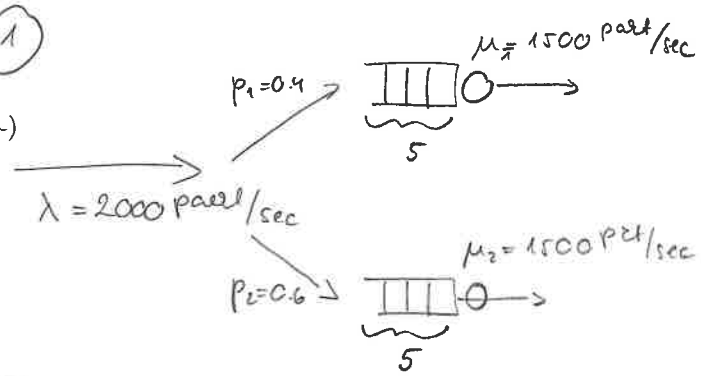
EP2200 Make-up exam

2015, June 13.

(1)

a)



a) We need to prove, that random demultiplexing of Poisson arrivals gives Poisson process: eq. interarrivals to S_1:

$$P(\tau_1 > t) = P(\text{no arrival to } S_1 \text{ in } t) =$$

$$P(\text{no arrivals}) + \sum_{k=1}^{\infty} P(\text{all } k \text{ directed to } S_2 \mid k \text{ arrivals in } t) P(k \text{ arrival in } t) =$$

$$= \sum_{k=0}^{\infty} (1-p_1)^k \cdot \frac{(\lambda t)^k}{k!} e^{-\lambda t} = \sum_{k=0}^{\infty} \frac{((1-p_1)\lambda t)^k}{k!} e^{-\lambda t} = e^{(1-p_1)\lambda t} e^{-\lambda t} = e^{-p_1 \lambda t}$$

$\Rightarrow$ Interarrival time $\tau_1 \sim Exp(p_1 \lambda) \Rightarrow \lambda_1 = p_1 \lambda = 800$ pkt/sec

Similarly: $\Rightarrow \lambda_2 = p_2 \lambda = 1200$ pkt/sec

b) $P(\text{arriving pack is rejected}) = \sum_{i=1}^{2} P(\text{arriving packet to } S_i \text{ is rejected}) \underbrace{P(\text{packet sent to } S_i)}_{p_1,\, p_2}$

$P(\text{arriving packet to } S_i \text{ is rejected}) = P_6(\lambda_i, \mu_i) =$

Formula sheet $= \dfrac{(1-\rho_i)\rho_i^6}{1-\rho_i^7}$

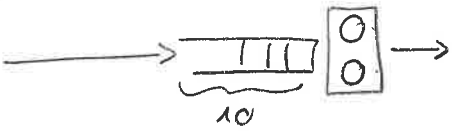
$P_{block,1} = P_6(800, 1500) = 0.01$

$P_{block,2} = P_6(1200, 1500) = 0.066$

$P(\text{rejection}) = 0.043$

$$P(\text{rejected at } S_1 \mid \text{rejected}) = \frac{P(\text{rejected at } S_1)}{P(\text{rejected})} = \frac{p_1 P_{block,1}}{P(\text{rejection})} = 0.1$$

1.c.

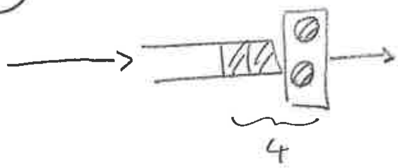


P(packet rejected | previous rejected as well) =

P(arrival before departure from any server) =

$$\int_0^\infty e^{-2\mu t}\lambda e^{-\lambda t}\,dt = \dots = 0.4$$

1.d



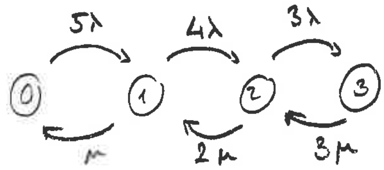
$P(w_4 > 1\,ms) = P(0, 1, \text{ or } 2 \text{ services in } 1\,ms) =$

$$\sum_{i=0}^{2} \frac{(2\mu t)^i}{i!} e^{-2\mu t} = \dots = 0.423$$

(2)

a) $M/M/3/3/5$

$$\left.\begin{aligned}\mu &= \frac{1}{3} \text{ call/min} \\ \lambda &= \frac{1}{20} \text{ call/min}\end{aligned}\right\} \rho = \frac{\lambda}{\mu} = \frac{3}{20}$$



b)
$$\left.\begin{aligned} p_0 \cdot 5\lambda &= p_1 \mu \\ p_1 \cdot 4\lambda &= p_2 \cdot 2\mu \\ p_2 \cdot 3\lambda &= p_3 \cdot 3\mu \\ p_0 + \dots p_3 &= 1 \end{aligned}\right\} \quad \begin{aligned} p_0 &\approx 0.49 \\ p_3 &\approx 0.016 \end{aligned}$$

$P(\text{all lines blocked}) = p_3 = 0.016$

$$P(\text{call is blocked}) = \frac{2\lambda p_3}{5\lambda p_0 + 4\lambda p_1 + 3\lambda p_2 + 2\lambda p_3} \approx 0.0072$$

c)

All lines are idle $\approx$ time to next call when all lines are idle $\approx \mathrm{Exp}(5\lambda) = \mathrm{Exp}\left(\frac{1}{4}\right) \Rightarrow \bar{\tau}_i = 4$ min

All lines are busy $\approx$ time to next service when 3 calls on $\approx$ $\mathrm{Exp}(3\mu) = \mathrm{Exp}(1) \Rightarrow \bar{\tau}_b = 1$ min

d) Average arrival intensity: $\bar{\lambda} = p_0 \cdot 5\lambda + p_1 \cdot 4\lambda + p_2 \cdot 3\lambda \approx 0.21$

Busy hours in $T = \bar{\lambda} \cdot T \cdot \frac{1}{\mu} = 5.22$

$\Rightarrow$ Pay ca. 522 SEK per day.

(3)

$C = 20 \cdot 10^3$ bits

$\lambda = 10$ pkt/s

Type 1 $\quad p_1 = 0.4 \quad Exp(\mu_1), \; L_1 = 1000$ bit

Type 2 $\quad p_2 = 0.6 \quad$ Erlang-2, $L_2 = 1000$ bits

a) $M/G/1$ (parallel service of an Exp and an Erlang-2 server)

$\bar{X}_1 = \frac{L_1}{C} = 0.05\,s, \quad \mu_1 = 20$ pkt/s, $\quad \overline{X_1^2} = \frac{2}{\mu^2} = 0.005, \quad Var[x] = 0.0025$

$\bar{X}_2 = 0.05\,s, \quad \mu_2 = 20$ pkt/s $\quad \overline{X_2^2} = Var(x_2) + \bar{x}_2^2 = 0.00375 \quad Var[x] = 0.00125$

$\bar{x} = p_1 \cdot \bar{x}_1 + p_2 \bar{x}_2 = 0.05\,s$

$\overline{x^2} = p_1 \overline{x_1^2} + p_2 \overline{x_2^2} = 0.00425\,s^2$

$C_x^2 = \frac{Var[x]}{\bar{x}^2} = \underline{\underline{0.7}}$

$Var[x] = \overline{x^2} - \bar{x}^2 = 0.00175$

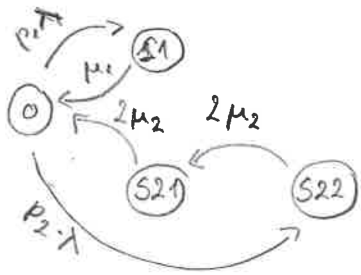
b). $P(\text{source busy}) = \{\text{single server, no loss}\} = \lambda \bar{x} = 0.5 = \rho$

Sorry, the second part was more complicated than planned. Not considered for grading. (Point given)

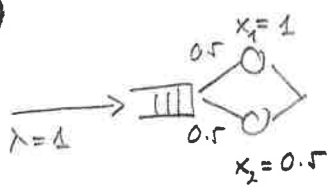
c) $W = \frac{\lambda \overline{x^2}}{2(1-\rho)} = \frac{10 \cdot 0.00425}{2(1-0.7)} = 0.0035\,s$

Since $\bar{X} = \bar{x}_1 = \bar{x}_2$, $T = T_1 = T_2 = 0.0535\,s$

d) States: $\{0, S1, S21, S22\}$



(4)



a) M/G/1

$\bar{x} = 0.5 \cdot 1 + 0.5 \cdot 0.5 = 0.75 = \frac{3}{4} \Rightarrow \rho = \lambda \bar{x} = \frac{3}{4}$

$x_1^2 = 1 \quad x_2^2 = 0.25 \quad \bar{x^2} = 0.5 \cdot 1 + 0.5 \cdot 0.25 = 0.625 = \frac{5}{8}$

$\bar{W} = \frac{\lambda \bar{x^2}}{2(1-\rho)} = 1.25 , \quad \bar{T} = \bar{W} + \bar{x} = 2$

b) Preemptive resume for type 2 packets.

P(type 1 served without interruption) = P(no type 2 in $\frac{1}{2}$ time unit)

$= e^{-0.5 \cdot \lambda \cdot x_1} = e^{-0.5 \cdot 1/2} \approx e^{-\frac{1}{4}}$

c) High priority can see its own queue only

M/G/1 with $\rho_2 = \frac{1}{2} \lambda \cdot x_2 = 0.25$

$W_2 = \frac{\lambda_1 \bar{x_2}^2}{2(1-\rho_2)} = \frac{0.5 \cdot 0.25}{2 \cdot 0.75} = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{12} , \quad T_2 = W_2 + x_2 = \frac{1}{12} + \frac{1}{2} = \frac{7}{12}$

d) $R_1 = \frac{1}{2}\left(\lambda_1 x_1^2 + \lambda_2 x_2^2\right) = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4}\right) = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$

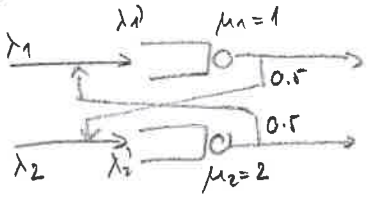
(Note, now type 2 is the high prio!)

$\bar{T}_1 = \frac{(1-\rho)\bar{x}_1 + \bar{R}_1}{(1-\rho_2)(1-\rho)} = \frac{\frac{1}{4} \cdot 1 + \frac{5}{16}}{(1-\frac{1}{4}) \cdot (1-\frac{3}{4})} = \frac{\frac{9}{16}}{\frac{3}{4} \cdot \frac{1}{4}} = 3$

$\bar{W}_1 = \frac{\bar{R}_1}{(1-\rho_2)(1-\rho_1)} = \frac{\frac{5}{16}}{\frac{3}{16}} = \frac{5}{3}$

5

a)



M/M/1

M/M/1

b) $\rho_1 = \frac{1}{2} = \lambda_1' / \mu_1 \Rightarrow \lambda_1' = \frac{1}{2}$

$\rho_2 = \frac{1}{4} = \lambda_2' / \mu_2 \Rightarrow \lambda_2' = \frac{1}{2}$

$\Rightarrow \left.\begin{array}{l} \lambda_1' = \lambda_1 + \frac{1}{2}\lambda_2' \\ \lambda_2' = \lambda_2 + \frac{1}{2}\cdot\lambda_1' \end{array}\right\} \Rightarrow \begin{array}{l} \lambda_1 = \frac{1}{4} \\ \lambda_2 = \frac{1}{4} \end{array}$

c) $P(\text{empty network}) = \prod_{i=1}^{2} P(\text{node } i \text{ is empty}) = (1-\rho_1)(1-\rho_2) = \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8}$

$N_1 = \frac{\rho_1}{1-\rho_1} = \frac{1/2}{1/2} = 1 \qquad N_2 = \frac{\rho_2}{1-\rho_2} = \frac{1/4}{3/4} = \frac{1}{3} \qquad N = N_1 + N_2 = \frac{4}{3}$

$T = \frac{N}{\lambda} = \frac{N_1 + N_2}{\lambda_1 + \lambda_2} = \frac{4/3}{1/2} = \frac{8}{3}$

d) Average number of services $= V = \frac{\lambda_1' + \lambda_2'}{\lambda_1 + \lambda_2} = \underline{\underline{2}}$