SF2722 Differential Geometry, spring 2017
Homework 4

Examiner: Mattias Dahl, Hans Ringström.
Code of conduct (Hederskodex): It is assumed that:

- you shall solve the problems on your own and write down your own solution,
- if you in spite of this are using something you have gotten from somewhere else for some reason (a friend, a book or the internet etc.) you should give a reference to the source.

Deadline: Hand in your solutions before Tuesday March 21 (we will not accept solutions handed in after Tuesday March 28).

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1. In homework problem 3.3 a Lie group homomorphism $T : \mathcal{S} \to SO(3)$ is described. Find the Lie algebras of $\mathcal{S}$ and $SO(3)$, and the map between them induced by T.

2. Let M be a connected smooth manifold. Show that the group of diffeomorphisms of M acts transitively on M: that is, for any $p, q \in M$, there is a diffeomorphism $F : M \to M$ such that $F(p) = q$. [Hint: first prove that if $p, q \in \mathbb{B}^n$ (the open unit ball in $\mathbb{R}^n$), there is a compactly supported smooth vector field on $\mathbb{B}^n$ whose flow θ satisfies $\theta_1(p) = q$.]

(Lee Problem 9-7 on page 246.)

3. For each k-tuple of vector fields on $\mathbb{R}^3$ shown below, either find smooth coordinates (s^1, s^2, s^3) in a neighborhood of $(1, 0, 0)$ such that $V_i = \partial/\partial s^i$ for $i = 1, \ldots, k$, or explain why there are none.

(a) $k = 2$; $V_1 = \frac{\partial}{\partial x}$, $V_2 = \frac{\partial}{\partial x} + \frac{\partial}{\partial y}$.

(b) $k = 2$; $V_1 = (x+1)\frac{\partial}{\partial x} - (y+1)\frac{\partial}{\partial y}$, $V_2 = (x+1)\frac{\partial}{\partial x} + (y+1)\frac{\partial}{\partial y}$.

(c) $k = 3$; $V_1 = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$, $V_2 = y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}$, $V_3 = z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}$.

(Lee Problem 9-17 on page 247.)