EP2200 Queuing Theory and Teletraffic Systems

Saturday, June 13, 2015, 09:00-14:00

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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas and Laplace transforms and Erlang tables.

1. Consider a system with two servers, S_1 and S_2. Packets arrive to the system following a Poisson Process with intensity $\lambda = 2000$ packets per second. With probabilities, $p_1 = 0.4$ and $p_2 = 1 - p_1$ an arbitrary packet is forwarded to server S_1 or S_2, respectively. The service times are exponential with rates $\mu_1 = \mu_2 = 1500\text{sec}^{-1}$. Each server has a dedicated buffer that can hold at most 5 packets. Packets arriving to a server with a buffer that is full are dropped. Waiting packets are served at each server on a first-come-first-served basis.

a) Provide the proof that packets arrive to a given server according to a Poisson Process, and give the arrival rates. (2p)

b) Calculate the probability that an arbitrary arriving packet is rejected. Consider an arbitrary rejected packet. What is the probability that the packet has been rejected by S_1?(3p)

Consider, now, that there is a common queue for the two servers and the waiting packets are served, on a first-come-first-served basis, by an available server. The buffer capacity is 10.

c) Calculate the probability that once a packet is rejected, the following arrival is rejected as well. (2p)

d) Assume that, upon arrival, a packet finds 2 other packets waiting in the common queue. Derive the probability that the waiting time of this packet is larger than 1msec. (3p)

2. Consider a local telephone switch with 3 outgoing lines. The lines are used by 5 secretaries. When on the phone, secretaries talk for an exponentially distributed time with an average of 3 minutes. Between phone calls they work at their computers for an exponentially distributed time for an average of 20 minutes. If a secretary tries to make a call when all the lines are busy she goes back to her computer, to work again for an exponentially distributed time with an average of 20 minutes.

a) Give the Kendall notation of the system and draw the state transition diagram. (2p)

b) Calculate the probability that all the phone lines are busy and the probability that a secretary finds all lines busy when she tries to make a call. (3p)

c) Give the distribution and the mean length of the time periods when all phone lines are idle, and when all phone lines are busy. (2p)

d) Assume that calls cost 100 SEK per hour on each phone line. How much does the company pay per day (assume 8 working hours per day)? (3p)

3. A source node attempts to transmit data packets to a destination node. The capacity of the link between the source and the destination is 20000 bits/s. Packets arrive at the source as a Poisson process with a mean of 10 packets/s, and are waiting in a FIFO queue with infinite capacity if there is a packet on transmission. 40% of packets are type I packets, whereas the remaining packets are type II. The length of every type I packet is exponentially distributed with a mean of 1000 bits, whereas the length of every type II packet is Erlang-2 distributed, again with a mean of 1000 bits.

a) Give the queuing model of the system with Kendall notation, and the mean and the coefficient of variation of the packet transmission times. (3p)

b) Calculate the probability that the source is busy (i.e. transmitting a packet)? Give the probability density function of the waiting time of an arriving packet that finds the source busy but the queue empty.(2p)

c) Calculate the mean delay (waiting time + transmission time) of an arbitrary packet, and of the two types of packets, respectively. (2p)

d) Assume now that the source can not buffer any packets. If a packet is generated, when another packet is still under transmission, the new packet is dropped. Define suitable system states, and give the Markov-chain of the system. (3p)

4. Jobs arrive to a server according to a Poisson process with intensity 1 jobs per time unit. Half of the jobs has deterministic service time of 1 time unit, the other half of the jobs has deterministic service time of 0.5 time units. Jobs that arrive when the server is busy are waiting in an infinite buffer.

a) Give the model of the queuing system with Kendall notation, and calculate the average waiting time and the average system time (that is, waiting + service time). (3p)

Assume, that jobs with the shorter service time are served with preemptive resume priority.

b) Calculate the probability that a low priority job is served without interruption. (2p)

c) Calculate the mean waiting and system time of the high priority jobs. (2p)

d) Calculate the mean waiting and system time of the low priority jobs. (3p)

5. Consider an open queuing network with two nodes. Each of the nodes has a single server, and infinite buffer capacity. The service times are exponential with $\mu_1 = 1$ service per time unit at node 1, and $\mu_2 = 2$ services per time unit at node 2. Jobs arrive to the queuing network, according to a Poisson process, both to node 1 and to node 2, with intensities λ_1 and λ_2. After service at a node, jobs leave the queuing network with probability 0.5. Otherwise they proceed to the other node, for further service.

a) Draw the queuing network, and give all the known parameters. (2p)

b) You measure the utilization of the two nodes, and you find that node 1 is busy in $\frac{1}{2}$ part of the time, while node 1 is busy in $\frac{1}{4}$ part of the time. Based on this information, calculate λ_1 and λ_2. (3p)

c) Calculate the probability that the queuing network is empty, the average number of jobs in the network, and the average time a job spends in the network. (3p)

d) Calculate the average number of services a job receives, before it leaves the network. (2p)