EP2200 Queuing Theory and Teletraffic Systems

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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas, Laplace transforms and Erlang tables.

1. Consider a zoo park where arriving visitors have to wait in a common queue in order to buy a ticket. Only one clerk is working and it takes on average two minutes to serve a visitor, with an exponentially distributed time. Visitors are served in a FIFO fashion and each visitor buys only one ticket. We consider that a visitor buying a ticket still takes up a place in the queue. Visitors arrive in a Poisson fashion with an intensity of one per four minutes.
a) Give the Kendall notation of the above system and draw the state transition diagram. (2p)
b) Calculate the probability that an arriving visitor has to wait to buy a ticket, and the average waiting time considering all visitors. (2p)
c) Calculate the probability that an arriving visitor will need to wait for more than 10 minutes in the queue. Calculate also the probability that a visitor will need less than 15 minutes to buy a ticket (including the waiting time and the time that the clerk needs to serve the visitor). (2p)
Assume now that only seven visitors can stay in the queue.
d) What is the probability that an arriving visitor is not able to join the queue? What is the probability that an arriving visitor finds exactly five others already in the queue? What is the average waiting time of such visitor? (2p)
e) Consider the periods when the queue is full. Calculate the probability that no visitors arrive and have to be rejected during such a period. (2p)

2. Consider an Internet Café with five computers connected to the Internet. Customers who would like to use a computer arrive with exponentially distributed interarrival times, with a mean of a ten minutes. Each customer uses the Internet for a time that is exponentially distributed with a mean of 30 minutes. Customers arriving when all computers are occupied decide to order a coffee and wait for one of the computers to become free. Assume that the number of seats in the waiting area is very large and can be approximated as infinite.
a) Give the Kendall notation of the system. Calculate the mean number of customers in the café. Assuming that the computers are selected randomly, what is the percentage of time a computer is busy? (2p)
b) Calculate the probability that a customer does not have to wait to access the Internet. Consider a customer that arrives when all computers are busy but noone is waiting. What is the expected waiting time of such a customer? What is the probability that she needs to wait more than ten minutes? (3p)
Assume now that customers arriving when all computers are occupied decide to leave the café immediately.
c) Calculate the probability that an arriving customer will leave the café, without using a computer. Assuming that the computers are selected randomly, what is the percentage of time a computer is busy in this case? (2p)
d) Calculate the mean length of the time periods when the café can not accept the arriving customers (that is, in blocking state) and the mean length of the periods when it can accept the arriving customers. (3p)

3. A computing system serves computational tasks that arrive according to a Poisson process, with an intensity of one task per minute. The probability density function of the service time of the tasks can be estimated as $b(t) = 0.2\mu_1 e^{-\mu_1 t} + 0.8\mu_2 e^{-\mu_2 t}$, where $\mu_1 = 1\text{min}^{-1}$ and $\mu_2 = 4\text{min}^{-1}$.
a) Calculate the mean and the variance of the service time, and give the Laplace transform of the service time distribution. (3p)
b) Tasks are served by a single processor, and can wait in an infinite buffer. Give the Kendall notation of the system. Calculate the utilization of the processor, and the average time a task spends in the computing system, from arrival until completed service. (2p)

To increase the utilization of the processor without introducing significant additional delays, the administrators decide that in addition to serve all the previously considered tasks even low priority tasks should be computed by the same processor, under preemptive resume policy. These low priority tasks arrive with an intensity of one per five minutes, and require a constant service time of two minutes.
c) Is the queuing system stable? What is the probability that a low priority task is served without interruption? (2p)
d) What is the average time from the start of the service of a low priority task until it is finally completed? (3p)

4. A surveillance system works as follows: every time the camera detects a significant change in the area, it captures an image and attempts to forward this image to a central controller through a low capacity wireless link. The image transmission time is Erlang-2 distributed with a mean of 12 seconds. The camera does not have lots of memory, and therefore can store only one additional image if one image is still under transmission. If an additional image is captured when one is already stored, the new image is dropped. Due to random happenings in the monitored area the camera captures images according to a Poisson process, in average one image per minute.
a) Give the Kendall notation of the system, and the state transition diagram. (2p)
b) Identify the blocking states and calculate the probability that an image needs to be dropped. (3p)
c) Calculate the mean waiting time, considering all the transmitted images. (2p)
d) Since it is more valuable to transmit recent images, the system is changed slightly: if an image is already stored when a new image is generated, the old image is dropped and the new one is stored instead. Do your solutions for (a)-(c) hold for this case as well? Motivate your answer. (3p)

5. Answer the following short questions.
a) Consider an arrival process where the inter-arrival times are independent and uniformly distributed in the range of one to two minutes. You follow the process from a random point of time. Give the expected time until the first arrival you see. (2p)
b) A finite population system has three users who would like to access one server. Each user behaves as follows. The user is idle for an exponentially distributed time with an average of ten minutes, and then tries to access the server. If the server is busy, the user waits in a FIFO queue. The service time is exponentially distributed with an average of five minutes. After completed service the user becomes idle again, and so on. Give the Kendall notation, and the state transition diagram of the system. Calculate the probability that a user accesses the server without waiting. (3p)
c) Consider a queuing system where, after service, half of the requests need to be sent back to the queue for further processing. The processing times are exponentially distributed with a mean of 0.1 sec, and new requests arrive according to a Poisson process. What is the maximum intensity of new arrivals that keeps the queuing system stable? Consider an arrival rate of 2sec^{-1}. How long time do the requests spend in the queuing system in average, and how many times do they visit the server in average? (2p)
d) We would like to model the number of visitors in an exhibition area. We assume that visitors arrive according to a Poisson process with parameter $\lambda = 3\text{min}^{-1}$, and stay for an exponentially distributed time with parameter $\mu = 2\text{hours}^{-1}$. Construct the Markov-chain that describes how the number of visitors evolves in time. Motivate your decisions. Derive the probability that the exhibition area is empty. (3p)