metric spaces.Prove that f also must be uniformly continuous.

4 Suppose that (p_n) and (q_n) are Cauchy sequences in a metric space. Does the sequence $(d(p_n,q_n)$ converge ?

5 Find an example of a bijective continuous function which is not a homeomorphism.

6 Let X be the real vector space of functions from the natural numbers to the real numbers, such that $f(n) \to 0$ as $n \to \infty$. Define a norm on X by
$\| f \|$ = the maximum of $| f(n) |$ as n varies over N.
Prove that X is complete.

7 Let $f : X \times Y \to R$ be continuous where X and Y are compact metric spaces. Let for each y in Y g(y) denote the minimum of f (x,y) as x varies over X. Prove that g is continuous.

8 Let L denote the set of linear operators from R^n to itself endowed with the operator norm. Define $f : L \to L$ by $f(T)=\exp(T(I-T))$, where exp is the exponential function defined by its usual power series expansion.
Is f locally invertible near the identity map I ?

GOOD LUCK ! *Lasse*