Exam in Foundations of Analysis. SF2713

Tuesday January 12^{th} 2016 14.00-19.00

Each problem gives at most 3 points

The grades are as follow:
A: 22-24, B: 19-21, C: 16-18, D: 14-15, E: 12-13, Fx: 11

1. Define the following concepts:
   a) a separable metric space
   b) uniform convergence of a sequence of functions between metric spaces
   c) the derivative at a point of a function between normed spaces

2. Formulate the following theorems:
   a) the Heine-Borel theorem
   b) the Stone-Weierstrasse theorem
   c) The inverse function theorem

3. Prove the contraction principle

4. Show that in any compact metric space X there exists a countable set S of open subsets of X, such that every open subset of X is a union of some of the sets in S.

5. Prove that any metric space, in which every infinite subset has a limit point, is compact.

6. Prove that every open mapping from R to R is monotonic. (a mapping is open if it maps open sets to open sets).

7. Suppose that we are given an equicontinuous sequence of functions which converges pointwise on a compact set K. Prove that the convergence has to be uniform on K.

8. Let X denote the set of complex n by n matrices endowed with operator norm and concider the equation.

$$ABA - BCB + CAC = I$$

Show that this equation can be solved for A in terms of B and C locally near $(A,B,C) = (I, I, I)$

Good Luck!

Lasse