SF1624 Algebra och geometri
Exam
Monday, 13 March 2017

KTH Teknikvetenskap

Time: 08:00-11:00
No books/notes/calculators etc. allowed
Examiner: Tilman Bauer

This exam consists of six problems, each worth 6 points.

Part A comprises the first two problems. The bonus points from the seminars will be automatically added to the total score of this part, which however cannot exceed 12 points.

The next two problems constitute part B, and the last two problems part C. The latter is mostly for achieving a high grade.

The thresholds for the respective grades are as follows:

Grade	A	B	C	D	E	Fx
Total sum	27	24	21	18	16	15
of which in part C	6	3	–	–	–	–

To get full score on a problem, your solution must be well-presented and easy to follow. In particular, you should define your notation; clearly explain the logical structure of your argument in words or symbols; and motivate and explain your argument. Solutions severely lacking in these respects will achieve at most 3 points.

PART A

1. Consider the vectors $\vec{P} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \vec{Q} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \vec{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Let l_1 be the line through $\vec{P}$ and $\vec{Q}$, and let l_2 be the line parallel to $\vec{u}$ which goes through $\vec{P}$.

(a) Find vector equations for the lines l_1 och l_2. **(3 p)**

(b) Find an equation for the plane containing the lines l_1 and l_2. **(3 p)**

2. Consider the following matrix:

$$A(t) = \begin{bmatrix} t & 1 & 10-t \\ 1 & 0 & -1 \\ 10 & 1 & t \end{bmatrix}$$

(a) For which values of t is $A(t)$ invertible? **(2 p)**

(b) Give some value of t such that the corresponding system

$$A(t)\vec{x} = \vec{0}$$

has infinitely many solutions and find the solution set. **(3 p)**

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p.t.o.

Part B

3. The subspace V of $\mathbb{R}^4$ is spanned by the vectors

$$\vec{u} = \frac{1}{3}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \quad \vec{v} = \frac{1}{5}\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{w} = \frac{1}{7}\begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \end{bmatrix}.$$

(a) Find a nontrivial linear relation between the vectors $\vec{u}, \vec{v}$ and $\vec{w}$. **(3 p)**

(b) Compute the distance between the point $\vec{P} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$ and V. **(3 p)**

4. Consider the following map:

$$F : \mathbb{R}^2 \to \mathbb{R}^2, \quad F(x, y) = (0, x)$$

(a) Find all eigenvalues and corresponding eigenspaces of F. **(3 p)**

(b) Determine if the matrix for F is diagonalizable. **(3 p)**

Part C

5. The subspace V of $\mathbb{R}^3$ is given by the equation $x - 2y + z = 0$. Let $T \colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - 2y + 3z \\ 2x - 4y + 6z \\ 3x - 6y + 9z \end{bmatrix}.$$

(a) Find a basis $\mathcal{B} = \{\vec{u}, \vec{v}, \vec{w}\}$ for $\mathbb{R}^3$ such that V is spanned by $\vec{u}$ and $\vec{v}$. **(2 p)**

(b) Show that $T(\vec{x})$ lies in V for all $\vec{x}$ which lie in V. **(2 p)**

(c) Determine the matrix representation for the map T with respect to the basis $\mathcal{B}$ of $\mathbb{R}^3$. **(2 p)**

6. The multiplication table M is defined as the 9×9 matrix whose entries are given by the formula $M_{ij} = i \cdot j$, där $i, j = 1, 2, \ldots, 9$.

(a) Compute the rank of M. **(2 p)**

(b) Show that the number $1^2 + 2^2 + \cdots + 9^2 = 285$ is an eigenvalue for M. What are the corresponding eigenvectors? Then, determine all remaining eigenvalues for M (you do not need to find eigenvectors). **(4 p)**