Recitation 11

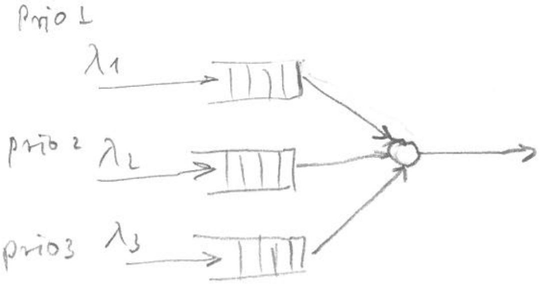
Chapter 11:

$\boxed{11.3}$

$\boxed{11.5}$

M/G/1 with priority:

- Low priority customer selected only if high priority queues are empty:



Non-preemptive: the service is completed even if higher priority customer arrives

Preemptive: the service is interrupted if higher priority customer arrives, and the service continues from the point of interruption

Non-preemptive:

Waiting time for a customer of priority i = Residual service time, R_s +

+ Service time of customers already waiting in queue i +
+ Service time of customers already waiting in queues $j<i$ (higher priority) +
+ Service time of customers arriving to queues $j<i$ while our customer is waiting

$$R_s = \frac{1}{2}\sum_{i=1}^{K} \lambda_i \cdot E[X_i^2], \quad W_i = \frac{R_s}{\left(1-\sum_{j=1}^{i-1}\rho_j\right)\cdot\left(1-\sum_{j=1}^{i}\rho_j\right)}, \quad W = \sum_{i=1}^{K} p_i \cdot W_i = \sum_{i=1}^{K}\frac{\lambda_i}{\lambda} W_i$$

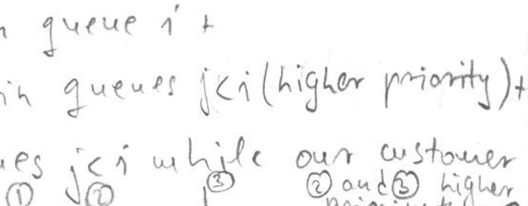
$$T_i = W_i + E[X_i]$$

Preemptive:

Waiting time for a customer of priority i = Time from arrival to the first service attempt =

= Residual service time, $R_{s,i}$ (now priority dependant) +

+ Service time of customers already waiting in queue i +
+ Service time of customers already waiting in queues $j<i$ (higher priority) +
+ Service time of customers arriving to queues $j<i$ while our customer is waiting



$$R_{s,i} = \frac{1}{2}\sum_{j=1}^{i} \lambda_j \cdot E[X_j^2], \quad W_i = \frac{R_{s,i}}{\left(1-\sum_{j=1}^{i-1}\rho_j\right)\cdot\left(1-\sum_{j=1}^{i}\rho_j\right)}$$

$$\left.\begin{aligned} T_i &= W_i + E[X_i'] \\ E[X_i'] &= \frac{E[X_i]}{1-\sum_{j=1}^{i-1}\rho_j}\end{aligned}\right\} \Rightarrow$$ Mean system time = Waiting time + service time including interruptions by arriving high priority customers

11.3 (1.)

Type A: Poisson (λ_A), $\lambda_A = 100 \frac{\text{packets}}{s}$, $L_A = 20$ bits, $E[X_A] = \frac{L_A}{C} = 2 \cdot 10^{-3} s$

Type B: Poisson (λ_B), $\lambda_B = 20 \frac{\text{packets}}{s}$, $L_B \sim Exp$, $\bar{L_B} = 100$bits, $E[X_B] = 10 \cdot 10^{-3} s$

$C = 10\,000 \frac{\text{bits}}{s}$

$\rho_A = \lambda_A \cdot E[X_A] = 0.2$, $\rho_B = \lambda_B \cdot E[X_B] = 0.2$

$E[X_A^2] = E[X_A]^2 = 4 \cdot 10^{-6}$, $E[X_B^2] = 2 \cdot E[X_B]^2 = 200 \cdot 10^{-6} s^2$

a.) Non-preemptive priority system, W_A, W_B, T_A, T_B, $\bar{W}$

a.1 Type A has prio 1

$$\bar{R}_S = \frac{1}{2} \cdot \left[\lambda_A \cdot E[X_A^2] + \lambda_B \cdot E[X_B^2]\right] = \frac{1}{2} \cdot \left[400 \cdot 10^{-6} + 4000 \cdot 10^{-6}\right] = 2.2 \cdot 10^{-3} [s]$$

$$W_A = \frac{\bar{R}_S}{1-\rho_A} = \frac{2.2 \cdot 10^{-3}}{0.8} = 2.75 \cdot 10^{-3} [s], \quad T_A = E[X_A] + W_A = 4.75 \cdot 10^{-3} [s]$$

$$W_B = \frac{R_S}{(1-\rho_A) \cdot (1-\rho_A-\rho_B)} = \frac{2.2 \cdot 10^{-3}}{(1-0.2) \cdot (1-0.4)} = 4.58 \cdot 10^{-3} [s], \quad T_B = E[X_B] + W_B = 14.58 \cdot 10^{-3} [s]$$

$$\bar{W} = \frac{\lambda_A}{\lambda_A + \lambda_B} \cdot W_A + \frac{\lambda_B}{\lambda_A + \lambda_B} \cdot W_B = 3.055 \cdot 10^{-3} [s]$$

a.2 Type B has prio. 1

$$W_A = \frac{\bar{R}_S}{(1-\rho_B) \cdot (1-\rho_B-\rho_A)} = 4.58 \cdot 10^{-3} [s]$$

$$W_B = \frac{\bar{R}_S}{(1-\rho_B)} = 2.75 \cdot 10^{-3} [s]$$

b.) Preemptive priority system, Type A is prio 1., W_A, W_B, T_A, T_B = ?

$$\bar{R}_A = \frac{1}{2} \cdot \lambda_A \cdot E[X_A^2] = 0.2 \cdot 10^{-3} [s] \quad \bar{R}_B = \frac{1}{2} \cdot \left[\lambda_A \cdot E[X_A^2] + \lambda_B \cdot E[X_B^2]\right] = 2.2 \cdot 10^{-3} [s]$$

$$W_A = \frac{\bar{R}_A}{(1-\rho_A)} = 0.25 \cdot 10^{-3} [s] \quad W_B = \frac{\bar{R}_B}{(1-\rho_A) \cdot (1-\rho_A-\rho_B)} = 4.58 \cdot 10^{-3} [s]$$

$$T_A = E[X_A] + W_A = 2.25 \cdot 10^{-3} [s] \quad T_B = E[X_B'] + W_B = 17.08 \cdot 10^{-3} [s]$$

$$E[X_B'] = \frac{E[X_B]}{1-\rho_A} = \frac{10 \cdot 10^{-3}}{0.8} = 12.5 \cdot 10^{-3}$$

M/G/1 with vacation:

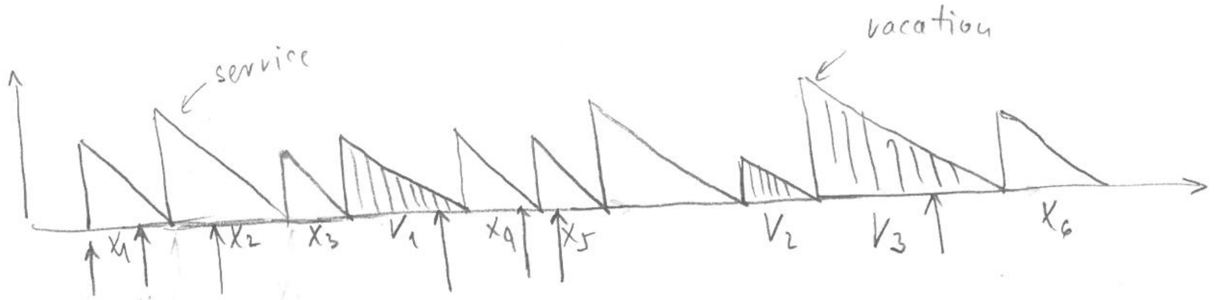
- Vacation: the server is not available for a while after the system gets idle. (no idle periods)
  vacation periods: independent, identicly distributed V

Arrival: Poisson(λ)

Service time: $E[X]$, $E[X^2]$

Vacation time: $E[V]$, $E[V^2]$

$\rho = \lambda \cdot E[X]$



$R_s(t)$ - remaining service time

$R_v(t)$ - remaining vacation time

$$R_v = \frac{(1-\rho)}{2} \cdot \frac{E[V^2]}{E[V]}$$

average remaining vacation time (includes also the periods when the system is busy)

$$R_s = \frac{\lambda \cdot E[X^2]}{2}$$

average remaining service time (includes also the periods when the system is on the vacation)

$$W = \frac{R_s}{1-\rho} + \frac{R_v}{1-\rho}$$

$$W = \frac{\lambda \cdot E[X^2]}{2 \cdot (1-\rho)} + \frac{E[V^2]}{2E[V]}$$

$$T = W + E[X]$$

mean system time

11.5 M/G/1 with vacation

indep.

Type 1: Poisson(λ_1), $\lambda_1 = 60 \frac{jobs}{s}$, $X_1 \sim Exp$, $E[X_1] = 12 \cdot 10^{-3} s$

Type 2: Poisson(λ_2), $\lambda_2 = 40 \frac{jobs}{s}$, $X_2 \sim Exp$, $E[X_2] = 6 \cdot 10^{-3} s$

$\Rightarrow \rho = \lambda_1 \cdot E[X_1] + \lambda_2 \cdot E[X_2] = 0.96$

Vacation: $V \sim Exp$, $E[V] = 2 \cdot 10^{-3} s$, $E[V^2] = 2 \cdot E[V]^2 = 8 \cdot 10^{-6} s$

FIFO, inf. buff.

a.) Remaining vacation time, including the ϕ at the busy periods?
(derived in class)

$$R_V = \frac{(1-\rho)}{2} \cdot \frac{E[V^2]}{E[V]} = \frac{0.04}{2} \cdot \frac{8 \cdot 10^{-6}}{2 \cdot 10^{-3}} = 0.08 \cdot 10^{-3} s$$

- Remaining vacation time, when job arrives to an empty system?
- Vacation period is exponential, so the remaining time after the first arrival is still exponential.

$\Rightarrow R_{V|empty} = E[V] = 2 \cdot 10^{-3} s$

b.) T, T_1, T_2

$T = \bar{W} + E[X]$, $T_1 = \bar{W} + E[X_1]$, $T_2 = W \cdot E[X_2]$

$$\bar{W} = \frac{\lambda \cdot E[X^2]}{2 \cdot (1-\rho)} + \frac{E[V^2]}{2 \cdot E[V]} = \dots = 0.254 s$$

$\lambda = \lambda_1 + \lambda_2$

$$E[X^2] = \frac{\lambda_1}{\lambda_1 + \lambda_2} \cdot E[X_1^2] + \frac{\lambda_2}{\lambda_1 + \lambda_2} \cdot E[X_2^2]$$

$\rho = \lambda_1 \cdot E[X_1] + \lambda_2 \cdot E[X_2] = \lambda \cdot E[X]$