EP2200 Queuing Theory and Teletraffic Systems

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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas, Laplace transforms and Erlang tables.

1. Consider a sensor network node that transmits packets with measurement results. The packets are of exponential size, the average packet transmission time is 10^{-3} seconds. Packets are generated randomly according to a Poisson process, 100 packets per second in average. The sensor node has little memory and therefore its transmission buffer can store up to 2 packets (in addition to the one under transmission).
a) Give the Kendall notation, define the states, and draw the state transmission diagram. (1p)
b) Calculate the packet loss probability, that is, the probability that an arbitrary packet needs to be dropped because the buffer is full, and the utilization of the output link. (3p)
c) Calculate the average waiting time of accepted packets and the probability that an accepted packet needs to wait more than 10^{-3} seconds. (2p)
Now assume, that the node saves energy by moving to *sleep mode* if the buffer becomes empty, and staying there for an exponentially distributed time with an average of 0.5 seconds. Packets that are generated during this time need to be stored. Again a maximum of two packets can be stored, others are dropped. When the node wakes up, it starts to serve the packets accumulated in the buffer. If the buffer is still empty, it moves immediately back to sleep mode.
d) Define the states and draw the state transmission diagram. (2p)
e) Calculate the probability that a sleep period is followed immediately by another one. (2p)

2. A call center has 10 clerks answering calls. The average call inter-arrival time is 1 minute, the average call holding time is 7 minutes. The call holding time has exponential distribution.
a) To simplify analysis, we assume that calls arrive according to ta Poisson process. Motivate why this assumption is reasonable. Give the Kendall notation of the system under this assumption. (2p)
Assume that calls arriving when all the clerks are busy are blocked.
b) Calculate the average number of calls blocked per hour and the average number of calls answered per hour. Give the probability that an arbitrary clerk is busy at a random point of time. (3p)
Now assume that arriving calls wait in an infinite buffer if all the clerks are busy.
c) Calculate the probability that an arriving call has to wait, the average waiting time and the probability that the call has to wait more than 1 minute. (3p)
d) Consider the calls that arrive when there is one call already waiting. Give the expected waiting and system times of these calls. Give the probability that these calls need to wait more than 1 minute. (2p)

3. The 4 secretaries of a small office share two copy machines. The secretaries work at their computers for 1 hour in average and then use the copy machine for 15 minutes in average. If both of the machines are occupied the secretaries go back to their computer again.
a) What assumptions do we need to make to model the system with a Markov chain with states representing the number of used copy machines? Motivate your answer. What would be the Kendall notation of the system? (2p)
b) Give the Markov chain of the system under the assumptions you made in part (a). Calculate the probability that both of the copy machines are used at an arbitrary point of time and the probability that a secretary coming to use them finds both machines occupied. (4p)
c) Give the utilization of one of the copy machines. (2p)
d) Consider a secretary who starts to copy when both of the machines are idle. What is the probability that noone comes to the other machine while he is copying? (2p)

4. The jobs in a batch-processing computer with a single CPU has been measured. Jobs arrive according to a Poisson process. There seems to be two kinds of jobs randomly intermixed. A third of the jobs have an average execution time of 62.5 milliseconds, while the other two thirds have 25 milliseconds expected execution time. The distribution is exponential for both types. Jobs wait in an infinite buffer if the processor is busy.
a) At what rate can jobs arrive to keep the system stable? (2p)
b) At what rate can jobs arrive to keep the average waiting time below 100 milliseconds? (3p)
c) Assume that the system starts garbage collection every time it becomes empty. The garbage collection has a fixed time of 10 milliseconds. What is the maximum arrival rate to keep the average waiting time below 100 milliseconds in this case? (3p)
d) Consider the jobs that arrive when the CPU is performing garbage collection. What is the remaining garbage collection time observed by these jobs? (2p)

5. Consider a queuing network of two nodes with exponential service times with $\mu_1 = \mu_2 = 1$ service per time unit and infinite buffers. Customers enter the network at the first node. They arrive according to a Poisson process with intensity $\lambda = 0.5$ arrivals per time unit. After service at the first node all customers join the second node. After service at the second node the customers leave the network with probability 0.7, otherwise they join the first node again.
a) Draw the queuing network and calculate the arrival intensities at the queues. (2p)
b) Prove that the second node can be modeled as an M/M/1 system. (3p)
c) Calculate the utilization of the servers and the probability that there are no customers in the entire network. (2p)
d) Calculate the average number of nodes visited by a customer, and the average time customers spend in the network. (3p)