1 Preparation for the Oral Exam.

The oral exam will take approximately half an hour. It's main purpose is to make sure that you have thought trough the course material once with care - not that you should remember all the details.

As a preparation you should think through the following questions carefully. During the exam you will randomly pick some questions and answer them to the best of your knowledge. I might ask follow up questions or ask you to expand certain parts of your answers. It is therefore important that you also look at other parts of the course that are not explicitly included in these questions.

1. An important concept in measure theory is "$\sigma-$*algebra*". What is a $\sigma-$algebra, why are they important and how are they used in the theory of the integral.
2. What does it mean for a set to be "*measurable*"? Why do we introduce the concept of measurability?
3. Important properties of the lebesgue measure is that it is sub-additive, and even additive. What does that mean? One of additivity/sub-additivity is easy to prove, the other difficult. Can you say something about the proofs (at least the easy one)?
4. We have an outer lebesgue measure m^* and a lebesgue measure m. What is the relation between the two? Why aren't we satisfied with the outer measure?
5. One of the more important theorems in intergartion theory is Fatou's Lemma. State the Lemma and say something about it's proof.
6. What is the monotone convergence theorem? Say something about its proof?
7. Why do we bother to define the Lebesgue integral, wasn't the Riemann integral good enough? Explain!
8. What does the Egoroff and the Lusin Theorem say?
9. What does the Fubini Theorem say?
10. Outline the proof (no details) of Fubini's Theorem.
11. What is a dual space? What is the dual space of $L^p(D)$ for $1 < p < \infty$?
12. Give a very rough outline of the proof of the fact that $L^q(D)$ is dual to $L^p(D)$ for $1 < p < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$.
13. What is weak convergence in $L^p(D)$? Why is it needed? Can you give a very rough sketch of the proof of weak compactness in L^p?
14. What is a weak derivative? Why do we introduce them?
15. We know that C^∞ is dense in $W^{1,p}(D)$. Can you give a rough outline of how one can approximate $u \in W^{1,p}(D)$ by a $C^\infty-$ function v so that $\|u - v\|_{W^{1,p}(D)} < \epsilon$ - you may assume that u has compact support in D.

16. Give a very rough sketch of a proof that any non-decreasing function $f : [a, b] \mapsto \mathbb{R}$ is differentiable a.e.

17. What is a Vitali covering of a set $S \subset \mathbb{R}$? How are Vitali coverings related to the fundamental Theorem of calculus?

18. What does it mean for a function f to be absolutely continuous? How is absolute continuity used to prove the fundamental theorem of calculus: $\int_a^b f'(x)dx = f(b) - f(a)$ (very rough sketch).

19. Say something very brief about the relation between a function $u \in W^{1,2}(Q)$, $Q = \{x \in \mathbb{R}^2;\ |x_1| < 1, |x_2| < 1\}$, and the absolute continuity in t of the function $f(t) = u(t, x_2)$. Is $f(t)$ absolutely continuous for any, some , all $x_2 \in [-1, 1]$?

20. It was not straight forward to define boundary values for a Sobolev function $u \in W^{1,2}(Q)$, $Q = \{x \in \mathbb{R}^2;\ |x_1| < 1, |x_2| < 1\}$. How was that done?