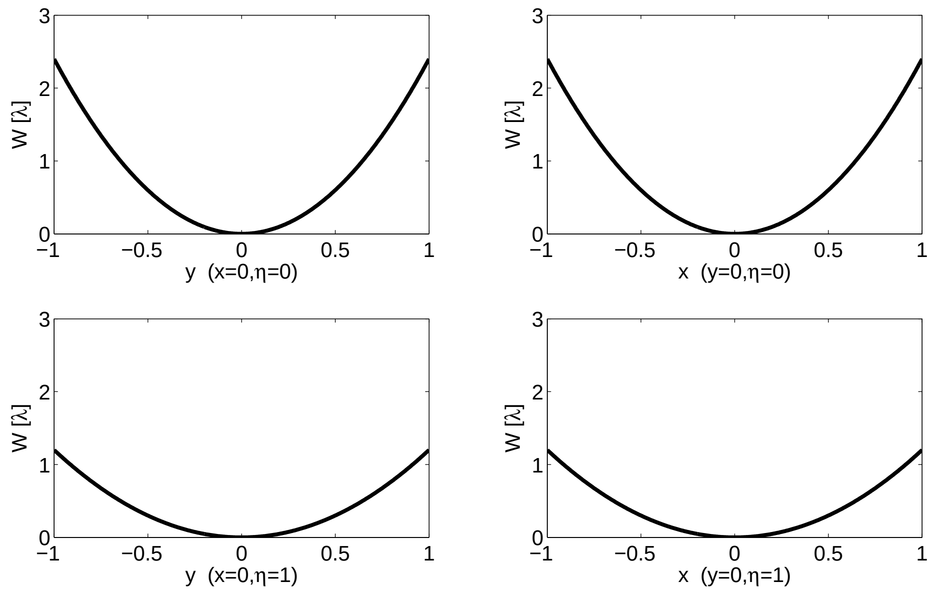
Solutions SK2330 Optical Design 2015-01-14 14-19 FD41

Grading limits: 0-10p F, 10p or over Fx, 12p or over D, 15p or over C, 19p or over B, 22–24p A

1. a) The straight lines indicate there is no spherical aberration or coma, as these are higher-order curves. Distortion and tilt would show up as offsets, and there are none. The remaining three - defocus, astigmatism and field curvature - all show up as straight lines. However, if there was astigmatism, there vould be a dfiference between tangential and sagittal plots for off-axis image points, and there is no such difference. The aberrations must be defocus, showing up at straight lines for the on-axis points, and field curvature, as the amount of aberration is different for no- and off-axis points.
   b) For defocus, the transverse aberration is given by $TA_x = -\frac{R}{nh} \cdot 2xW_{020}$. From the on-axis graph we see that $TA_x = -40\lambda$ at x=1, yielding $-40\lambda = -\frac{2R}{nh}W_{00}$. Using $R = 125$ mm, $h = 15$ mm and $n = 1$ yields $W_{020} = 2.4\lambda$. The transverse aberration for the off-axis point will be $TA_x = -\frac{R}{nh} \cdot 2x\left(W_{020} + \eta^2 W_{220}\right)$, as there is both defocus and field curvature. Using $TA_x = -20\lambda$ at $x = 1$ yields $W_{220} = -1.2\lambda$. Plotting the wavefront aberration gives the figure below.



2. Marginal ray: u is the angle of the ray leaving the object. After the surface the angle is $u' = nu$. The refraction invariant at the surface is $A = nu$. The change in u/n at the surface is

$$\Delta\left\{\frac{u}{n}\right\} = \frac{u'}{n'} - \frac{u}{n} = nu - \frac{u}{n} = u\frac{n^2 - 1}{n} \ . \tag{1}$$

   The height at the surface is $h = du$.
   Principal ray: $\bar{u}$ is the angle of the ray leaving the object, and $\bar{u}' = n\bar{u}$ is the angle

after the surface. The refraction invariant is $\bar{A} = n\bar{u}$.
Others: P=0 as the surface has no curvature.

The Seidel sums then become:

$$S_{\mathrm{I}} = -A^2 h \Delta \left\{ \frac{u}{n} \right\} = -\left(nu\right)^2 duu \frac{n^2-1}{n} = -du^4 n \left(n^2 - 1\right) \tag{2}$$

$$S_{\mathrm{II}} = -A\bar{A} h \Delta \left\{ \frac{u}{n} \right\} = -nun\bar{u}duu \frac{n^2-1}{n} = -d\bar{u}u^3 n \left(n^2 - 1\right) \tag{3}$$

$$S_{\mathrm{III}} = -\bar{A}^2 h \Delta \left\{ \frac{u}{n} \right\} = -\left(n\bar{u}\right)^2 duu \frac{n^2-1}{n} = -d\bar{u}^2 u^2 n \left(n^2 - 1\right) \tag{4}$$

$$S_{\mathrm{VI}} = 0 \tag{5}$$

$$S_{\mathrm{V}} = -\frac{\bar{A}}{A} \left[H^2 P + \bar{A}^2 h \Delta \left\{ \frac{u}{n} \right\} \right] = -\frac{\bar{A}}{A} \bar{A}^2 h \Delta \left\{ \frac{u}{n} \right\} \tag{6}$$

$$= -(n\bar{u})^3 \cdot \frac{1}{nu} duu \frac{n^2-1}{n} = -d\bar{u}^3 un \left(n^2 - 1\right) \tag{7}$$

3. For a thin lens with object at infinity, the conjugate factor is $Y = -1$, and hence

$$S_{\mathrm{I}} = \frac{h^4 K^3}{AX^2 - BX + C + D} \ . \tag{8}$$

As all coefficients before X^2 are positive, S_{I} as a function of X must be an X^2 curve which is positive on the edges, and which has a minimum value. If this minimum is positive, S_{I} must be positive for all X. If it is zero or negative, S_{I} will be zero for some x values. Taking the derivative with respect to X yields

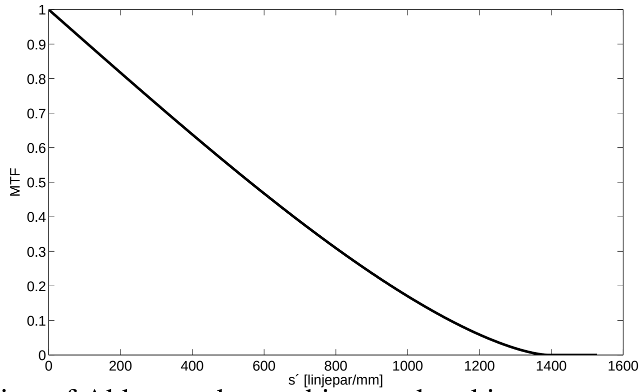
$$\frac{\partial S_{\mathrm{I}}}{\partial X} = \frac{h^4 K^3}{4} \left(2AX - B\right) \ . \tag{9}$$

Setting the derivative to zero yields $X = fracB2A$ at the minimum, and insertion yields the minumum value as

$$S_{\mathrm{I}}\big|_{\min} = \frac{h^4 K^3}{4} \left(C + D - \frac{B^2}{4A} \right) \tag{10}$$

which is positive at the given refractive index. (In fact, it is negative at all physically possible refractive indeces.) Hence S_{I} will always be positive.

4. Some ray construction in the figure yields the back principal plane located 55 mm before the image plane. The height of the marginal ray at this plane is 21 mm. Hence we can use $l' = 55$ mm and $D = 42$ mm, yielding the MTF curve below.



5. a) A reasonable definition of Abbe number at this waveband is

$$V = \frac{n_{4\mu\mathrm{m}} - 1}{n_{3\mu\mathrm{m}} - n_{5\mu\mathrm{m}}}. \tag{11}$$

Then the Abbe numbers of the materials are

Material	V
Silicon	235
Germanium	107
Zinc selenide	177
Zinc sulphide	113

For a doublet, two materials with different Abbe numbers should be used. As silicon differs a lot from the others, one material should be silicon, and the other for example germanium .

b) Choosing the first lens to be made of silicon and the second of germanium, the power of the first lens is given by

$$K_1 = \frac{V_1}{V_1 - V_2} K = \frac{235}{235 - 107} 10\,\mathrm{D} = 18.36\,\mathrm{D} \tag{12}$$

and the second by

$$K_2 = \frac{V_2}{V_2 - V_1} K = \frac{107}{107 - 235} 10\,\mathrm{D} = -8.36\,\mathrm{D}\,. \tag{13}$$

Choosing, for example, to let the last surface of the second lens be flat, $r_{22} = \infty$, the first surface must have the radius of curvature

$$r_{21} = \frac{n_2 - 1}{K_2} = \frac{4.0254 - 1}{-8.36}\mathrm{D} = -362\,\mathrm{mm}. \tag{14}$$

The second radius of the first lens must be the same, $r_{12} = r_{21} = -362\,\mathrm{mm}$. The first radius of the firs lens must then be given by

$$\frac{1}{r_{11}} = \frac{K_1}{n_1 - 1} + \frac{1}{r_{12}} \tag{15}$$

yielding a radius of $r_{11} = 97\,\mathrm{mm}$.