2015 March 16

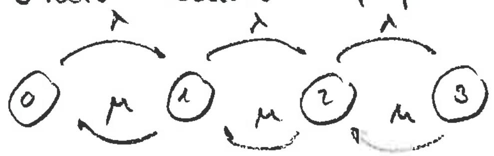
① $\lambda = 100$ packets/s

$\bar{x} = 10^{-3}$ sec. $\Rightarrow \mu = \underbrace{1000}$ packets/sec. $\Big\}$ $\rho = \frac{\lambda}{\mu} = 0.1$

buffer = 2

a) M / M / 1 / 3

State: number of packets in the system



b) Steady state, state probabilities:

$$\left.\begin{aligned} \lambda p_0 &= \mu p_1 \\ \lambda p_1 &= \mu p_2 \\ \lambda p_2 &= \mu p_3 \\ p_0 + p_1 + p_2 + p_3 &= 0 \end{aligned}\right\} \quad p_0(1+\rho+\rho^2+\rho^3) = 1$$

$$p_0 = \frac{1}{1.111}, \quad p_3 = \frac{0.001}{1.111} = 0.0009$$

$p(\text{dropp}) \overset{\text{PASTA}}{=} \underline{p_3 = 0.0009}$, utilization $= \lambda_{eff} \cdot \bar{x} = (1-p_3)\lambda\bar{x} = 0.099$

c) $N_q = 1 \cdot p_2 + 2 \cdot p_3 = \frac{0.012}{1.111} = 0.00108$

$\bar{W} = \frac{N_q}{\lambda_{eff}} \approx 10^{-4}$ s

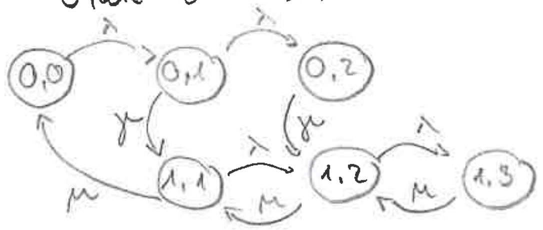
P(accepted packet needs to wait more than 10^{-3} s) =

$\frac{p_1}{p_0+p_1+p_2} P(w > 10^{-3}\text{s} \mid p_1) + \frac{p_2}{p_0+p_1+p_2} P(w > 10^{-3}\text{s} \mid p_2) =$

$\frac{0.1}{1.11} P(\emptyset \text{ service in } 10^{-3}\text{s}) + \frac{0.01}{1.11} P(0 \text{ or } 1 \text{ service in } 10^{-3}\text{s}) =$

$\frac{0.1}{1.11} \cdot e^{\overbrace{-\mu t}^{-1}} + \frac{0.01}{1.11}\left(e^{-\mu t} + \overbrace{\mu t}^{1} e^{-\mu t}\right) = \frac{0.12}{1.11} \cdot e^{-1} \approx 0.039$

d) State: {mode, packets} mode: 0 sleep, 1: awake, $\gamma = \frac{1}{0.5} = 2$, $\bar{s} = 0.5$ s



e) P(sleep followed by sleep) =

P(no arrival in sleep) =

$\int_0^\infty e^{-\lambda t} \gamma e^{-\gamma t} dt = \ldots = \frac{\gamma}{\lambda + \gamma} = 0.019$

(2)

$\lambda = 1$ call/min

$\bar{x} = 7\text{min} \Rightarrow \mu = \frac{1}{7}$ call/min $\Big\}$ $a = \lambda \bar{x} = 7$

a) Poisson arrival process: good assumption if calls arrive from a large population, and are generated independently from each other. This holds for call center traffic. $\Rightarrow$ M/M/10/10

b) $P(\text{block}) = E_{10}(7) \overset{\text{Erlang table}}{=} = 0.078$

Average number of calls blocked per hour $= \lambda \cdot 60\text{min} \cdot P(\text{block}) = 4.68$

answered per hour $= \lambda \cdot 60 \cdot (1 - P\text{block}) = 55.32$

$P(\text{clerc busy}) = \text{utilization} = \dfrac{a \cdot (1 - p(\text{block}))}{m} = 0.6454$

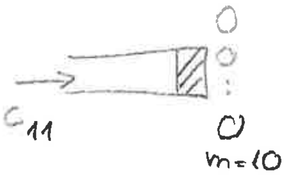
c) M/M/10

$$P(\text{wait}) = \frac{m \cdot E_m(a)}{m - \varrho(1 - E_m(a))} = 0.22$$

$$\bar{W} = \frac{1}{m\mu - \lambda} P(\text{wait}) = 0.513 \text{ min}$$

$$P(w > 1\text{min}) = P(\text{wait}) e^{-\mu(m-a)\cdot 1} = 0.22\, e^{-\frac{1}{7}\cdot 3} = 0.143$$

d)



C_{11} has to wait for 2 finished services. average time between services: $\bar{\tau} = \frac{\bar{x}}{m} = 0.7\text{min}$

$$\bar{w}_{11} = 2 \cdot \bar{\tau} = 1.4\text{ min}$$

$$\bar{T}_{11} = 2 \cdot \bar{\tau} + \bar{x} = 8.4 \text{ min}$$

$P(w_{11} > 1\text{min}) = P(\emptyset \text{ or } 1 \text{ service in } 1 \text{ minute}) =$

$$e^{-m\mu} + m\mu e^{-m\mu} = \left(1 + \frac{10}{7}\right) e^{-\frac{10}{7}} = 0.582$$

③ $K=4$

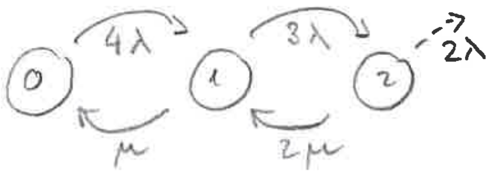
$M=2$

a) We can model the system with an MC where the states represent the number of customers under service, if the service and interarrival processes are memoryless ⇒ Exponential.

⇒ $M/M/2/2/4$

b) $\lambda = 1$ arrival / hour / per secretary

$\bar{x} = 0.25$ hour → $\mu = 4$ service/hour



$p_0 \cdot 4 = p_1 \cdot 4 \Rightarrow p_1 = p_0$

$p_1 \cdot 3 = p_2 \cdot 8 \Rightarrow p_2 = \frac{3}{8} p_1$

$p_0 + p_1 + p_2 = 1 \Rightarrow p_0 (1 + 1 + \frac{3}{8}) = 1$

$p_0 = \frac{8}{19}$, $p_1 = \frac{8}{19}$, $p_2 = \frac{3}{19}$

P(both machine used) = {time blocking} = $p_2 = \frac{3}{19}$

P(arriving secretary finds the machines busy) = {call blocking} =

$$= \frac{2\lambda p_2}{4\lambda p_0 + 3\lambda p_1 + 2\lambda p_2} = \frac{6}{32+24+6} = \underline{\underline{\frac{6}{62}}} = \frac{3}{31} \quad (\ll p_2 \,!)$$

c) $U = \frac{\bar{N}}{2} = \frac{p_1 + 2p_2}{2} = \frac{7}{19}$ $\left(\text{Also: } \frac{p_0 4\lambda\bar{x} + p_1 3\lambda\bar{x}}{2} = \frac{4}{19} \cdot \frac{7}{4} = \frac{7}{19}\right)$

d) P(secretary finishes copying alone) = P(no arrivals while copying) =

$$\int_0^\infty e^{-3\lambda t} \cdot \mu \cdot e^{-\mu t}\, dt = \frac{\mu}{\mu + 3\lambda} = \frac{4}{7}$$

4. $M/H_2/1$

A: $E[x_A] = 62.5\ ms$, $Exp(\mu_A)$ $\mu_A = 16$ jobs/s $p_A = \frac{1}{3}$

B: $E[x_B] = 25ms$, $Exp(\mu_B)$ $\mu_B = 40$ jobs/s $p_B = \frac{2}{3}$

a) $E[x] = \frac{1}{3}E[x_A] + \frac{2}{3}E[x_B] = \frac{62.5 + 50}{3} = \frac{112.5}{3} = 37.5\ ms$

Stability: $a = \lambda E[x] < 1$

$\lambda < \frac{1}{37.5}$ arrival/ms $= 26{,}66$ arrival/s

b)

$\bar{W} = \frac{\lambda E[x^2]}{2(1-\lambda E[x])} < 0.1\,s = W_M$

Exp: $E[x] = \frac{1}{a}$

$E[x^2] = \frac{2}{a^2} = 2 \cdot E[x]^2$

$E[x^2] = \frac{2}{3}E[x_A^2] + \frac{4}{3}E[x_B^2] =$

$= 3437.5\ ms^2$

$2W_M - 2\lambda E[x]\bar{W}_M > \lambda E[x^2]$

$\frac{2W_M}{2E[x]W_M + E[x^2]} > \lambda$

$\frac{200}{2 \cdot 3750 + 3437.5} > \lambda$

$\lambda < 18.2$ arrivals/sec

c) The garbage collection gives a fixed term to the average waiting time:

$\bar{W} = \frac{\lambda E[x^2]}{2(1-\lambda E[x])} + \frac{\overline{V^2}}{2\bar{V}}$

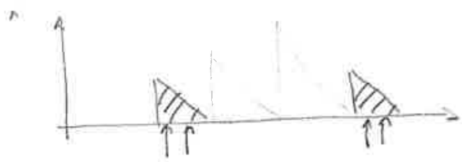
$V \sim Det(10ms) \Rightarrow \frac{\overline{V^2}}{2\bar{V}} = \frac{V^2}{2V} = 5ms$

$\Rightarrow$ Same as above, but $W_M = 95ms$

$\Rightarrow \lambda < \frac{2 \cdot 95}{2 \cdot 37.5 \cdot 95 + 3437.5} = 17.9$ arrivals/sec.

d) 

Arrivals happen according to a uniform distribution.



$\bar{R}_S = \frac{n \cdot \frac{V^2}{2}}{n \cdot V} = \frac{V}{2} = 5ms$

(5)

$\mu_1 = \mu_2 = 1$
$\lambda = 0.5$

a) $\lambda_2 = \lambda_1$

$\lambda_1 = \lambda + 0.3\lambda_2 = \lambda + 0.3\lambda_1$

$\lambda_1 = \frac{\lambda}{1-0.3} = \frac{\lambda}{0.7} = \frac{5}{7} = 0.71$

$\lambda_2 = \frac{\lambda}{0.7}$

b) Second node can be modeled as M/M/1 if arrival is Poisson:
Laplace transform of the inter-arrival time = dep. time at the first node:

$$T(s) = \rho_1 \frac{\mu_1}{s+\mu_1} + (1-\rho_1)\frac{\lambda_1}{s+\lambda_1}\frac{\mu_1}{s+\mu_1} =$$

$$= \frac{\lambda_1}{\mu_1}\frac{\mu_1}{s+\mu_1} + \frac{\lambda_1\mu_1}{(s+\lambda_1)(s+\mu_1)} - \frac{\lambda_1}{\cancel{\mu_1}}\frac{\lambda_1}{s+\lambda_1}\frac{\cancel{\mu_1}}{s+\mu_1} =$$

$$= \frac{\lambda_1(s+\cancel{\lambda_1}) + \lambda_1\mu_1 - \cancel{\lambda_1^2}}{(s+\mu_1)(s+\lambda_1)} = \frac{\lambda_1}{s+\lambda_1} \Rightarrow$$ Interarrival is $Exp(\lambda_1) \Rightarrow M/M/1$

c) $\rho_1 = \rho_2 = \frac{\lambda_1}{\mu} = \frac{5}{7}$, $P(\text{empty}) = p_{10} \cdot p_{20} = (1-\rho_1)(1-\rho_2) = \frac{4}{49} = 0.081$

d) Average number of visited nodes: $V = \frac{\sum_i \lambda_i}{\lambda} = \frac{2 \cdot \frac{5}{7}}{1/2} = \frac{20}{7}$

Average time in the network:

$T = \frac{N}{\lambda} = 2\left(\frac{\rho_1}{1-\rho_1}\right) \cdot 2 = 4 \cdot \frac{\frac{5}{7}}{\frac{2}{7}} = 10$ time units