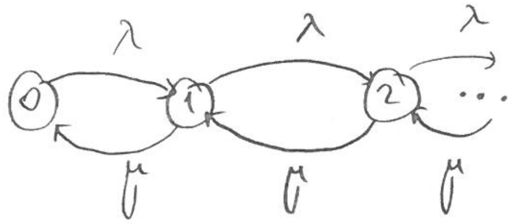
(1.) $\lambda = \frac{1}{4}\left[\frac{\text{visitors}}{\text{min}}\right]$, $\mu = \frac{1}{2}\cdot\left[\frac{1}{\text{min}}\right]$ $\quad \rho = \frac{\lambda}{\mu} = 0.5$

(2p) a.) M / M / 1 / ∞



(2p) b.) P(wait) = P (finding the clerk busy) = $1 - P_0$

M/M/1: $P_k = (1-\rho)\cdot\rho^k$

$P(\text{wait}) = 1 - (1-\rho) = \rho = 0.5$

$$\overline{W} = \frac{\rho}{\mu - \lambda} = \frac{0.5}{0.5 - 0.25} = 2 \text{ min}$$

(2p) c.) P (wait more than 10 min) = $P(W > 10 \text{ min}) = 1 - \overbrace{F_W(10\text{ min})}^{P(W \le 10\text{ min})}$

↑ ccDF ↑ CDF

$= \rho \cdot e^{-(\mu-\lambda)\cdot t} \approx 0.041$

P (total time less than 15 min) = $P(T \le 15 \text{ min}) = F_T(15) =$

↑ CDF

$= 1 - e^{-(\mu-\lambda)\cdot t} \approx 0.976$

(2p) d.) M / M / 1 / 7 (a visitor buying a ticket still takes up a place in the queue)

$$P(\text{blocking}) = P_7 = \frac{(1-\rho)\cdot\rho^7}{1-\rho^{7+1}} = \frac{1}{255} \approx 0.0039$$

$$P(\text{finding 5 visitors in the queue}) = P_5 = \frac{(1-\rho)\cdot\rho^5}{1-\rho^{7+1}} = \frac{4}{255} \approx 0.015$$

Average waiting time of a visitor who finds five visitors in the system = Average service time of five visitors =

$= 5 \cdot \frac{1}{\mu} = 10$ min

(2p) e.) P(no visitor arrives when the queue is full) = P(current service finishes before the next arrival) = P(service time < inter-arrival time)

$$= \int^{\infty} e^{-\lambda t}\cdot \mu \cdot e^{-\mu t}\, dt = \dots = \frac{\mu}{\lambda + \mu}$$

(2.) $\lambda = \frac{1}{10}\left[\frac{\text{customers}}{\text{min}}\right]$ $\mu = \frac{1}{30}\left[\frac{1}{\text{min}}\right]$ $\rho = \frac{\lambda}{\mu} = 3$, $m = 5$

2p) a.) Erlang wait system M/M/5

$\bar{N} = \rho + \frac{\lambda}{m\mu - \lambda} \cdot P(\text{wait}) = 3.354$

$P(\text{wait}) = D_m(\rho) = \frac{m \cdot E_m(\rho)}{m - \rho \cdot (1 - E_m(\rho))}$

$E_m(\rho) = E_5(3) \approx 0.110054$

$D_m(\rho) = \frac{5 \cdot 0.110054}{5 - 3 \cdot (1 - 0.110054)} \approx 0.236$

Percentage of time a computer is busy = utilization $= \frac{\lambda}{m\mu} = \frac{\rho}{m} = 0.6$

3p) b.) P(customer does not have to wait) $= 1 - P(\text{wait}) = 1 - D_m(\rho) \approx 0.764$

Expected waiting time of a customer who finds all computers busy, but the queue is empty?

She will wait for the first computer to be free

$X = \min(X_1 + X_2 + X_3 + X_4 + X_5)$ $X_i \sim Exp(\mu) \Rightarrow X \sim Exp(5\mu)$

Expected waiting time $= \frac{1}{5\mu} = 6$ min

P(She has to wait more than 10 min) = P(in 10 min 0 customers served)

$= \frac{(5\mu \cdot t)^0}{0!} \cdot e^{-5\mu t} = e^{-\frac{5}{3}} \approx 0.189$

2p) c.) M/M/5/5

$P(\text{blocking}) = P_5 = E_m(\rho) = E_5(3) \approx 0.110054$

Percentage of time a computer is busy = utilization $= \frac{\lambda_{eff}}{m\mu} =$

$= \frac{\lambda \cdot (1 - P(\text{blocking}))}{m\mu} \approx 0.534$

3p) d.) $T_{\text{blocking}} \sim Exp(5\mu) \Rightarrow T_{\text{blocking}} = \frac{1}{5\mu} = 6$ min

$P(\text{blocking}) = \frac{T_{\text{blocking}}}{T_{\text{blocking}} + T_{\text{non-blocking}}} \Rightarrow$

$T_{\text{non-blocking}} = \frac{T_{\text{blocking}}}{P(\text{blocking})} - T_{\text{blocking}} \approx 48.52$ min

(3) $\lambda = 1$ tar/min

$b(t) = 0.2\mu_1 e^{-\mu_1 t} + 0.8\mu_2 e^{-\mu_2 t}$

$\mu_1 = 1\ \text{min}^{-1}$ $\quad E[x_1] = 1$ min $\quad E[x_1^2] = \frac{2}{\mu^2} = 2\ \text{min}^2$

$\mu_2 = 4\ \text{min}^{-1}$ $\quad E[x_2] = \frac{1}{4}$ min $\quad E[x_2^2] = \frac{1}{8}\ \text{min}^2$

a) b(t) is the linear combination of two Exp distributions.

$E[x] = 0.2\,E[x_1] + 0.8\,E[x_2] = \frac{1}{5} + \frac{4}{5}\cdot\frac{1}{4} = \frac{2}{5}$ min

$E[x^2] = 0.2\,E[x_1^2] + 0.8\,E[x_2^2] = \frac{1}{5}\cdot 2 + \frac{4}{5}\cdot\frac{1}{8_2} = \frac{5}{10} = \frac{1}{2}\ \text{min}^2$ (3)

$V[x] = E[x^2] - E[x]^2 = \frac{1}{2} - \frac{4}{25} = \frac{17}{50}$

$B(s) = \frac{1}{5}\cdot\frac{1}{s+1} + \frac{4}{5}\cdot\frac{4}{s+4}$

b) M / H2 / 1 (or M/G/1)

$\rho = \lambda E[x] = \frac{2}{5}$ (2)

$T = E[x] + \frac{\lambda E[x^2]}{2(1-\rho)} = \frac{49}{60} \approx 0.81$

c) Preemptive resume

$\lambda_L = \frac{1}{5}$ tar/min

$x_L = 2$ min (deterministic)

$\Big\}\ \rho_L = \lambda_L x = \frac{2}{5}$

$\rho = \rho_h + \rho_L = \frac{2}{5} + \frac{2}{5} = \frac{4}{5}$ $\Rightarrow$ The system is stable.

P(low priority serves without interruption) = P(no high prio arrival in 2 min) = (2)

P(no arrival with $\lambda = 1\ \text{min}^{-1}$ in $t = 2$ min) $= \frac{(\lambda t)^0}{0!} e^{-\lambda t} = e^{-2}$

d) $E[x_L'] = E[x_L] + E[x_L']\underbrace{\lambda_h \cdot E[x_h]}_{\rho_h}$

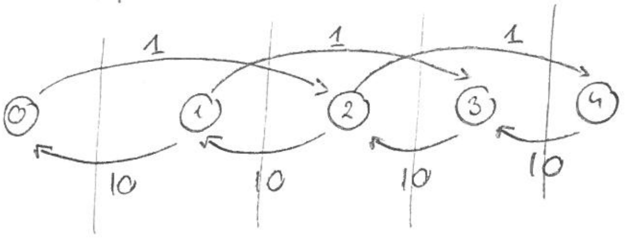
$E[x_L'] = \frac{E[x_L]}{1-\rho_h} = \frac{5}{3}\cdot 2 = \frac{10}{3}$ min. (3)

④

$X \sim$ Erlang-2 $\quad E[X] = 12s = \frac{1}{5}$ min $\quad \mu = 5 \text{ min}^{-1}$

$\lambda = 1$ image/min

a) $M / E_2 / 1 / 2$



(2)

b)
$$\left.\begin{aligned} p_0 &= 10 p_1 \\ p_0 + p_1 &= 10 p_2 \\ p_1 + p_2 &= 10 p_3 \\ p_2 &= 10 p_4 \\ \sum_i p_i &= 1 \end{aligned}\right\} \quad \left\{ \frac{500}{621}, \frac{50}{621}, \frac{55}{621}, \frac{105}{6210}, \frac{55}{6210} \right\}$$

(3)

Blocking states: p_3, p_4

$$P(\text{blocking}) = p_3 + p_4 = \frac{160}{6210} = \frac{16}{621}$$

c) $$W = \left(0 \cdot p_0 + \frac{1}{2\mu} p_1 + \frac{1}{\mu} p_2\right) \cdot \frac{1}{p_0 + p_1 + p_2} = \ldots = \frac{16}{605} \text{ min}$$ (2)

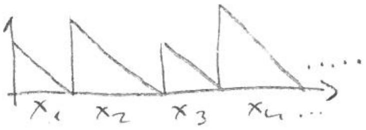
d) (a): Solution will not change, the state transition diagram does not show which image is dropped. (3)

(b): The state transition diagram, and thus the state probabilities will not change, neither the prob. of ~~past~~ image drops.

(c): This solution will change, the average waiting time of transmitted images will decrease, since the dropped images "take over" some of the waiting times.

(5)

a)



I will see the average remaining inter-arrival time.

$X_i \sim U[1,2]$ [min]

$\underline{E[R_x]} = \frac{1}{\sum x_i} \sum_{i=1}^{n} x_i^2/2 = \frac{E[x^2]}{2E[x]} =$

$= \frac{7}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \underline{\underline{\frac{7}{9}}}$ min

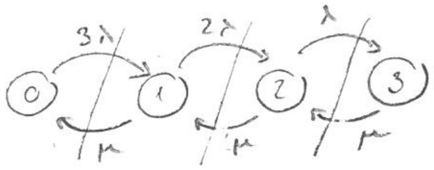
$E[x] = \frac{2+1}{2} = \frac{3}{2}$

$E(x^2) = \int_1^2 x^2\,dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$ (2)

b) M/M/1/3/3

$\lambda = \frac{1}{10}$ min^{-1}

$\mu = \frac{1}{5}$ min^{-1}



$p_0 3\lambda = p_1 \mu$

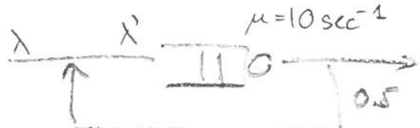
$p_1 2\lambda = p_2 2\mu$

$p_2 \lambda = p_3 2\mu$

$\Rightarrow \{p_0, p_1, p_2, p_3\} = \left\{\frac{4}{19}, \frac{6}{19}, \frac{6}{19}, \frac{3}{19}\right\}$ (3)

$P(\text{no waiting}) = a_0 = \frac{3\lambda p_0}{3\lambda p_0 + 2\lambda p_1 + \lambda p_2} = \underline{\underline{\frac{2}{5}}}$

c)



$\lambda' = \lambda + 0.5\lambda'$

$\lambda' = \frac{\lambda}{0.5} = 2\lambda$

$\rho = \lambda'/\mu < 1$

$\underline{\underline{\lambda_m < \frac{\mu}{2} = 5\ \text{sec}^{-1}}}$

If $\lambda = 2$ sec^{-1}

$\lambda' = 4$ sec^{-1}

$\rho = \frac{\lambda'}{\mu} = \frac{4}{10}$

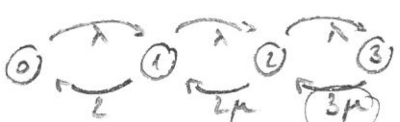
$N = \frac{\rho}{1-\rho} = \frac{2}{3}$ (2)

$T = \frac{N}{\lambda} = \underline{\underline{\frac{2}{6}\ \text{sec}}}$, $\underline{\underline{V = \frac{\lambda'}{\lambda} = 2}}$

d) $\lambda = 3$ min^{-1}

$\mu = 2$ hours^{-1} $= \frac{1}{30}$ min^{-1}

This is an ∞ server system (M/M/∞)



← any of the visitors can leave.

$\underline{\rho = \frac{\lambda}{\mu} = 90}$

$p_k = \frac{\rho^k}{k!} p_0 \Rightarrow p_0 = \frac{1}{\sum_{k=0}^{\infty} \frac{\rho^k}{k!}} = \frac{1}{e^{\rho}} = e^{-\rho}$ (3)

$\underline{\underline{P_{empty} = p_0 = e^{-\rho}}}$